AnaFlow Documentation

Release 1.0.1

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CONTENTS

CHAPTER 1

ANAFLOW QUICKSTART

AnaFlow provides several analytical and semi-analytical solutions for the groundwater-flow equation.

1.1 Installation

The package can be installed via pip. On Windows you can install WinPython to get Python and pip running.

```
pip install anaflow
```

1.2 Provided Functions

The following functions are provided directly

- `thiem` Thiem solution for steady state pumping
- `theis` Theis solution for transient pumping
- `ext_thiem_2d` extended Thiem solution in 2D from *Zech 2013*
- `ext_theis_2d` extended Theis solution in 2D from *Mueller 2015*
- `ext_thiem_3d` extended Thiem solution in 3D from *Zech 2013*
- `ext_theis_3d` extended Theis solution in 3D from *Mueller 2015*
- `neuman2004` transient solution from *Neuman 2004*
- `neuman2004_steady` steady solution from *Neuman 2004*
- `grf` "General Radial Flow" Model from *Barker 1988*

- *ext_grf* the transient extended GRF model
- *ext_grf_steady* the steady extended GRF model
- *ext_thiem_tpl* extended Thiem solution for truncated power laws
- *ext_theis_tpl* extended Theis solution for truncated power laws
- *ext_thiem_tpl_3d* extended Thiem solution in 3D for truncated power laws
- *ext_theis_tpl_3d* extended Theis solution in 3D for truncated power laws

1.3 Laplace Transformation

We provide routines to calculate the laplace-transformation as well as the inverse laplace-transformation of a given function

- *get_lap* Get the laplace transformation of a function
- *get_lap_inv* Get the inverse laplace transformation of a function

1.4 Requirements

- NumPy >= 1.14.5
- SciPy >= 1.1.0
- pentapy

1.5 License

MIT

CHAPTER 2

ANAFLOW TUTORIAL

In the following you will find several Tutorials on how to use AnaFlow to explore its whole beauty and power.

2.1 Tutorial 1: The Theis solution

In the following the well known Theis function is called an plotted for three different time-steps.

Reference: Theis 1935

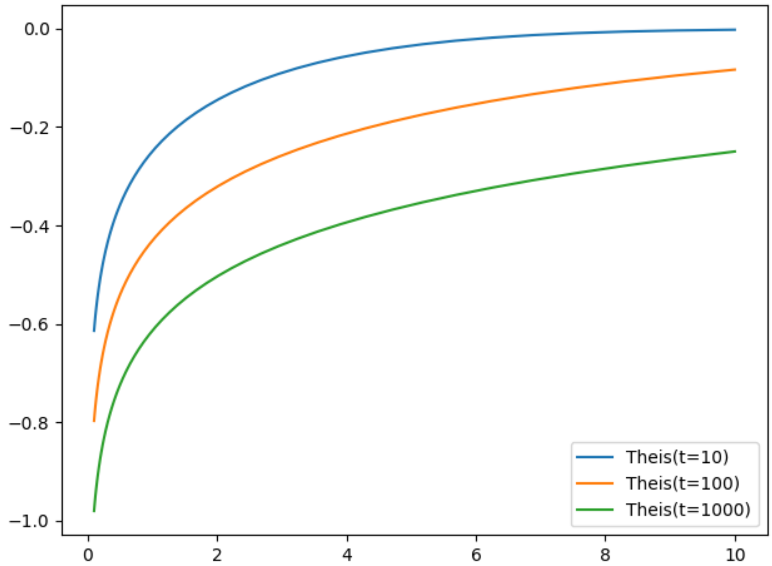
```
import numpy as np
from matplotlib import pyplot as plt
from anaflow import theis


time = [10, 100, 1000]
rad = np.geomspace(0.1, 10)

head = theis(time=time, rad=rad, storage=1e-4, transmissivity=1e-4, rate=-1e-4)

for i, step in enumerate(time):
    plt.plot(rad, head[i], label="Theis(t={})".format(step))

plt.legend()
plt.tight_layout()
plt.show()
```



2.2 Tutorial 2: The extended Theis solution in 2D

We provide an extended theis solution, that incorporates the effectes of a heterogeneous transmissivity field on a pumping test.

In the following this extended solution is compared to the standard theis solution for well flow. You can nicely see, that the extended solution represents a transition between the theis solutions for the geometric- and harmonic-mean transmissivity.

Reference: Zech et. al. 2016

```
import numpy as np
from matplotlib import pyplot as plt
from anaflow import theis, ext_theis_2d


time_labels = ["10 s", "10 min", "10 h"]
time = [10, 600, 36000]        # 10s, 10min, 10h
rad = np.geomspace(0.05, 4)    # radius from the pumping well in [0, 4]
var = 0.5                      # variance of the log-transmissivity
len_scale = 10.0               # correlation length of the log-transmissivity
TG = 1e-4                      # the geometric mean of the transmissivity
TH = TG * np.exp(-var / 2.0)   # the harmonic mean of the transmissivity
S = 1e-4                       # storativity
rate = -1e-4                   # pumping rate

head_TG = theis(time, rad, S, TG, rate)
head_TH = theis(time, rad, S, TH, rate)
head_ef = ext_theis_2d(time, rad, S, TG, var, len_scale, rate)
time_ticks=[]
for i, step in enumerate(time):
    label_TG = "Theis($T_G$)" if i == 0 else None
    label_TH = "Theis($T_H$)" if i == 0 else None
    label_ef = "extended Theis" if i == 0 else None
    plt.plot(rad, head_TG[i], label=label_TG, color="C"+str(i), linestyle="--")
    plt.plot(rad, head_TH[i], label=label_TH, color="C"+str(i), linestyle=":")
    plt.plot(rad, head_ef[i], label=label_ef, color="C"+str(i))
    time_ticks.append(head_ef[i][-1])
```

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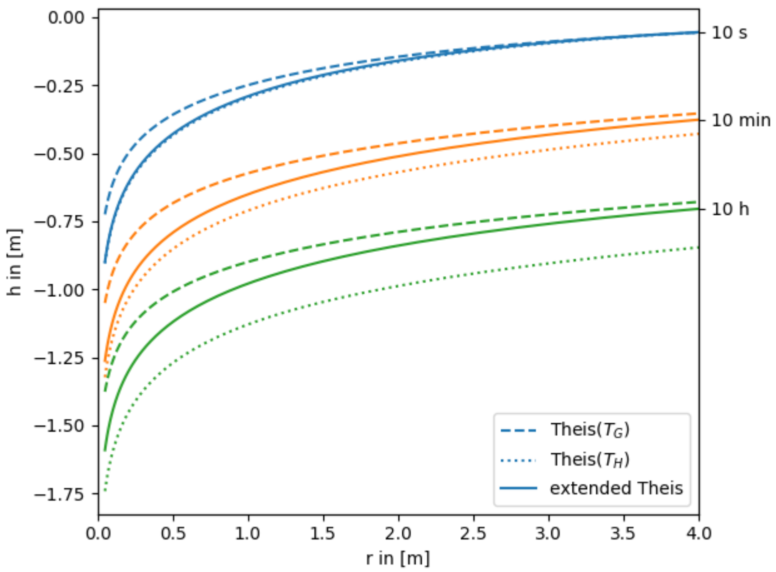
```
plt.xlabel("r in [m]")
plt.ylabel("h in [m]")
plt.legend()
ylim = plt.gca().get_ylim()
plt.gca().set_xlim([0, rad[-1]])
ax2 = plt.gca().twinx()
ax2.set_yticks(time_ticks)
ax2.set_yticklabels(time_labels)
ax2.set_ylim(ylim)
plt.tight_layout()
plt.show()
```



2.3 Tutorial 3: The extended Theis solution in 3D

We provide an extended theis solution, that incorporates the effectes of a heterogeneous conductivity field on a pumping test. It also includes an anisotropy ratio of the horizontal and vertical length scales.

In the following this extended solution is compared to the standard theis solution for well flow. You can nicely see, that the extended solution represents a transition between the theis solutions for the effective and harmonic-mean conductivity.

Reference: Müller 2015

```
import numpy as np
from matplotlib import pyplot as plt
from anaflow import theis, ext_theis_3d
from anaflow.tools.special import aniso


time_labels = ["10 s", "10 min", "10 h"]
time = [10, 600, 36000]                    # 10s, 10min, 10h
rad = np.geomspace(0.05, 4)                # radial distance from the pumping well␣
↪in [0, 4]
var = 0.5                                  # variance of the log-conductivity
len_scale = 10.0                           # correlation length of the log-
↪conductivity
anis = 0.75                                # anisotropy ratio of the log-conductivity
KG = 1e-4                                  # the geometric mean of the conductivity
```

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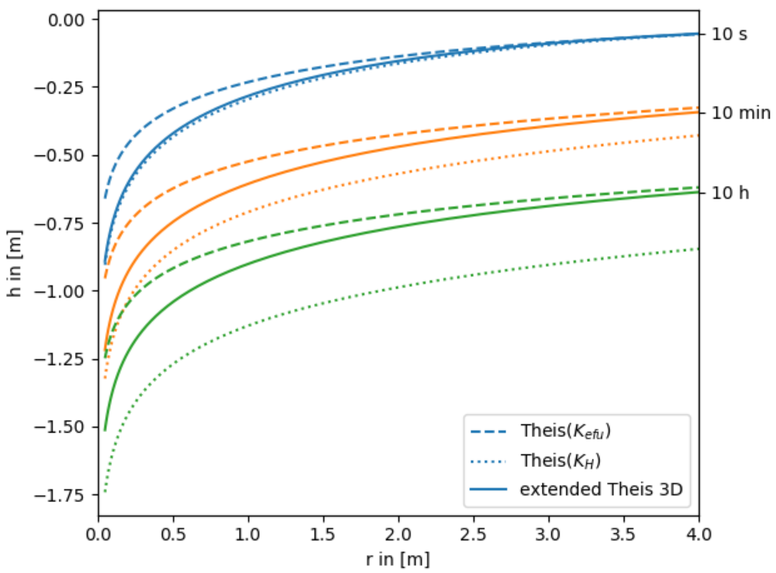
```
Kefu = KG*np.exp(var*(0.5-aniso(anis)))  # the effective conductivity for uniform␣
↪flow
KH = KG*np.exp(-var/2.0)                 # the harmonic mean of the conductivity
S = 1e-4                                 # storage
rate = -1e-4                             # pumping rate
L = 1.0                                  # vertical extend of the aquifer

head_Kefu = theis(time, rad, S, Kefu*L, rate)
head_KH = theis(time, rad, S, KH*L, rate)
head_ef = ext_theis_3d(time, rad, S, KG, var, len_scale, anis, L, rate)
time_ticks=[]
for i, step in enumerate(time):
    label_TG = "Theis($K_{efu}$)" if i == 0 else None
    label_TH = "Theis($K_H$)" if i == 0 else None
    label_ef = "extended Theis 3D" if i == 0 else None
    plt.plot(rad, head_Kefu[i], label=label_TG, color="C"+str(i), linestyle="--")
    plt.plot(rad, head_KH[i], label=label_TH, color="C"+str(i), linestyle=":")
    plt.plot(rad, head_ef[i], label=label_ef, color="C"+str(i))
    time_ticks.append(head_ef[i][-1])

plt.xlabel("r in [m]")
plt.ylabel("h in [m]")
plt.legend()
ylim = plt.gca().get_ylim()
plt.gca().set_xlim([0, rad[-1]])
ax2 = plt.gca().twinx()
ax2.set_yticks(time_ticks)
ax2.set_yticklabels(time_labels)
ax2.set_ylim(ylim)
plt.tight_layout()
plt.show()
```



2.4 Tutorial 4: The extended Theis solution for truncated power laws

We provide an extended theis solution, that incorporates the effectes of a heterogeneous conductivity field following a truncated power law. In addition, it incorporates the assumptions of the general radial flow model and provides an arbitrary flow dimension.

In the following this extended solution is compared to the standard theis solution for well flow. You can nicely see, that the extended solution represents a transition between the theis solutions for the well- and farfield-conductivity.

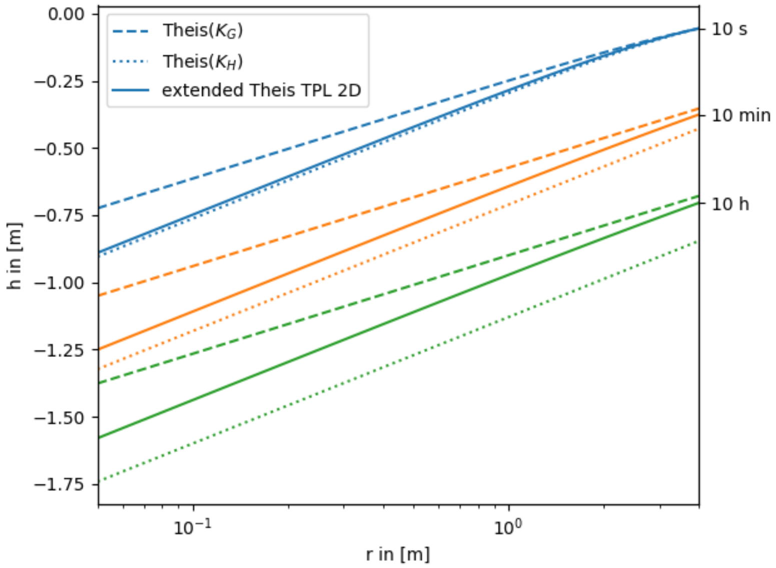
Reference: (not yet published)

```
import numpy as np
from matplotlib import pyplot as plt
from anaflow import theis, ext_theis_tpl


time_ticks = []
time_labels = ["10 s", "10 min", "10 h"]
time = [10, 600, 36000]       # 10s, 10min, 10h
rad = np.geomspace(0.05, 4)  # radial distance from the pumping well in [0, 4]
S = 1e-4                      # storage
KG = 1e-4                     # the geometric mean of the conductivity
len_scale = 20.0              # upper bound for the length scale
hurst = 0.5                   # hurst coefficient
var = 0.5                     # variance of the log-conductivity
rate = -1e-4                  # pumping rate
KH = KG * np.exp(-var / 2)    # the harmonic mean of the conductivity

head_KG = theis(time, rad, S, KG, rate)
head_KH = theis(time, rad, S, KH, rate)
head_ef = ext_theis_tpl(
    time=time,
    rad=rad,
    storage=S,
    cond_gmean=KG,
    len_scale=len_scale,
    hurst=hurst,
    var=var,
    rate=rate,
)
for i, step in enumerate(time):
    label_TG = "Theis($K_G$)" if i == 0 else None
    label_TH = "Theis($K_H$)" if i == 0 else None
    label_ef = "extended Theis TPL 2D" if i == 0 else None
    plt.plot(rad, head_KG[i], label=label_TG, color="C"+str(i), linestyle="--")
    plt.plot(rad, head_KH[i], label=label_TH, color="C"+str(i), linestyle=":")
    plt.plot(rad, head_ef[i], label=label_ef, color="C"+str(i))
    time_ticks.append(head_ef[i][-1])

plt.xscale("log")
plt.xlabel("r in [m]")
plt.ylabel("h in [m]")
plt.legend()
ylim = plt.gca().get_ylim()
plt.gca().set_xlim([rad[0], rad[-1]])
ax2 = plt.gca().twinx()
ax2.set_yticks(time_ticks)
ax2.set_yticklabels(time_labels)
ax2.set_ylim(ylim)
plt.tight_layout()
plt.show()
```



2.5 Tutorial 5: The transient heterogeneous Neuman solution from 2004

We provide the transient pumping solution for the apparent transmissivity from Neuman 2004. This solution is build on the apparent transmissivity from Neuman 2004, which represents a mean drawdown in an ensemble of pumping tests in heterogeneous transmissivity fields following an exponential covariance.

In the following this solution is compared to the standard theis solution for well flow. You can nicely see, that the extended solution represents a transition between the theis solutions for the well- and farfield-conductivity.

Reference: Neuman 2004

```
import numpy as np
from matplotlib import pyplot as plt
from anaflow import theis, neuman2004


time_labels = ["10 s", "10 min", "10 h"]
time = [10, 600, 36000]       # 10s, 10min, 10h
rad = np.geomspace(0.05, 4)   # radius from the pumping well in [0, 4]
var = 0.5                     # variance of the log-transmissivity
len_scale = 10.0              # correlation length of the log-transmissivity
TG = 1e-4                     # the geometric mean of the transmissivity
TH = TG*np.exp(-var/2.0)      # the harmonic mean of the transmissivity
S = 1e-4                      # storativity
rate = -1e-4                    # pumping rate

head_TG = theis(time, rad, S, TG, rate)
head_TH = theis(time, rad, S, TH, rate)
head_ef = neuman2004(
    time=time,
    rad=rad,
    trans_gmean=TG,
    var=var,
    len_scale=len_scale,
    storage=S,
    rate=rate,
)
time_ticks = []
```

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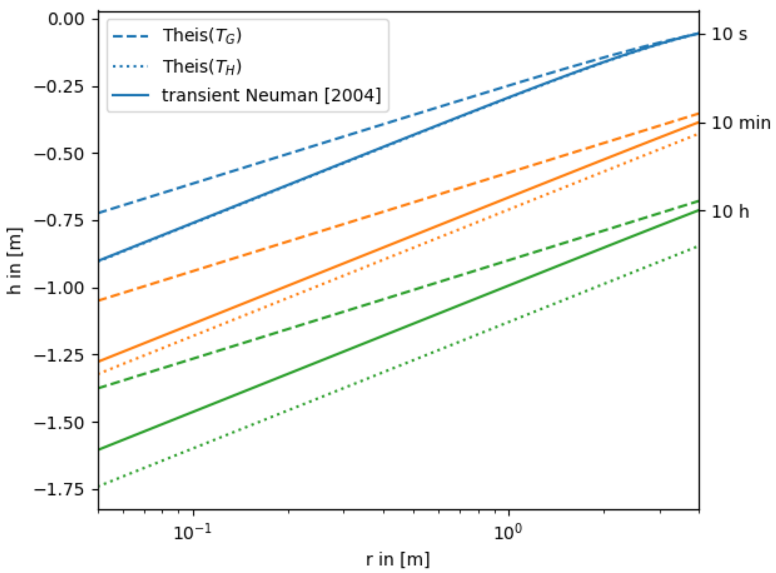
```
for i, step in enumerate(time):
    label_TG = "Theis($T_G$)" if i == 0 else None
    label_TH = "Theis($T_H$)" if i == 0 else None
    label_ef = "transient Neuman [2004]" if i == 0 else None
    plt.plot(rad, head_TG[i], label=label_TG, color="C"+str(i), linestyle="--")
    plt.plot(rad, head_TH[i], label=label_TH, color="C"+str(i), linestyle=":")
    plt.plot(rad, head_ef[i], label=label_ef, color="C"+str(i))
    time_ticks.append(head_ef[i][-1])

plt.xscale("log")
plt.xlabel("r in [m]")
plt.ylabel("h in [m]")
plt.legend()
ylim = plt.gca().get_ylim()
plt.gca().set_xlim([rad[0], rad[-1]])
ax2 = plt.gca().twinx()
ax2.set_yticks(time_ticks)
ax2.set_yticklabels(time_labels)
ax2.set_ylim(ylim)
plt.tight_layout()
plt.show()
```



2.6 Tutorial 6: Comparison of different solutions

In the following we compare a set of different solutions of the groundwater flow equation.

1. extended Thiem 2D vs. steady solution for coarse graining transmissivity

The extended Thiem 2D solutions is the analytical solution of the groundwater flow equation for the coarse graining transmissivity for pumping tests. Therefore the results should coincide.

References:

- Schneider & Attinger 2008
- Zech & Attinger 2016

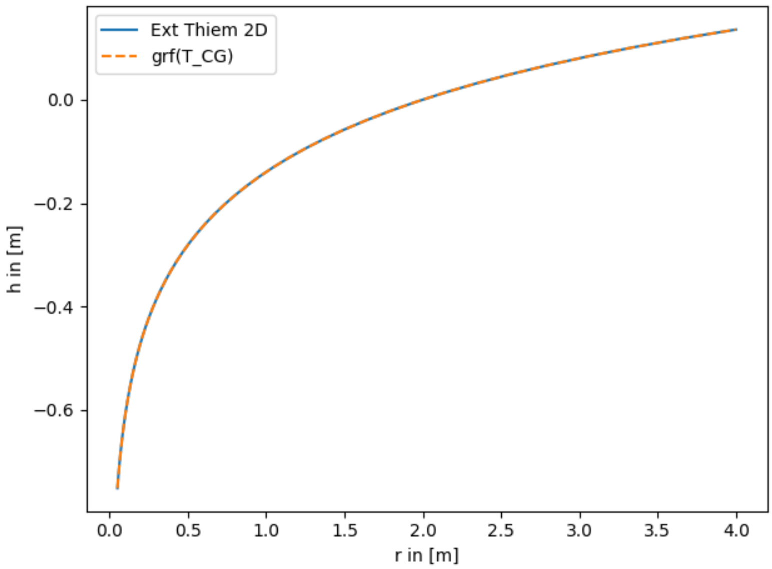
```
import numpy as np
from matplotlib import pyplot as plt
from anaflow import ext_thiem_2d, ext_grf_steady
from anaflow.tools.coarse_graining import T_CG


rad = np.geomspace(0.05, 4)  # radius from the pumping well in [0, 4]
r_ref = 2.0                  # reference radius
var = 0.5                    # variance of the log-transmissivity
len_scale = 10.0             # correlation length of the log-transmissivity
TG = 1e-4                    # the geometric mean of the transmissivity
rate = -1e-4                 # pumping rate

head1 = ext_thiem_2d(rad, r_ref, TG, var, len_scale, rate)
head2 = ext_grf_steady(rad, r_ref, T_CG, rate=rate, trans_gmean=TG, var=var, len_
↪scale=len_scale)

plt.plot(rad, head1, label="Ext Thiem 2D")
plt.plot(rad, head2, label="grf(T_CG)", linestyle="--")

plt.xlabel("r in [m]")
plt.ylabel("h in [m]")
plt.legend()
plt.tight_layout()
plt.show()
```



2. extended Thiem 3D vs. steady solution for coarse graining conductivity

The extended Thiem 3D solutions is the analytical solution of the groundwater flow equation for the coarse graining conductivity for pumping tests. Therefore the results should coincide.

Reference: Zech et. al. 2012

```
import numpy as np
from matplotlib import pyplot as plt
from anaflow import ext_thiem_3d, ext_grf_steady
from anaflow.tools.coarse_graining import K_CG
```

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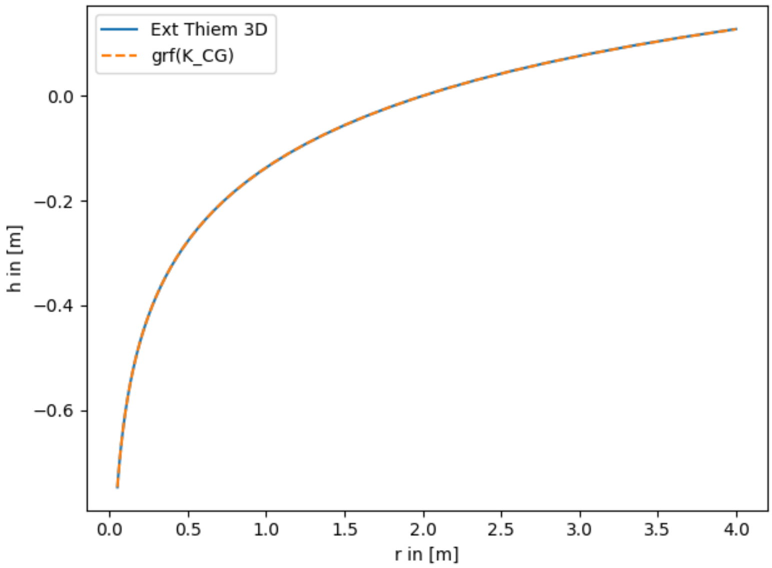
```
rad = np.geomspace(0.05, 4)  # radius from the pumping well in [0, 4]
r_ref = 2.0                  # reference radius
var = 0.5                    # variance of the log-transmissivity
len_scale = 10.0             # correlation length of the log-transmissivity
KG = 1e-4                    # the geometric mean of the transmissivity
anis = 0.7                   # aniso ratio
rate = -1e-4                 # pumping rate

head1 = ext_thiem_3d(rad, r_ref, KG, var, len_scale, anis, 1, rate)
head2 = ext_grf_steady(rad, r_ref, K_CG, rate=rate, cond_gmean=KG, var=var, len_
↪scale=len_scale, anis=anis)

plt.plot(rad, head1, label="Ext Thiem 3D")
plt.plot(rad, head2, label="grf(K_CG)", linestyle="--")

plt.xlabel("r in [m]")
plt.ylabel("h in [m]")
plt.legend()
plt.tight_layout()
plt.show()
```



3. extended Thiem 2D vs. steady solution for apparent transmissivity from Neuman

Both, the extended Thiem and the Neuman solution, represent an effective steady drawdown in a heterogeneous aquifer. In both cases the heterogeneity is represented by two point statistics, characterized by mean, variance and length scale of the log transmissivity field. Therefore these approaches should lead to similar results.

References:

- Neuman 2004
- Zech & Attinger 2016

```
import numpy as np
from matplotlib import pyplot as plt
from anaflow import ext_thiem_2d, neuman2004_steady
```

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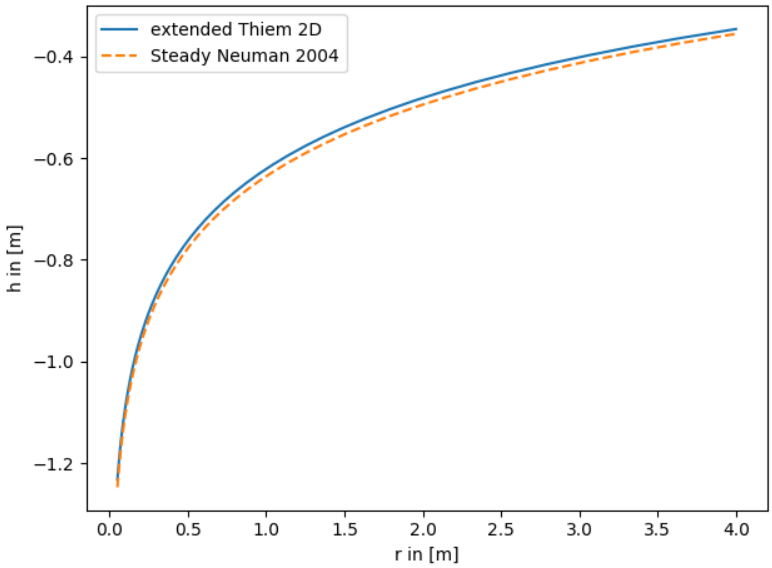
```
rad = np.geomspace(0.05, 4)  # radius from the pumping well in [0, 4]
r_ref = 30.0                 # reference radius
var = 0.5                    # variance of the log-transmissivity
len_scale = 10.0             # correlation length of the log-transmissivity
TG = 1e-4                    # the geometric mean of the transmissivity
rate = -1e-4                 # pumping rate

head1 = ext_thiem_2d(rad, r_ref, TG, var, len_scale, rate)
head2 = neuman2004_steady(rad, r_ref, TG, var, len_scale, rate)

plt.plot(rad, head1, label="extended Thiem 2D")
plt.plot(rad, head2, label="Steady Neuman 2004", linestyle="--")

plt.xlabel("r in [m]")
plt.ylabel("h in [m]")
plt.legend()
plt.tight_layout()
plt.show()
```



4. extended Theis 2D vs. transient solution for apparent transmissivity from Neuman

Both, the extended Theis and the Neuman solution, represent an effective transient drawdown in a heterogeneous aquifer. In both cases the heterogeneity is represented by two point statistics, characterized by mean, variance and length scale of the log transmissivity field. Therefore these approaches should lead to similar results.

References:

- Neuman 2004
- Zech et. al. 2016

```
import numpy as np
from matplotlib import pyplot as plt
from anaflow import ext_theis_2d, neuman2004


time_labels = ["10 s", "10 min", "10 h"]
time = [10, 600, 36000]       # 10s, 10min, 10h
```

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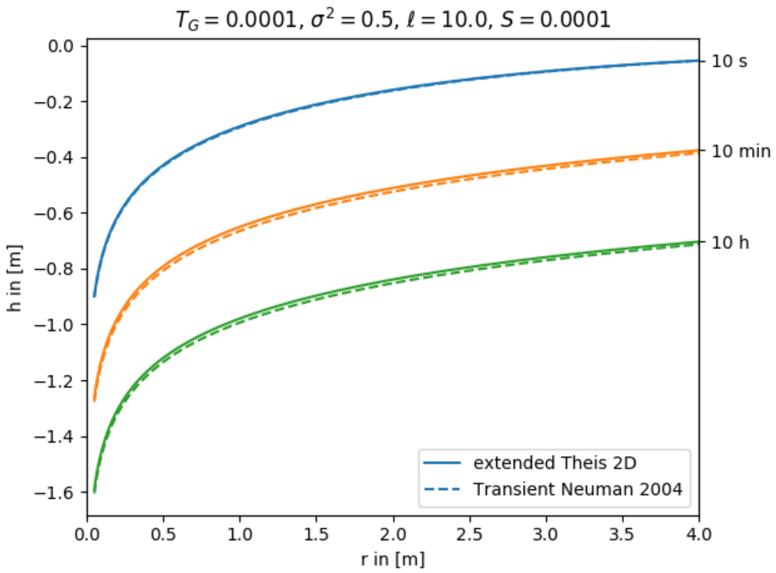
```
rad = np.geomspace(0.05, 4)  # radius from the pumping well in [0, 4]
TG = 1e-4                    # the geometric mean of the transmissivity
var = 0.5                    # correlation length of the log-transmissivity
len_scale = 10.0             # variance of the log-transmissivity
S = 1e-4                     # storativity
rate = -1e-4                 # pumping rate

head1 = ext_theis_2d(time, rad, S, TG, var, len_scale, rate)
head2 = neuman2004(time, rad, S, TG, var, len_scale, rate)
time_ticks=[]
for i, step in enumerate(time):
    label1 = "extended Theis 2D" if i == 0 else None
    label2 = "Transient Neuman 2004" if i == 0 else None
    plt.plot(rad, head1[i], label=label1, color="C"+str(i))
    plt.plot(rad, head2[i], label=label2, color="C"+str(i), linestyle="--")
    time_ticks.append(head1[i][-1])

plt.title("$T_G={}$, $\sigma^2={}$, $\ell={}$, $S={}$".format(TG, var, len_scale,␣
↪S))
plt.xlabel("r in [m]")
plt.ylabel("h in [m]")
plt.legend()
ylim = plt.gca().get_ylim()
plt.gca().set_xlim([0, rad[-1]])
ax2 = plt.gca().twinx()
ax2.set_yticks(time_ticks)
ax2.set_yticklabels(time_labels)
ax2.set_ylim(ylim)
plt.tight_layout()
plt.show()
```



2.7 Tutorial 7: Convergence of different solutions

In the following we analyze the convergence of some solutions of the groundwater flow equation.

1. Convergence of the extended Theis solutions for truncated power laws

Here we set an outer boundary to the transient solution, so this condition coincides with the references head of the steady solution.

Reference: (not yet published)

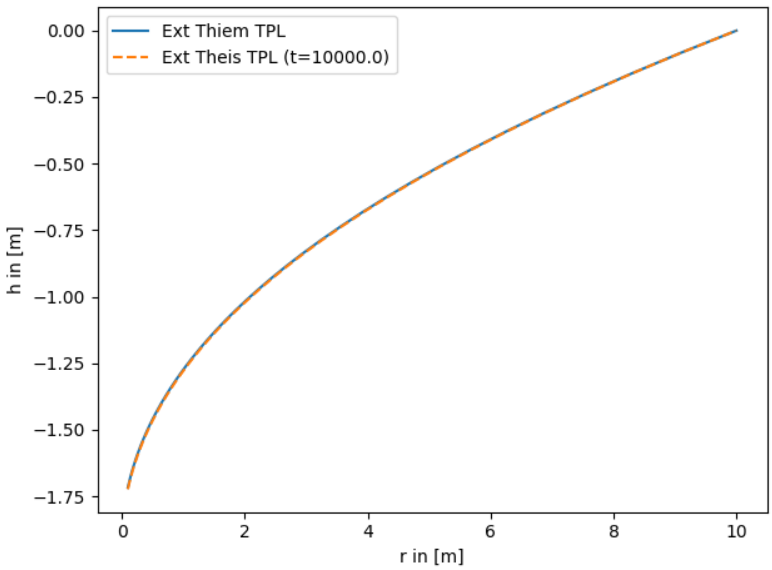
```
import numpy as np
from matplotlib import pyplot as plt
from anaflow import ext_theis_tpl, ext_thiem_tpl

time = 1e4                    # time point for steady state
rad = np.geomspace(0.1, 10)  # radius from the pumping well in [0, 4]
r_ref = 10.0                  # reference radius
KG = 1e-4                     # the geometric mean of the transmissivity
len_scale = 5.0               # correlation length of the log-transmissivity
hurst = 0.5                   # hurst coefficient
var = 0.5                     # variance of the log-transmissivity
rate = -1e-4                  # pumping rate
dim = 1.5                     # using a fractional dimension

head1 = ext_thiem_tpl(rad, r_ref, KG, len_scale, hurst, var, dim=dim, rate=rate)
head2 = ext_theis_tpl(time, rad, 1e-4, KG, len_scale, hurst, var, dim=dim,␣
↪rate=rate, r_bound=r_ref)

plt.plot(rad, head1, label="Ext Thiem TPL")
plt.plot(rad, head2, label="Ext Theis TPL (t={})".format(time), linestyle="--")

plt.xlabel("r in [m]")
plt.ylabel("h in [m]")
plt.legend()
plt.tight_layout()
plt.show()
```



2. Convergence of the general radial flow model (GRF)

The GRF model introduces an arbitrary flow dimension and was presented to analyze groundwater flow in rock formations. In the following we compare the bounded transient solution for late times, the unbounded quasi steady solution and the steady state.

Reference: Barker 1988

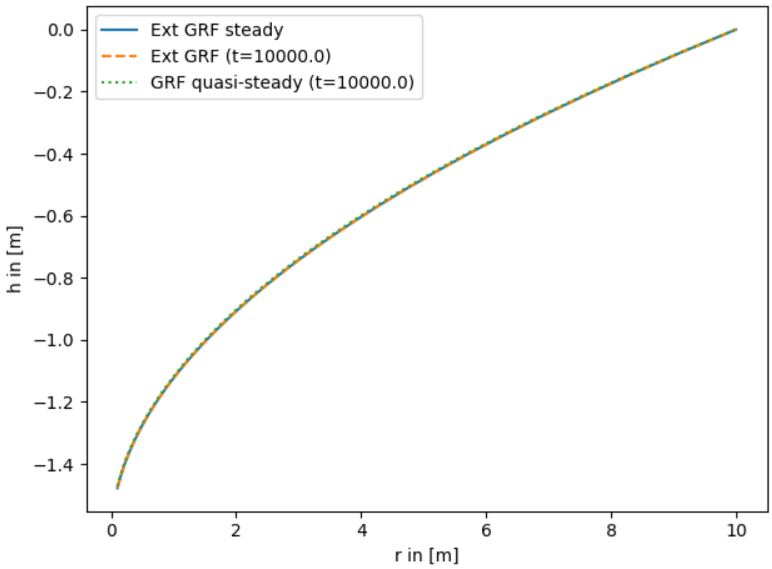
```
import numpy as np
from matplotlib import pyplot as plt
from anaflow import ext_grf, ext_grf_steady, grf

time = 1e4                    # time point for steady state
rad = np.geomspace(0.1, 10)   # radius from the pumping well in [0, 4]
r_ref = 10.0                  # reference radius
K = 1e-4                      # the geometric mean of the transmissivity
rate = -1e-4                  # pumping rate
dim = 1.5                     # using a fractional dimension

head1 = ext_grf_steady(rad, r_ref, K, dim=dim, rate=rate)
head2 = ext_grf(time, rad, [1e-4], [K], [0, r_ref], dim=dim, rate=rate)
head3 = grf(time, rad, 1e-4, K, dim=dim, rate=rate)
head3 -= head3[-1]  # quasi-steady

plt.plot(rad, head1, label="Ext GRF steady")
plt.plot(rad, head2, label="Ext GRF (t={})".format(time), linestyle="--")
plt.plot(rad, head3, label="GRF quasi-steady (t={})".format(time), linestyle=":")

plt.xlabel("r in [m]")
plt.ylabel("h in [m]")
plt.legend()
plt.tight_layout()
plt.show()
```



3. Quasi steady Theis vs. Thiem

Since a lot of pumping test analysis is done by interpreting the so called quasi steady state, we will compare the quasi steady state of theis, a late time head of the bounded theis and the thiem solution.

References:

- Theis 1935
- Thiem 1906

```
import numpy as np
from matplotlib import pyplot as plt
from anaflow import theis, thiem
```

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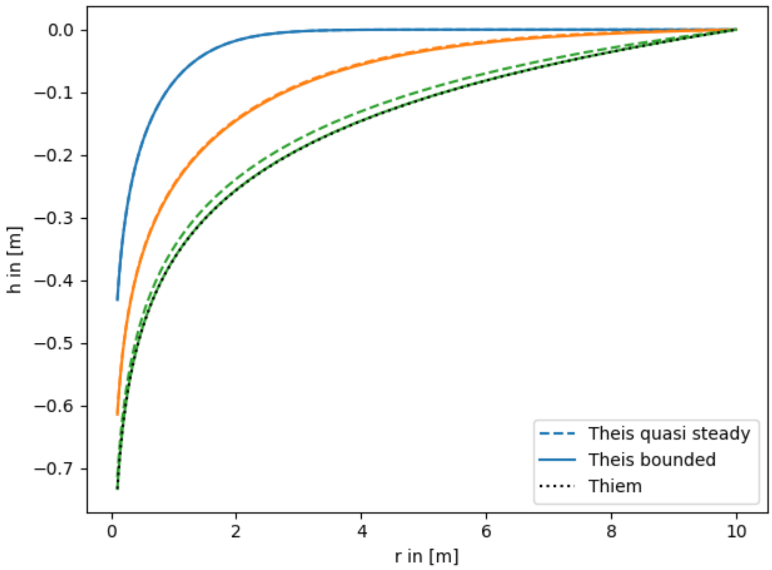
```
time = [10, 100, 1000]
rad = np.geomspace(0.1, 10)
r_ref = 10.0

head_ref = theis(time, np.full_like(rad, r_ref), storage=1e-3, transmissivity=1e-4,
↪ rate=-1e-4)
head1 = theis(time, rad, storage=1e-3, transmissivity=1e-4, rate=-1e-4) - head_ref
head2 = theis(time, rad, storage=1e-3, transmissivity=1e-4, rate=-1e-4, r_bound=r_
↪ref)
head3 = thiem(rad, r_ref, transmissivity=1e-4, rate=-1e-4)

for i, step in enumerate(time):
    label_1 = "Theis quasi steady" if i == 0 else None
    label_2 = "Theis bounded" if i == 0 else None
    plt.plot(rad, head1[i], label=label_1, color="C"+str(i), linestyle="--")
    plt.plot(rad, head2[i], label=label_2, color="C"+str(i))

plt.plot(rad, head3, label="Thiem", color="k", linestyle=":")

plt.xlabel("r in [m]")
plt.ylabel("h in [m]")
plt.legend()
plt.tight_layout()
plt.show()
```



2.8 Tutorial 8: Advanced stuff

Beside the implementations of solutions form literatur, AnaFlow provides some advanced features to pimp the solutions for the groundwater flow equation.

1. Self defined radial conductivity or transmissivity

All heterogeneous solutions of AnaFlow are derived by calculating an equivalent step function of a radial symmetric transmissivity resp. conductivity function.

The following code shows how to apply this workflow to a self defined transmissivity function. The function in use represents a linear transition from a local to a far field value of transmissivity within a given range.

The step function is calculated as the harmonic mean within given bounds, since the groundwater flow under a pumping condition is perpendicular to the different annular regions of transmissivity.

Reference: (not yet published)

```python
import numpy as np
from matplotlib import pyplot as plt
import matplotlib.gridspec as gridspec
from anaflow import ext_grf, ext_grf_steady
from anaflow.tools import specialrange_cut, annular_hmean, step_f


def cond(rad, K_far, K_well, len_scale):
    """Conductivity with linear increase from K_well to K_far."""
    return np.minimum(np.abs(rad) / len_scale, 1.0) * (K_far - K_well) + K_well


time_labels = ["10 s", "100 s", "1000 s"]
time = [10, 100, 1000]
rad = np.geomspace(0.1, 6)
S = 1e-4
K_well = 1e-5
K_far = 1e-4
len_scale = 5.0
rate = -1e-4
dim = 1.5

cut_off = len_scale
parts = 30
r_well = 0.0
r_bound = 50.0

# calculate a disk-distribution of "trans" by calculating harmonic means
R_part = specialrange_cut(r_well, r_bound, parts, cut_off)
K_part = annular_hmean(cond, R_part, ann_dim=dim, K_far=K_far, K_well=K_well, len_
↪scale=len_scale)
S_part = np.full_like(K_part, S)
# calculate transient and steady heads
head1 = ext_grf(time, rad, S_part, K_part, R_part, dim=dim, rate=rate)
head2 = ext_grf_steady(rad, r_bound, cond, dim=dim, rate=-1e-4, K_far=K_far, K_
↪well=K_well, len_scale=len_scale)

# plotting
gs = gridspec.GridSpec(2, 1, height_ratios=[1, 3])
ax1 = plt.subplot(gs[0])
ax2 = plt.subplot(gs[1], sharex=ax1)
time_ticks=[]
for i, step in enumerate(time):
    label = "Transient" if i == 0 else None
    ax2.plot(rad, head1[i], label=label, color="C"+str(i))
    time_ticks.append(head1[i][-1])

ax2.plot(rad, head2, label="Steady", color="k", linestyle=":")

rad_lin = np.linspace(rad[0], rad[-1], 1000)
ax1.plot(rad_lin, step_f(rad_lin, R_part, K_part), label="step Conductivity")
ax1.plot(rad_lin, cond(rad_lin, K_far, K_well, len_scale), label="Conductivity")
ax1.set_yticks([K_well, K_far])
ax1.set_ylabel(r"$K$ in $[\frac{m}{s}]$")
plt.setp(ax1.get_xticklabels(), visible=False)
```

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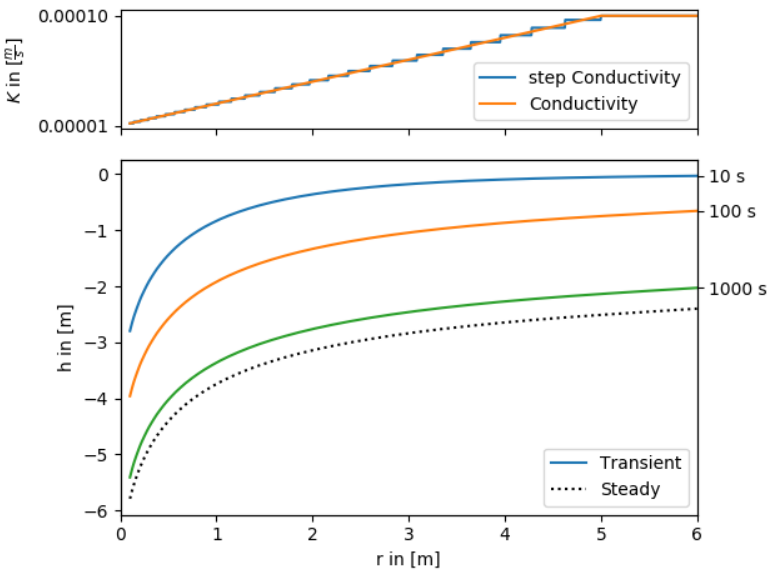
```
ax1.legend()
ax2.set_xlabel("r in [m]")
ax2.set_ylabel("h in [m]")
ax2.legend()
ax2.set_xlim([0, rad[-1]])
ax3 = ax2.twinx()
ax3.set_yticks(time_ticks)
ax3.set_yticklabels(time_labels)
ax3.set_ylim(ax2.get_ylim())

plt.tight_layout()
plt.show()
```



2. Accruing pumping rate

AnaFlow provides different representations for the pumping condition. One is an accruing pumping rate represented by the error function. This could be interpreted as that the water pump needs a certain time to reach its constant rate state.

```
import numpy as np
from scipy.special import erf
from matplotlib import pyplot as plt
import matplotlib.gridspec as gridspec
from anaflow import theis


time = np.geomspace(1, 600)
rad = [1, 5]

# Q(t) = Q * erf(t / a)
a = 120
lap_kwargs = {"cond": 4, "cond_kw": {"a": a}}
head1 = theis(
    time=time,
    rad=rad,
    storage=1e-4,
    transmissivity=1e-4,
    rate=-1e-4,
```

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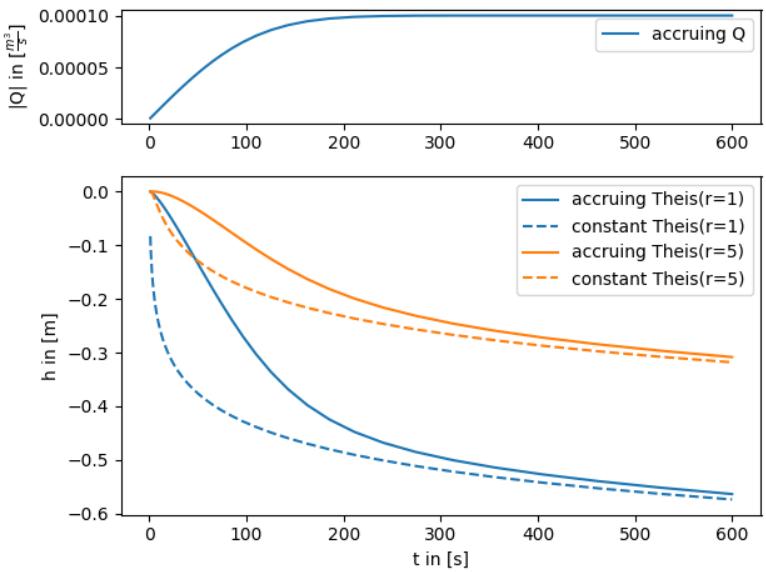
```
    lap_kwargs=lap_kwargs,
)
head2 = theis(
    time=time,
    rad=rad,
    storage=1e-4,
    transmissivity=1e-4,
    rate=-1e-4,
)
gs = gridspec.GridSpec(2, 1, height_ratios=[1, 3])
ax1 = plt.subplot(gs[0])
ax2 = plt.subplot(gs[1], sharex=ax1)

for i, step in enumerate(rad):
    ax2.plot(
        time,
        head1[:, i],
        color="C" + str(i),
        label="accruing Theis(r={})".format(step),
    )
    ax2.plot(
        time,
        head2[:, i],
        color="C" + str(i),
        label="constant Theis(r={})".format(step),
        linestyle="--"
    )
ax1.plot(time, 1e-4 * erf(time / a), label="accruing Q")
ax2.set_xlabel("t in [s]")
ax2.set_ylabel("h in [m]")
ax1.set_ylabel(r"|Q| in [$\frac{m^3}{s}$]")
ax1.legend()
ax2.legend()
plt.tight_layout()
plt.show()
```



3. Interval pumping

Another case often discussed in literatur is interval pumping, where the pumping is just done in a certain time frame.

Unfortunatly the Stehfest algorithm is not suitable for this kind of solution, which is demonstrated in the following script.

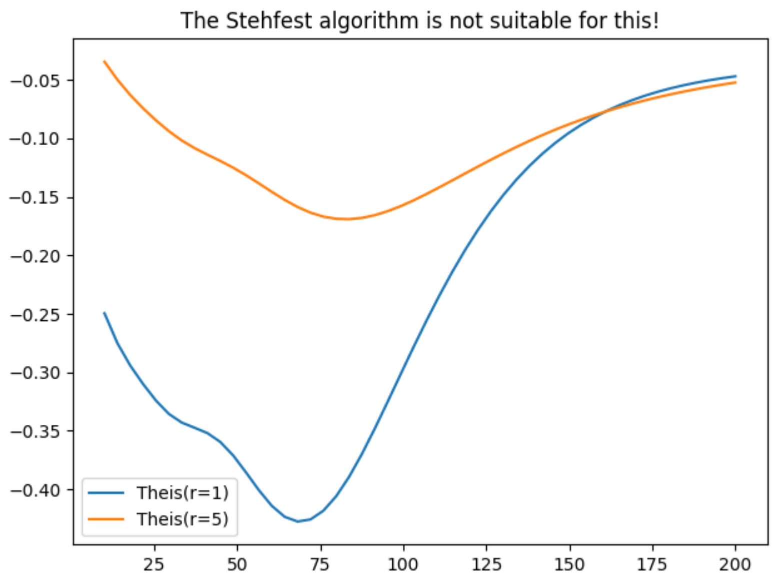
```
import numpy as np
from matplotlib import pyplot as plt
from anaflow import theis


time = np.linspace(10, 200)
rad = [1, 5]

# Q(t) = Q * characteristic([0, T])
lap_kwargs = {"cond": 3, "cond_kw": {"a": 100}}
head = theis(
    time=time,
    rad=rad,
    storage=1e-4,
    transmissivity=1e-4,
    rate=-1e-4,
    lap_kwargs=lap_kwargs,
)

for i, step in enumerate(rad):
    plt.plot(time, head[:, i], label="Theis(r={})".format(step))

plt.title("The Stehfest algorithm is not suitable for this!")
plt.legend()
plt.tight_layout()
plt.show()
```



CHAPTER 3

ANAFLOW API

3.1 Purpose

Anaflow provides several analytical and semi-analytical solutions for the groundwater-flow-equation.

3.2 Subpackages

`flow`	Anaflow subpackage providing flow-solutions for the groundwater flow equation.
`tools`	Anaflow subpackage providing miscellaneous tools.

3.3 Solutions

Homogeneous

Solutions for homogeneous aquifers

`thiem`(rad, r_ref, transmissivity[, rate, h_ref])	The Thiem solution.
`theis`(time, rad, storage, transmissivity[, . . .])	The Theis solution.
`grf`(time, rad, storage, conductivity[, dim, . . .])	The general radial flow (GRF) model for a pumping test.

Heterogeneous

Solutions for heterogeneous aquifers

`ext_thiem_2d`(rad, r_ref, trans_gmean, var, . . .)	The extended Thiem solution in 2D.
`ext_thiem_3d`(rad, r_ref, cond_gmean, var, . . .)	The extended Thiem solution in 3D.
`ext_thiem_tpl`(rad, r_ref, cond_gmean, . . . [, . . .])	The extended Thiem solution for truncated power-law fields.
`ext_thiem_tpl_3d`(rad, r_ref, cond_gmean, . . .)	The extended Theis solution for truncated power-law fields in 3D.

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Table 3 – continued from previous page

ext_theis_2d(time, rad, storage, ...[, ...])	The extended Theis solution in 2D.
ext_theis_3d(time, rad, storage, cond_gmean, ...)	The extended Theis solution in 3D.
ext_theis_tpl(time, rad, storage, ...[, ...])	The extended Theis solution for truncated power-law fields.
ext_thiem_tpl_3d(rad, r_ref, cond_gmean, ...)	The extended Theis solution for truncated power-law fields in 3D.
neuman2004(time, rad, storage, trans_gmean, ...)	The transient solution for the apparent transmissivity from [Neuman2004].
neuman2004_steady(rad, r_ref, trans_gmean, ...)	The steady solution for the apparent transmissivity from [Neuman2004].

Extended GRF

The extended general radial flow model.

ext_grf(time, rad, S_part, K_part, R_part[, ...])	The extended "General radial flow" model for transient flow.
ext_grf_steady(rad, r_ref, conductivity[, ...])	The extended "General radial flow" model for steady flow.

3.4 Laplace

Helping functions related to the laplace-transformation

get_lap(func[, arg_dict])	Callable Laplace transform.
get_lap_inv(func[, method, method_dict, ...])	Callable Laplace inversion.

3.5 Tools

Helping functions.

step_f(rad, r_part, f_part)	Callalbe step function.
specialrange(val_min, val_max, steps[, typ])	Calculation of special point ranges.
specialrange_cut(val_min, val_max, steps[, ...])	Calculation of special point ranges.

3.6 anaflow.flow

Anaflow subpackage providing flow-solutions for the groundwater flow equation.

Subpackages

homogeneous	Anaflow subpackage providing flow solutions in homogeneous aquifers.
heterogeneous	Anaflow subpackage providing flow solutions in heterogeneous aquifers.
ext_grf_model	Anaflow subpackage providing the extended GRF Model.
laplace	Anaflow subpackage providing flow solutions in laplace space.

Solutions

Homogeneous

Solutions for homogeneous aquifers

thiem(rad, r_ref, transmissivity[, rate, h_ref])	The Thiem solution.
theis(time, rad, storage, transmissivity[, . . .])	The Theis solution.
grf(time, rad, storage, conductivity[, dim, . . .])	The general radial flow (GRF) model for a pumping test.

Heterogeneous

Solutions for heterogeneous aquifers

ext_thiem_2d(rad, r_ref, trans_gmean, var, . . .)	The extended Thiem solution in 2D.
ext_thiem_3d(rad, r_ref, cond_gmean, var, . . .)	The extended Thiem solution in 3D.
ext_thiem_tpl(rad, r_ref, cond_gmean, . . . [, . . .])	The extended Thiem solution for truncated power-law fields.
ext_thiem_tpl_3d(rad, r_ref, cond_gmean, . . .)	The extended Theis solution for truncated power-law fields in 3D.
ext_theis_2d(time, rad, storage, . . . [, . . .])	The extended Theis solution in 2D.
ext_theis_3d(time, rad, storage, cond_gmean, . . .)	The extended Theis solution in 3D.
ext_theis_tpl(time, rad, storage, . . . [, . . .])	The extended Theis solution for truncated power-law fields.
ext_thiem_tpl_3d(rad, r_ref, cond_gmean, . . .)	The extended Theis solution for truncated power-law fields in 3D.
neuman2004(time, rad, storage, trans_gmean, . . .)	The transient solution for the apparent transmissivity from [Neuman2004].
neuman2004_steady(rad, r_ref, trans_gmean, . . .)	The steady solution for the apparent transmissivity from [Neuman2004].

Extended GRF

The extended general radial flow model.

ext_grf(time, rad, S_part, K_part, R_part[, ...])	The extended "General radial flow" model for transient flow.
ext_grf_steady(rad, r_ref, conductivity[, ...])	The extended "General radial flow" model for steady flow.

anaflow.flow.homogeneous

Anaflow subpackage providing flow solutions in homogeneous aquifers.

The following functions are provided

thiem(rad, r_ref, transmissivity[, rate, h_ref])	The Thiem solution.
theis(time, rad, storage, transmissivity[, …])	The Theis solution.
grf(time, rad, storage, conductivity[, dim, …])	The general radial flow (GRF) model for a pumping test.

thiem(*rad*, *r_ref*, *transmissivity*, *rate=-0.0001*, *h_ref=0.0*)
The Thiem solution.

The Thiem solution for steady-state flow under a pumping condition in a confined and homogeneous aquifer. This solution was presented in [Thiem1906].

Parameters

- **rad** (`numpy.ndarray`) – Array with all radii where the function should be evaluated.
- **r_ref** (`float`) – Reference radius with known head (see *h_ref*).
- **transmissivity** (`float`) – Transmissivity of the aquifer.
- **rate** (`float`, optional) – Pumpingrate at the well. Default: -1e-4
- **h_ref** (`float`, optional) – Reference head at the reference-radius *r_ref*. Default: `0.0`

Returns **head** – Array with all heads at the given radii.

Return type `numpy.ndarray`

References

Notes

The parameters `rad`, `r_ref` and `transmissivity` will be checked for positivity. If you want to use cartesian coordiantes, just use the formula `r = sqrt(x**2 + y**2)`

Examples

```
>>> thiem([1,2,3], 10, 0.001, -0.001)
array([-0.3664678 , -0.25615   , -0.19161822])
```

theis(*time*, *rad*, *storage*, *transmissivity*, *rate=-0.0001*, *r_well=0.0*, *r_bound=inf*, *h_bound=0.0*, *struc_grid=True*, *lap_kwargs=None*)
The Theis solution.

The Theis solution for transient flow under a pumping condition in a confined and homogeneous aquifer. This solution was presented in [Theis35].

Parameters

- **time** (`numpy.ndarray`) – Array with all time-points where the function should be evaluated
- **rad** (`numpy.ndarray`) – Array with all radii where the function should be evaluated
- **storage** (`float`) – Storage coefficient of the aquifer.
- **conductivity** (`float`) – Conductivity of the aquifer.

- **rate** (float, optional) – Pumpingrate at the well. Default: -1e-4
- **r_well** (float, optional) – Inner radius of the pumping-well. Default: `0.0`
- **r_bound** (float, optional) – Radius of the outer boundariy of the aquifer. Default: `np.inf`
- **h_bound** (float, optional) – Reference head at the outer boundary, as well as initial condition. Default: `0.0`
- **struc_grid** (bool, optional) – If this is set to `False`, the *rad* and *time* array will be merged and interpreted as single, r-t points. In this case they need to have the same shapes. Otherwise a structured r-t grid is created. Default: `True`
- **lap_kwargs** (dict or None optional) – Dictionary for *get_lap_inv* containing *method* and *method_dict*. The default is equivalent to `lap_kwargs = {"method": "stehfest", "method_dict": None}`. Default: `None`

Returns **head** – Array with all heads at the given radii and time-points.

Return type numpy.ndarray

References

grf (*time, rad, storage, conductivity, dim=2, lat_ext=1.0, rate=-0.0001, r_well=0.0, r_bound=inf, h_bound=0.0, struc_grid=True, lap_kwargs=None*)

The general radial flow (GRF) model for a pumping test.

This solution was presented in [Barker88].

Parameters

- **time** (numpy.ndarray) – Array with all time-points where the function should be evaluated.
- **rad** (numpy.ndarray) – Array with all radii where the function should be evaluated.
- **storage** (float) – Storage coefficient of the aquifer.
- **conductivity** (float) – Conductivity of the aquifer.
- **dim** (float, optional) – Fractional dimension of the aquifer. Default: `2.0`
- **lat_ext** (float, optional) – Lateral extend of the aquifer. Default: `1.0`
- **rate** (float, optional) – Pumpingrate at the well. Default: -1e-4
- **r_well** (float, optional) – Inner radius of the pumping-well. Default: `0.0`
- **r_bound** (float, optional) – Radius of the outer boundary of the aquifer. Default: `np.inf`
- **h_bound** (float, optional) – Reference head at the outer boundary, as well as initial condition. Default: `0.0`
- **struc_grid** (bool, optional) – If this is set to "False", the "rad" and "time" array will be merged and interpreted as single, r-t points. In this case they need to have the same shapes. Otherwise a structured r-t grid is created. Default: `True`
- **lap_kwargs** (dict or None optional) – Dictionary for *get_lap_inv* containing *method* and *method_dict*. The default is equivalent to `lap_kwargs = {"method": "stehfest", "method_dict": None}`. Default: `None`

Returns **head** – Array with all heads at the given radii and time-points.

Return type numpy.ndarray

References

anaflow.flow.heterogeneous

Anaflow subpackage providing flow solutions in heterogeneous aquifers.

The following functions are provided

`ext_thiem_2d`(rad, r_ref, trans_gmean, var, ...)	The extended Thiem solution in 2D.
`ext_thiem_3d`(rad, r_ref, cond_gmean, var, ...)	The extended Thiem solution in 3D.
`ext_thiem_tpl`(rad, r_ref, cond_gmean, ...[, ...])	The extended Thiem solution for truncated power-law fields.
`ext_thiem_tpl_3d`(rad, r_ref, cond_gmean, ...)	The extended Theis solution for truncated power-law fields in 3D.
`ext_theis_2d`(time, rad, storage, ...[, ...])	The extended Theis solution in 2D.
`ext_theis_3d`(time, rad, storage, cond_gmean, ...)	The extended Theis solution in 3D.
`ext_theis_tpl`(time, rad, storage, ...[, ...])	The extended Theis solution for truncated power-law fields.
`ext_theis_tpl_3d`(time, rad, storage, ...[, ...])	The extended Theis solution for truncated power-law fields in 3D.
`neuman2004`(time, rad, storage, trans_gmean, ...)	The transient solution for the apparent transmissivity from [Neuman2004].
`neuman2004_steady`(rad, r_ref, trans_gmean, ...)	The steady solution for the apparent transmissivity from [Neuman2004].

ext_thiem_2d(*rad, r_ref, trans_gmean, var, len_scale, rate=-0.0001, h_ref=0.0, T_well=None, prop=1.6*)

The extended Thiem solution in 2D.

The extended Thiem solution for steady-state flow under a pumping condition in a confined aquifer. The type curve is describing the effective drawdown in a 2D statistical framework, where the transmissivity distribution is following a log-normal distribution with a gaussian correlation function.

Parameters

- **rad** (`numpy.ndarray`) – Array with all radii where the function should be evaluated
- **r_ref** (`float`) – Radius of the reference head.
- **trans_gmean** (`float`) – Geometric-mean transmissivity.
- **var** (`float`) – Variance of log-transmissivity.
- **len_scale** (`float`) – Correlation-length of log-transmissivity.
- **rate** (`float`, optional) – Pumpingrate at the well. Default: -1e-4
- **h_ref** (`float`, optional) – Reference head at the reference-radius *r_ref*. Default: `0.0`
- **T_well** (`float`, optional) – Explicit transmissivity value at the well. Default: `None`
- **prop** (`float`, optional) – Proportionality factor used within the upscaling procedure. Default: `1.6`

Returns **head** – Array with all heads at the given radii.

Return type `numpy.ndarray`

References

Notes

If you want to use cartesian coordiantes, just use the formula `r = sqrt(x**2 + y**2)`

Examples

```
>>> ext_thiem_2d([1,2,3], 10, 0.001, 1, 10, -0.001)
array([-0.53084596, -0.35363029, -0.25419375])
```

ext_thiem_3d(*rad*, *r_ref*, *cond_gmean*, *var*, *len_scale*, *anis=1.0*, *lat_ext=1.0*, *rate=-0.0001*, *h_ref=0.0*, *K_well='KH'*, *prop=1.6*)

The extended Thiem solution in 3D.

The extended Thiem solution for steady-state flow under a pumping condition in a confined aquifer. The type curve is describing the effective drawdown in a 3D statistical framework, where the conductivity distribution is following a log-normal distribution with a gaussian correlation function and taking vertical anisotropy into account.

Parameters

- **rad** (`numpy.ndarray`) – Array with all radii where the function should be evaluated
- **r_ref** (`float`) – Reference radius with known head (see *h_ref*)
- **cond_gmean** (`float`) – Geometric-mean conductivity.
- **var** (`float`) – Variance of the log-conductivity.
- **len_scale** (`float`) – Corralation-length of log-conductivity.
- **anis** (`float`, optional) – Anisotropy-ratio of the vertical and horizontal corralation-lengths. Default: 1.0
- **lat_ext** (`float`, optional) – Lateral extend of the aquifer (thickness). Default: `1.0`
- **rate** (`float`, optional) – Pumpingrate at the well. Default: -1e-4
- **h_ref** (`float`, optional) – Reference head at the reference-radius *r_ref*. Default: `0.0`
- **K_well** (*string/float, optional*) – Explicit conductivity value at the well. One can choose between the harmonic mean (`"KH"`), the arithmetic mean (`"KA"`) or an arbitrary float value. Default: `"KH"`
- **prop** (`float`, optional) – Proportionality factor used within the upscaling procedure. Default: `1.6`

Returns **head** – Array with all heads at the given radii.

Return type `numpy.ndarray`

References

Notes

If you want to use cartesian coordiantes, just use the formula `r = sqrt(x**2 + y**2)`

Examples

```
>>> ext_thiem_3d([1,2,3], 10, 0.001, 1, 10, 1, 1, -0.001)
array([-0.48828026, -0.31472059, -0.22043022])
```

ext_thiem_tpl(*rad*, *r_ref*, *cond_gmean*, *len_scale*, *hurst*, *var=None*, *c=1.0*, *dim=2.0*, *lat_ext=1.0*, *rate=-0.0001*, *h_ref=0.0*, *K_well='KH'*, *prop=1.6*)

The extended Thiem solution for truncated power-law fields.

The extended Theis solution for steady flow under a pumping condition in a confined aquifer. The type curve is describing the effective drawdown in a d-dimensional statistical framework, where the conductivity distribution is following a log-normal distribution with a truncated power-law correlation function build on superposition of gaussian modes.

Parameters

- **rad** (numpy.ndarray) – Array with all radii where the function should be evaluated
- **r_ref** (float) – Reference radius with known head (see *h_ref*)
- **cond_gmean** (float) – Geometric-mean conductivity. You can also treat this as transmissivity by leaving 'lat_ext=1'.
- **len_scale** (float) – Corralation-length of log-conductivity.
- **hurst** (float) – Hurst coefficient of the TPL model. Should be in (0, 1).
- **var** (float) – Variance of the log-conductivity. If var is given, c will be calculated accordingly. Default: None
- **c** (float, optional) – Intensity of variation in the TPL model. Is overwritten if var is given. Default: `1.0`
- **dim** (float, optional) – Dimension of space. Default: `2.0`
- **lat_ext** (float, optional) – Lateral extend of the aquifer:
  - sqare-root of cross-section in 1D
  - thickness in 2D
  - meaningless in 3D

  Default: `1.0`
- **rate** (float, optional) – Pumpingrate at the well. Default: -1e-4
- **h_ref** (float, optional) – Reference head at the reference-radius *r_ref*. Default: `0.0`
- **K_well** (float, optional) – Explicit conductivity value at the well. One can choose between the harmonic mean (`"KH"`), the arithmetic mean (`"KA"`) or an arbitrary float value. Default: `"KH"`
- **prop** (float, optional) – Proportionality factor used within the upscaling procedure. Default: `1.6`

Returns **head** – Array with all heads at the given radii and time-points.

Return type numpy.ndarray

Notes

If you want to use cartesian coordiantes, just use the formula `r = sqrt(x**2 + y**2)`

---

ext_thiem_tpl_3d(*rad*, *r_ref*, *cond_gmean*, *len_scale*, *hurst*, *var=None*, *c=1.0*, *anis=1*, *lat_ext=1.0*, *rate=-0.0001*, *h_ref=0.0*, *K_well='KH'*, *prop=1.6*)

The extended Theis solution for truncated power-law fields in 3D.

The extended Theis solution for transient flow under a pumping condition in a confined aquifer with anisotropy in 3D. The type curve is describing the effective drawdown in a 3-dimensional statistical framework, where the conductivity distribution is following a log-normal distribution with a truncated power-law correlation function build on superposition of gaussian modes.

Parameters

- **time** (numpy.ndarray) – Array with all time-points where the function should be evaluated
- **rad** (numpy.ndarray) – Array with all radii where the function should be evaluated
- **storage** (float) – Storage of the aquifer.

- **cond_gmean** (float) – Geometric-mean conductivity.
- **len_scale** (float) – Corralation-length of log-conductivity.
- **hurst** (float) – Hurst coefficient of the TPL model. Should be in (0, 1).
- **var** (float) – Variance of the log-conductivity. If var is given, c will be calculated accordingly. Default: `None`
- **c** (float, optional) – Intensity of variation in the TPL model. Is overwritten if var is given. Default: `1.0`
- **anis** (float, optional) – Anisotropy-ratio of the vertical and horizontal corralation-lengths. Default: 1.0
- **lat_ext** (float, optional) – Lateral extend of the aquifer (thickness). Default: `1.0`
- **rate** (float, optional) – Pumpingrate at the well. Default: -1e-4
- **r_well** (float, optional) – Radius of the pumping-well. Default: `0.0`
- **r_bound** (float, optional) – Radius of the outer boundary of the aquifer. Default: `np.inf`
- **h_bound** (float, optional) – Reference head at the outer boundary as well as initial condition. Default: `0.0`
- **K_well** (float, optional) – Explicit conductivity value at the well. One can choose between the harmonic mean (`"KH"`), the arithmetic mean (`"KA"`) or an arbitrary float value. Default: `"KH"`
- **prop** (float, optional) – Proportionality factor used within the upscaling procedure. Default: `1.6`
- **far_err** (float, optional) – Relative error for the farfield transmissivity for calculating the cutoff-point of the solution. Default: `0.01`
- **struc_grid** (bool, optional) – If this is set to `False`, the *rad* and *time* array will be merged and interpreted as single, r-t points. In this case they need to have the same shapes. Otherwise a structured r-t grid is created. Default: `True`
- **parts** (int, optional) – Since the solution is calculated by setting the transmissivity to local constant values, one needs to specify the number of partitions of the transmissivity. Default: `30`
- **lap_kwargs** (dict or None optional) – Dictionary for *get_lap_inv* containing *method* and *method_dict*. The default is equivalent to `lap_kwargs = {"method": "stehfest", "method_dict": None}`. Default: `None`

Returns **head** – Array with all heads at the given radii and time-points.

Return type numpy.ndarray

Notes

If you want to use cartesian coordiantes, just use the formula `r = sqrt(x**2 + y**2)`

ext_theis_2d(*time*, *rad*, *storage*, *trans_gmean*, *var*, *len_scale*, *rate=-0.0001*, *r_well=0.0*, *r_bound=inf*, *h_bound=0.0*, *T_well=None*, *prop=1.6*, *struc_grid=True*, *far_err=0.01*, *parts=30*, *lap_kwargs=None*)

The extended Theis solution in 2D.

The extended Theis solution for transient flow under a pumping condition in a confined aquifer. The type curve is describing the effective drawdown in a 2D statistical framework, where the transmissivity distribution is following a log-normal distribution with a gaussian correlation function.

Parameters

- **time** (numpy.ndarray) – Array with all time-points where the function should be evaluated
- **rad** (numpy.ndarray) – Array with all radii where the function should be evaluated
- **storage** (float) – Storage of the aquifer.
- **trans_gmean** (float) – Geometric-mean transmissivity.
- **var** (float) – Variance of log-transmissivity.
- **len_scale** (float) – Correlation-length of log-transmissivity.
- **rate** (float, optional) – Pumpingrate at the well. Default: -1e-4
- **r_well** (float, optional) – Radius of the pumping-well. Default: `0.0`
- **r_bound** (float, optional) – Radius of the outer boundary of the aquifer. Default: `np.inf`
- **h_bound** (float, optional) – Reference head at the outer boundary as well as initial condition. Default: `0.0`
- **T_well** (float, optional) – Explicit transmissivity value at the well. Harmonic mean by default.
- **prop** (float, optional) – Proportionality factor used within the upscaling procedure. Default: `1.6`
- **far_err** (float, optional) – Relative error for the farfield transmissivity for calculating the cutoff-point of the solution. Default: `0.01`
- **struc_grid** (bool, optional) – If this is set to `False`, the *rad* and *time* array will be merged and interpreted as single, r-t points. In this case they need to have the same shapes. Otherwise a structured r-t grid is created. Default: `True`
- **parts** (int, optional) – Since the solution is calculated by setting the transmissivity to local constant values, one needs to specify the number of partitions of the transmissivity. Default: `30`
- **lap_kwargs** (dict or None optional) – Dictionary for *get_lap_inv* containing *method* and *method_dict*. The default is equivalent to `lap_kwargs = {"method": "stehfest", "method_dict": None}`. Default: None

Returns **head** – Array with all heads at the given radii and time-points.

Return type numpy.ndarray

Notes

If you want to use cartesian coordiantes, just use the formula `r = sqrt(x**2 + y**2)`

Examples

```
>>> ext_theis_2d([10,100], [1,2,3], 0.001, 0.001, 1, 10, -0.001)
array([[-0.33737576, -0.17400123, -0.09489812],
       [-0.58443489, -0.40847176, -0.31095166]])
```

ext_theis_3d(*time, rad, storage, cond_gmean, var, len_scale, anis=1.0, lat_ext=1.0, rate=-0.0001, r_well=0.0, r_bound=inf, h_bound=0.0, K_well='KH', prop=1.6, far_err=0.01, struc_grid=True, parts=30, lap_kwargs=None*)

The extended Theis solution in 3D.

The extended Theis solution for transient flow under a pumping condition in a confined aquifer. The type curve is describing the effective drawdown in a 3D statistical framework, where the transmissivity distribution is following a log-normal distribution with a gaussian correlation function and taking vertical anisotropy into account.

Parameters

- **time** (`numpy.ndarray`) – Array with all time-points where the function should be evaluated
- **rad** (`numpy.ndarray`) – Array with all radii where the function should be evaluated
- **storage** (`float`) – Storage of the aquifer.
- **cond_gmean** (`float`) – Geometric-mean conductivity.
- **var** (`float`) – Variance of the log-conductivity.
- **len_scale** (`float`) – Corralation-length of log-conductivity.
- **anis** (`float`, optional) – Anisotropy-ratio of the vertical and horizontal corralation-lengths. Default: 1.0
- **lat_ext** (`float`, optional) – Lateral extend of the aquifer (thickness). Default: `1.0`
- **rate** (`float`, optional) – Pumpingrate at the well. Default: -1e-4
- **r_well** (`float`, optional) – Radius of the pumping-well. Default: `0.0`
- **r_bound** (`float`, optional) – Radius of the outer boundary of the aquifer. Default: `np.inf`
- **h_bound** (`float`, optional) – Reference head at the outer boundary as well as initial condition. Default: `0.0`
- **K_well** (`float`, optional) – Explicit conductivity value at the well. One can choose between the harmonic mean (`"KH"`), the arithmetic mean (`"KA"`) or an arbitrary float value. Default: `"KH"`
- **prop** (`float`, optional) – Proportionality factor used within the upscaling procedure. Default: `1.6`
- **far_err** (`float`, optional) – Relative error for the farfield transmissivity for calculating the cutoff-point of the solution. Default: `0.01`
- **struc_grid** (`bool`, optional) – If this is set to `False`, the *rad* and *time* array will be merged and interpreted as single, r-t points. In this case they need to have the same shapes. Otherwise a structured r-t grid is created. Default: `True`
- **parts** (`int`, optional) – Since the solution is calculated by setting the transmissivity to local constant values, one needs to specify the number of partitions of the transmissivity. Default: `30`
- **lap_kwargs** (`dict` or `None` optional) – Dictionary for *`get_lap_inv`* containing *method* and *method_dict*. The default is equivalent to `lap_kwargs = {"method": "stehfest", "method_dict": None}`. Default: `None`

Returns **head** – Array with all heads at the given radii and time-points.

Return type `numpy.ndarray`

Notes

If you want to use cartesian coordiantes, just use the formula `r = sqrt(x**2 + y**2)`

Examples

```
>>> ext_theis_3d([10,100], [1,2,3], 0.001, 0.001, 1, 10, 1, 1, -0.001)
array([[-0.32756786, -0.16717569, -0.09141211],
       [-0.5416396 , -0.36982684, -0.27798614]])
```

ext_theis_tpl (*time*, *rad*, *storage*, *cond_gmean*, *len_scale*, *hurst*, *var=None*, *c=1.0*, *dim=2.0*, *lat_ext=1.0*, *rate=-0.0001*, *r_well=0.0*, *r_bound=inf*, *h_bound=0.0*, *K_well='KH'*, *prop=1.6*, *far_err=0.01*, *struc_grid=True*, *parts=30*, *lap_kwargs=None*)

The extended Theis solution for truncated power-law fields.

The extended Theis solution for transient flow under a pumping condition in a confined aquifer. The type curve is describing the effective drawdown in a d-dimensional statistical framework, where the conductivity distribution is following a log-normal distribution with a truncated power-law correlation function build on superposition of gaussian modes.

Parameters

- **time** (`numpy.ndarray`) – Array with all time-points where the function should be evaluated
- **rad** (`numpy.ndarray`) – Array with all radii where the function should be evaluated
- **storage** (`float`) – Storage of the aquifer.
- **cond_gmean** (`float`) – Geometric-mean conductivity. You can also treat this as transmissivity by leaving 'lat_ext=1'.
- **len_scale** (`float`) – Corralation-length of log-conductivity.
- **hurst** (`float`) – Hurst coefficient of the TPL model. Should be in (0, 1).
- **var** (`float`) – Variance of the log-conductivity. If var is given, c will be calculated accordingly. Default: `None`
- **c** (`float`, optional) – Intensity of variation in the TPL model. Is overwritten if var is given. Default: `1.0`
- **dim** (`float`, optional) – Dimension of space. Default: `2.0`
- **lat_ext** (`float`, optional) – Lateral extend of the aquifer:
  - sqare-root of cross-section in 1D
  - thickness in 2D
  - meaningless in 3D

  Default: `1.0`
- **rate** (`float`, optional) – Pumpingrate at the well. Default: -1e-4
- **r_well** (`float`, optional) – Radius of the pumping-well. Default: `0.0`
- **r_bound** (`float`, optional) – Radius of the outer boundary of the aquifer. Default: `np.inf`
- **h_bound** (`float`, optional) – Reference head at the outer boundary as well as initial condition. Default: `0.0`
- **K_well** (`float`, optional) – Explicit conductivity value at the well. One can choose between the harmonic mean (`"KH"`), the arithmetic mean (`"KA"`) or an arbitrary float value. Default: `"KH"`
- **prop** (`float`, optional) – Proportionality factor used within the upscaling procedure. Default: `1.6`
- **far_err** (`float`, optional) – Relative error for the farfield transmissivity for calculating the cutoff-point of the solution. Default: `0.01`

- **struc_grid** (`bool`, optional) – If this is set to `False`, the *rad* and *time* array will be merged and interpreted as single, r-t points. In this case they need to have the same shapes. Otherwise a structured r-t grid is created. Default: `True`
- **parts** (`int`, optional) – Since the solution is calculated by setting the transmissivity to local constant values, one needs to specify the number of partitions of the transmissivity. Default: `30`
- **lap_kwargs** (`dict` or `None` optional) – Dictionary for *get_lap_inv* containing *method* and *method_dict*. The default is equivalent to `lap_kwargs = {"method": "stehfest", "method_dict": None}`. Default: `None`

Returns **head** – Array with all heads at the given radii and time-points.

Return type `numpy.ndarray`

Notes

If you want to use cartesian coordiantes, just use the formula `r = sqrt(x**2 + y**2)`

ext_theis_tpl_3d(*time*, *rad*, *storage*, *cond_gmean*, *len_scale*, *hurst*, *var=None*, *c=1.0*, *anis=1*, *lat_ext=1.0*, *rate=-0.0001*, *r_well=0.0*, *r_bound=inf*, *h_bound=0.0*, *K_well='KH'*, *prop=1.6*, *far_err=0.01*, *struc_grid=True*, *parts=30*, *lap_kwargs=None*)

The extended Theis solution for truncated power-law fields in 3D.

The extended Theis solution for transient flow under a pumping condition in a confined aquifer with anisotropy in 3D. The type curve is describing the effective drawdown in a 3-dimensional statistical framework, where the conductivity distribution is following a log-normal distribution with a truncated power-law correlation function build on superposition of gaussian modes.

Parameters

- **time** (`numpy.ndarray`) – Array with all time-points where the function should be evaluated
- **rad** (`numpy.ndarray`) – Array with all radii where the function should be evaluated
- **storage** (`float`) – Storage of the aquifer.
- **cond_gmean** (`float`) – Geometric-mean conductivity.
- **len_scale** (`float`) – Corralation-length of log-conductivity.
- **hurst** (`float`) – Hurst coefficient of the TPL model. Should be in (0, 1).
- **var** (`float`) – Variance of the log-conductivity. If var is given, c will be calculated accordingly. Default: `None`
- **c** (`float`, optional) – Intensity of variation in the TPL model. Is overwritten if var is given. Default: `1.0`
- **anis** (`float`, optional) – Anisotropy-ratio of the vertical and horizontal corralation-lengths. Default: 1.0
- **lat_ext** (`float`, optional) – Lateral extend of the aquifer (thickness). Default: `1.0`
- **rate** (`float`, optional) – Pumpingrate at the well. Default: -1e-4
- **r_well** (`float`, optional) – Radius of the pumping-well. Default: `0.0`
- **r_bound** (`float`, optional) – Radius of the outer boundary of the aquifer. Default: `np.inf`
- **h_bound** (`float`, optional) – Reference head at the outer boundary as well as initial condition. Default: `0.0`

- **K_well** (float, optional) – Explicit conductivity value at the well. One can choose between the harmonic mean ("KH"), the arithmetic mean ("KA") or an arbitrary float value. Default: "KH"
- **prop** (float, optional) – Proportionality factor used within the upscaling procedure. Default: 1.6
- **far_err** (float, optional) – Relative error for the farfield transmissivity for calculating the cutoff-point of the solution. Default: 0.01
- **struc_grid** (bool, optional) – If this is set to False, the *rad* and *time* array will be merged and interpreted as single, r-t points. In this case they need to have the same shapes. Otherwise a structured r-t grid is created. Default: True
- **parts** (int, optional) – Since the solution is calculated by setting the transmissivity to local constant values, one needs to specify the number of partitions of the transmissivity. Default: 30
- **lap_kwargs** (dict or None optional) – Dictionary for *get_lap_inv* containing *method* and *method_dict*. The default is equivalent to `lap_kwargs = {"method": "stehfest", "method_dict": None}`. Default: None

Returns **head** – Array with all heads at the given radii and time-points.

Return type numpy.ndarray

Notes

If you want to use cartesian coordiantes, just use the formula `r = sqrt(x**2 + y**2)`

neuman2004 (*time*, *rad*, *storage*, *trans_gmean*, *var*, *len_scale*, *rate=-0.0001*, *r_well=0.0*, *r_bound=inf*, *h_bound=0.0*, *struc_grid=True*, *parts=30*, *lap_kwargs=None*)

The transient solution for the apparent transmissivity from [Neuman2004].

This solution is build on the apparent transmissivity from Neuman 2004, which represents a mean drawdown in an ensemble of pumping tests in heterogeneous transmissivity fields following an exponential covariance.

Parameters

- **time** (numpy.ndarray) – Array with all time-points where the function should be evaluated.
- **rad** (numpy.ndarray) – Array with all radii where the function should be evaluated.
- **storage** (float) – Storage of the aquifer.
- **trans_gmean** (float) – Geometric-mean transmissivity.
- **var** (float) – Variance of log-transmissivity.
- **len_scale** (float) – Correlation-length of log-transmissivity.
- **rate** (float, optional) – Pumpingrate at the well. Default: -1e-4
- **r_well** (float, optional) – Radius of the pumping-well. Default: 0.0
- **r_bound** (float, optional) – Radius of the outer boundary of the aquifer. Default: np.inf
- **h_bound** (float, optional) – Reference head at the outer boundary as well as initial condition. Default: 0.0
- **struc_grid** (bool, optional) – If this is set to False, the *rad* and *time* array will be merged and interpreted as single, r-t points. In this case they need to have the same shapes. Otherwise a structured r-t grid is created. Default: True

- **parts** (int, optional) – Since the solution is calculated by setting the transmissivity to local constant values, one needs to specify the number of partitions of the transmissivity. Default: `30`
- **lap_kwargs** (dict or None optional) – Dictionary for *get_lap_inv* containing *method* and *method_dict*. The default is equivalent to `lap_kwargs = {"method": "stehfest", "method_dict": None}`. Default: None

Returns **head** – Array with all heads at the given radii and time-points.

Return type numpy.ndarray

References

neuman2004_steady (*rad*, *r_ref*, *trans_gmean*, *var*, *len_scale*, *rate=-0.0001*, *h_ref=0.0*)

The steady solution for the apparent transmissivity from [Neuman2004].

This solution is build on the apparent transmissivity from Neuman 1994, which represents a mean drawdown in an ensemble of pumping tests in heterogeneous transmissivity fields following an exponential covariance.

Parameters

- **rad** (numpy.ndarray) – Array with all radii where the function should be evaluated
- **r_ref** (float) – Radius of the reference head.
- **trans_gmean** (float) – Geometric-mean transmissivity.
- **var** (float) – Variance of log-transmissivity.
- **len_scale** (float) – Correlation-length of log-transmissivity.
- **rate** (float, optional) – Pumpingrate at the well. Default: -1e-4
- **h_ref** (float, optional) – Reference head at the reference-radius *r_ref*. Default: `0.0`

Returns **head** – Array with all heads at the given radii.

Return type numpy.ndarray

References

anaflow.flow.ext_grf_model

Anaflow subpackage providing the extended GRF Model.

The following functions are provided

ext_grf(time, rad, S_part, K_part, R_part[, . . .])	The extended "General radial flow" model for transient flow.
ext_grf_steady(rad, r_ref, conductivity[, . . .])	The extended "General radial flow" model for steady flow.

ext_grf(*time, rad, S_part, K_part, R_part, dim=2, lat_ext=1.0, rate=-0.0001, h_bound=0.0, K_well=None, struc_grid=True, lap_kwargs=None*)

The extended "General radial flow" model for transient flow.

The general radial flow (GRF) model by Barker introduces an arbitrary dimension for radial groundwater flow. We introduced the possibility to define radial dependet conductivity and storage values.

This solution is based on the grf model presented in [Barker88].

Parameters

- **time** (`numpy.ndarray`) – Array with all time-points where the function should be evaluated
- **rad** (`numpy.ndarray`) – Array with all radii where the function should be evaluated
- **S_part** (`numpy.ndarray`) – Given storativity values for each disk
- **K_part** (`numpy.ndarray`) – Given conductivity values for each disk
- **R_part** (`numpy.ndarray`) – Given radii separating the disks (including r_well and r_bound).
- **dim** (`float`, optional) – Fractional dimension of the aquifer. Default: `2.0`
- **lat_ext** (`float`, optional) – Lateral extend of the aquifer. Default: `1.0`
- **rate** (`float`, optional) – Pumpingrate at the well. Default: -1e-4
- **h_bound** (`float`, optional) – Reference head at the outer boundary *R_part[-1]*. Default: `0.0`
- **K_well** (`float`, optional) – Conductivity at the well. Default: `K_part[0]`
- **struc_grid** (`bool`, optional) – If this is set to `False`, the *rad* and *time* array will be merged and interpreted as single, r-t points. In this case they need to have the same shapes. Otherwise a structured r-t grid is created. Default: `True`
- **lap_kwargs** (`dict` or `None` optional) – Dictionary for *get_lap_inv* containing *method* and *method_dict*. The default is equivalent to `lap_kwargs = {"method": "stehfest", "method_dict": None}`. Default: `None`

Returns Array with all heads at the given radii and time-points.

Return type `numpy.ndarray`

References

ext_grf_steady(*rad, r_ref, conductivity, dim=2, lat_ext=1.0, rate=-0.0001, h_ref=0.0, arg_dict=None, **kwargs*)

The extended "General radial flow" model for steady flow.

The general radial flow (GRF) model by Barker introduces an arbitrary dimension for radial groundwater flow. We introduced the possibility to define radial dependet conductivity.

This solution is based on the grf model presented in [Barker88].

Parameters

- **rad** (numpy.ndarray) – Array with all radii where the function should be evaluated
- **r_ref** (float) – Radius of the reference head.
- **conductivity** (float or callable) – Conductivity. Either callable function taking kwargs or float.
- **dim** (float, optional) – Fractional dimension of the aquifer. Default: `2.0`
- **lat_ext** (float, optional) – Lateral extend of the aquifer. Default: `1.0`
- **rate** (float, optional) – Pumpingrate at the well. Default: -1e-4
- **h_ref** (float, optional) – Reference head at the reference-radius *r_ref*. Default: `0.0`
- **arg_dict** (dict or None, optional) – Keyword-arguments given as a dictionary that are forwarded to the conductivity function. Will be merged with `**kwargs`. This is designed for overlapping keywords in `grf_steady` and `conductivity`. Default: `None`
- ****kwargs** – Keyword-arguments that are forwarded to the conductivity function. Will be merged with `arg_dict`

Returns Array with all heads at the given radii and time-points.

Return type numpy.ndarray

References

anaflow.flow.laplace

Anaflow subpackage providing flow solutions in laplace space.

The following functions are provided

grf_laplace(s[, rad, S_part, K_part, . . .])	The extended GRF-model for transient flow in Laplace-space.

grf_laplace(*s*, *rad=None*, *S_part=None*, *K_part=None*, *R_part=None*, *dim=2*, *lat_ext=1.0*, *rate=None*, *K_well=None*, *cut_off_prec=1e-20*, *cond=0*, *cond_kw=None*)

The extended GRF-model for transient flow in Laplace-space.

The General Radial Flow (GRF) Model allowes fractured dimensions for transient flow under a pumping condition in a confined aquifer. The solutions assumes concentric annuli around the pumpingwell, where each annulus has its own conductivity and storativity value.

Parameters

- **s** (`numpy.ndarray`) – Array with all Laplace-space-points where the function should be evaluated
- **rad** (`numpy.ndarray`) – Array with all radii where the function should be evaluated
- **S_part** (`numpy.ndarray` of length N) – Given storativity values for each disk
- **K_part** (`numpy.ndarray` of length N) – Given conductivity values for each disk
- **R_part** (`numpy.ndarray` of length N+1) – Given radii separating the disks as well as starting- and endpoints
- **dim** (`float`) – Flow dimension. Default: 3
- **lat_ext** (`float`) – The lateral extend of the flow-domain, used in *L^(3-dim)*. Default: 1
- **rate** (`float`) – Pumpingrate at the well
- **K_well** (`float`, optional) – Conductivity at the well. Default: `K_part[0]`
- **cut_off_prec** (`float`, optional) – Define a cut-off precision for the calculation to select the disks included in the calculation. Default `1e-20`
- **cond** (`int`, optional) – Type of the pumping condition:
  - 0 : constant
  - 1 : periodic (needs "w" as cond_kw)
  - 2 : slug (rate will be interpreted as slug-volume)
  - 3 : interval (needs "t" as cond_kw)
  - callable: laplace-transformation of the transient pumping-rate

  Default: 0
- **cond_kw** (`dict` optional) – Keyword args for the pumping condition. Default: None

Returns **grf_laplace** – Array with all values in laplace-space

Return type `numpy.ndarray`

Examples

```
>>> grf_laplace([5,10],[1,2,3],[1e-3,1e-3],[1e-3,2e-3],[0,2,10], 2, 1, -1)
array([[-2.71359196e+00, -1.66671965e-01, -2.82986917e-02],
       [-4.58447458e-01, -1.12056319e-02, -9.85673855e-04]])
```

3.7 anaflow.tools

Anaflow subpackage providing miscellaneous tools.

Subpackages

`laplace`	Anaflow subpackage providing functions concerning the laplace transformation.
`mean`	Anaflow subpackage providing several mean calculating routines.
`special`	Anaflow subpackage providing special functions.
`coarse_graining`	Anaflow subpackage providing helper functions related to coarse graining.

Functions

Annular mean

Functions to calculate dimension dependent annular means of a function.

`annular_fmean`(func, val_arr, f_def, f_inv[, . . .])	Calculating the annular generalized f-mean.
`annular_amean`(func, val_arr[, ann_dim, arg_dict])	Calculating the annular arithmetic mean.
`annular_gmean`(func, val_arr[, ann_dim, arg_dict])	Calculating the annular geometric mean.
`annular_hmean`(func, val_arr[, ann_dim, arg_dict])	Calculating the annular harmonic mean.
`annular_pmean`(func, val_arr[, p, ann_dim, . . .])	Calculating the annular p-mean.

Coarse Graining solutions

Effective Coarse Graining conductivity/transmissivity solutions.

`T_CG`(rad, trans_gmean, var, len_scale[, . . .])	The coarse-graining Transmissivity.
`K_CG`(rad, cond_gmean, var, len_scale, anis)	The coarse-graining conductivity.
`TPL_CG`(rad, cond_gmean, len_scale, hurst[, . . .])	The gaussian truncated power-law coarse-graining conductivity.

Special

Special functions.

`step_f`(rad, r_part, f_part)	Callalbe step function.
`specialrange`(val_min, val_max, steps[, typ])	Calculation of special point ranges.
`specialrange_cut`(val_min, val_max, steps[, . . .])	Calculation of special point ranges.
`neuman2004_trans`(rad, trans_gmean, var, . . .)	The apparent transmissivity from Neuman 2004.
`aniso`(e)	The anisotropy function.

Laplace

Helping functions related to the laplace-transformation

`get_lap`(func[, arg_dict])	Callable Laplace transform.
`get_lap_inv`(func[, method, method_dict, . . .])	Callable Laplace inversion.

anaflow.tools.laplace

Anaflow subpackage providing functions concerning the laplace transformation.

The following functions are provided

`get_lap`(func[, arg_dict])	Callable Laplace transform.
`get_lap_inv`(func[, method, method_dict, . . .])	Callable Laplace inversion.
`lap_trans`(func, phase[, arg_dict])	The laplace transform.
`stehfest`(func, time[, bound, arg_dict])	The stehfest-algorithm for numerical laplace inversion.

get_lap (*func*, *arg_dict=None*, ***kwargs*)

Callable Laplace transform.

Get the Laplace transform of a given function as a callable function.

Parameters

- **func** (`callable`) – function that shall be transformed. The first argument needs to be the time-variable: `func(t, **kwargs)`

  func should be capable of taking numpy arrays as input for *s* and the first shape component of the output of *func* should match the shape of *s*.

- **arg_dict** (`dict` or `None`, optional) – Keyword-arguments given as a dictionary that are forwarded to the function given in `func`. Will be merged with `**kwargs` This is designed for overlapping keywords. Default: `None`
- ****kwargs** – Keyword-arguments that are forwarded to the function given in `func`. Will be merged with `arg_dict`.

Returns The Laplace transformed of the given function.

Return type `callable`

Raises `ValueError` – If *func* is not callable.

lap_trans (*func*, *phase*, *arg_dict=None*, ***kwargs*)

The laplace transform.

Parameters

- **func** (`callable`) – function that shall be transformed. The first argument needs to be the time-variable: `func(s, **kwargs)`

  func should be capable of taking numpy arrays as input for *s* and the first shape component of the output of *func* should match the shape of *s*.

- **phase** (`float` or `numpy.ndarray`) – phase-points to evaluate the transformed function at
- **arg_dict** (`dict` or `None`, optional) – Keyword-arguments given as a dictionary that are forwarded to the function given in `func`. Will be merged with `**kwargs` This is designed for overlapping keywords in `stehfest` and `func`.Default: `None`
- ****kwargs** – Keyword-arguments that are forwarded to the function given in `func`. Will be merged with `arg_dict`

Returns Array with all evaluations in phase-space.

Return type `numpy.ndarray`

Raises `ValueError` – If *func* is not callable.

get_lap_inv (*func*, *method='stehfest'*, *method_dict=None*, *arg_dict=None*, ***kwargs*)

Callable Laplace inversion.

Get the Laplace inversion of a given function as a callable function.

Parameters

- **func** (`callable`) – function in laplace-space that shall be inverted. The first argument needs to be the laplace-variable: `func(s, **kwargs)`

  func should be capable of taking numpy arrays as input for *s* and the first shape component of the output of *func* should match the shape of *s*.

- **method** (`str`) – Method that should be used to calculate the inverse. One can choose between

  – `"stehfest"`: for the stehfest algorithm

  Default: `"stehfest"`

- **method_dict** (`dict` or `None`, optional) – Keyword arguments for the used method.
- **arg_dict** (`dict` or `None`, optional) – Keyword-arguments given as a dictionary that are forwarded to the function given in `func`. Will be merged with `**kwargs` This is designed for overlapping keywords. Default: `None`
- ****kwargs** – Keyword-arguments that are forwarded to the function given in `func`. Will be merged with `arg_dict`.

Returns The Laplace inverse of the given function.

Return type `callable`

Raises

- `ValueError` – If *func* is not callable.
- `ValueError` – If *method* is unknown.

stehfest (*func*, *time*, *bound=12*, *arg_dict=None*, ***kwargs*)

The stehfest-algorithm for numerical laplace inversion.

The Inversion was derivide in ''Stehfest 1970"[R1] and is given by the formula

$$f(t) = \frac{\ln 2}{t} \sum_{n=1}^{N} c_n \cdot \tilde{f}\left(n \cdot \frac{\ln 2}{t}\right)$$

$$c_n = (-1)^{n+\frac{N}{2}} \cdot \sum_{k=\left\lfloor \frac{n+1}{2} \right\rfloor}^{\min\left\{n, \frac{N}{2}\right\}} \frac{k^{\frac{N}{2}+1} \cdot \binom{2k}{k}}{\left(\frac{N}{2}-k\right)! \cdot (n-k)! \cdot (2k-n)!}$$

In the algorithm N corresponds to `bound`, $\tilde{f}$ to `func` and t to `time`.

Parameters

- **func** (`callable`) – function in laplace-space that shall be inverted. The first argument needs to be the laplace-variable: `func(s, **kwargs)`

  func should be capable of taking numpy arrays as input for *s* and the first shape component of the output of *func* should match the shape of *s*.

- **time** (`float` or `numpy.ndarray`) – time-points to evaluate the function at
- **bound** (`int`, optional) – Here you can specify the number of interations within this algorithm. Default: `12`
- **arg_dict** (`dict` or `None`, optional) – Keyword-arguments given as a dictionary that are forwarded to the function given in `func`. Will be merged with `**kwargs` This is designed for overlapping keywords in `stehfest` and `func`.Default: `None`
- ****kwargs** – Keyword-arguments that are forwarded to the function given in `func`. Will be merged with `arg_dict`

Returns Array with all evaluations in Time-space.

Return type `numpy.ndarray`

Raises

- `ValueError` – If *func* is not callable.
- `ValueError` – If *time* is not positive.
- `ValueError` – If *bound* is not positive.
- `ValueError` – If *bound* is not even.

References

Notes

The parameter `time` needs to be strictly positiv.

The algorithm gets unstable for `bound` values above 20.

Examples

```
>>> f = lambda x: x**-1
>>> stehfest(f, [1,10,100])
array([ 1.,   1.,   1.])
```

anaflow.tools.mean

Anaflow subpackage providing several mean calculating routines.

The following functions are provided

annular_fmean(func, val_arr, f_def, f_inv[, . . .])	Calculating the annular generalized f-mean.
annular_amean(func, val_arr[, ann_dim, arg_dict])	Calculating the annular arithmetic mean.
annular_gmean(func, val_arr[, ann_dim, arg_dict])	Calculating the annular geometric mean.
annular_hmean(func, val_arr[, ann_dim, arg_dict])	Calculating the annular harmonic mean.
annular_pmean(func, val_arr[, p, ann_dim, . . .])	Calculating the annular p-mean.

annular_fmean (*func*, *val_arr*, *f_def*, *f_inv*, *ann_dim=2*, *arg_dict=None*, ***kwargs*)

Calculating the annular generalized f-mean.

Calculating the annular generalized f-mean of a radial symmetric function for given consecutive radii defining annuli by the following formula

$$\varphi_i = f^{-1}\left(\frac{d}{r_{i+1}^d - r_i^d}\int\limits_{r_i}^{r_{i+1}} r^{d-1}\cdot f(\varphi(r))\,dr\right)$$

Parameters

- **func** (`callable`) – Function that should be used (φ in the formula). The first argument needs to be the radial variable: `func(r, **kwargs)`
- **val_arr** (`numpy.ndarray`) – Radii defining the annuli.
- **ann_dim** (`float`, optional) – The dimension of the annuli. Default: `2.0`
- **f_def** (`callable`) – Function defining the f-mean.
- **f_inv** (`callable`) – Inverse of the function defining the f-mean.
- **arg_dict** (`dict` or `None`, optional) – Keyword-arguments given as a dictionary that are forwarded to the function given in `func`. Will be merged with `**kwargs`. This is designed for overlapping keywords in `annular_fmean` and `func`. Default: `None`
- ****kwargs** – Keyword-arguments that are forwarded to the function given in `func`. Will be merged with `arg_dict`

Returns Array with all calculated arithmetic means

Return type `numpy.ndarray`

Raises

- `ValueError` – If *func* is not callable.
- `ValueError` – If *f_def* is not callable.
- `ValueError` – If *f_inv* is not callable.
- `ValueError` – If `val_arr` has less than 2 values.
- `ValueError` – If `val_arr` is not sorted in incresing order.

Notes

If the last value in val_arr is "inf", the given function should provide a value for "inf" as input: `func(float("inf"))`

annular_amean (*func*, *val_arr*, *ann_dim=2*, *arg_dict=None*, ***kwargs*)
Calculating the annular arithmetic mean.

Calculating the annular arithmetic mean of a radial symmetric function for given consecutive radii defining annuli by the following formula

$$\varphi_i = \frac{d}{r_{i+1}^d - r_i^d} \int\limits_{r_i}^{r_{i+1}} r^{d-1} \cdot \varphi(r)\, dr$$

Parameters

- **func** (callable) – Function that should be used (φ in the formula). The first argument needs to be the radial variable: `func(r, **kwargs)`
- **val_arr** (numpy.ndarray) – Radii defining the annuli.
- **ann_dim** (float, optional) – The dimension of the annuli. Default: `2.0`
- **arg_dict** (dict or None, optional) – Keyword-arguments given as a dictionary that are forwarded to the function given in `func`. Will be merged with `**kwargs`. This is designed for overlapping keywords in `annular_amean` and `func`. Default: `None`
- ****kwargs** – Keyword-arguments that are forwarded to the function given in `func`. Will be merged with `arg_dict`

Returns Array with all calculated arithmetic means

Return type numpy.ndarray

Raises

- ValueError – If *func* is not callable.
- ValueError – If `val_arr` has less than 2 values.
- ValueError – If `val_arr` is not sorted in incresing order.

Notes

If the last value in val_arr is "inf", the given function should provide a value for "inf" as input: `func(float("inf"))`

annular_gmean (*func*, *val_arr*, *ann_dim=2*, *arg_dict=None*, ***kwargs*)
Calculating the annular geometric mean.

Calculating the annular geometric mean of a radial symmetric function for given consecutive radii defining annuli by the following formula

$$\varphi_i = \exp\left(\frac{d}{r_{i+1}^d - r_i^d} \int\limits_{r_i}^{r_{i+1}} r^{d-1} \cdot \ln(\varphi(r))\, dr \right)$$

Parameters

- **func** (callable) – Function that should be used (φ in the formula). The first argument needs to be the radial variable: `func(r, **kwargs)`
- **val_arr** (numpy.ndarray) – Radii defining the annuli.
- **ann_dim** (float, optional) – The dimension of the annuli. Default: `2.0`
- **arg_dict** (dict or None, optional) – Keyword-arguments given as a dictionary that are forwarded to the function given in `func`. Will be merged with `**kwargs`. This is designed for overlapping keywords in `annular_gmean` and `func`. Default: `None`
- ****kwargs** – Keyword-arguments that are forwarded to the function given in `func`. Will be merged with `arg_dict`

Returns Array with all calculated geometric means

Return type `numpy.ndarray`

Raises

- `ValueError` – If *func* is not callable.
- `ValueError` – If `val_arr` has less than 2 values.
- `ValueError` – If `val_arr` is not sorted in incresing order.

Notes

If the last value in val_arr is "inf", the given function should provide a value for "inf" as input: `func(float("inf"))`

Examples

```
>>> f = lambda x: x**2
>>> annular_gmean(f, [1,2,3])
array([ 2.33588885,  6.33423311])
```

annular_hmean (*func*, *val_arr*, *ann_dim=2*, *arg_dict=None*, ***kwargs*)

Calculating the annular harmonic mean.

Calculating the annular harmonic mean of a radial symmetric function for given consecutive radii defining annuli by the following formula

$$\varphi_i = \left(\frac{d}{r_{i+1}^d - r_i^d} \int\limits_{r_i}^{r_{i+1}} r^{d-1} \cdot \varphi(r)^{-1} \, dr \right)^{-1}$$

Parameters

- **func** (`callable`) – Function that should be used (φ in the formula). The first argument needs to be the radial variable: `func(r, **kwargs)`
- **val_arr** (`numpy.ndarray`) – Radii defining the annuli.
- **ann_dim** (`float`, optional) – The dimension of the annuli. Default: `2.0`
- **arg_dict** (`dict` or `None`, optional) – Keyword-arguments given as a dictionary that are forwarded to the function given in `func`. Will be merged with `**kwargs`. This is designed for overlapping keywords in `annular_hmean` and `func`. Default: `None`
- ****kwargs** – Keyword-arguments that are forwarded to the function given in `func`. Will be merged with `arg_dict`

Returns Array with all calculated geometric means

Return type `numpy.ndarray`

Raises

- `ValueError` – If *func* is not callable.
- `ValueError` – If `val_arr` has less than 2 values.
- `ValueError` – If `val_arr` is not sorted in incresing order.

Notes

If the last value in val_arr is "inf", the given function should provide a value for "inf" as input: `func(float("inf"))`

annular_pmean (*func*, *val_arr*, *p=2.0*, *ann_dim=2*, *arg_dict=None*, ***kwargs*)
Calculating the annular p-mean.

Calculating the annular p-mean of a radial symmetric function for given consecutive radii defining annuli by the following formula

$$\varphi_i = \left(\frac{d}{r_{i+1}^d - r_i^d} \int\limits_{r_i}^{r_{i+1}} r^{d-1} \cdot \varphi(r)^p \, dr\right)^{\frac{1}{p}}$$

Parameters

- **func** (`callable`) – Function that should be used (φ in the formula). The first argument needs to be the radial variable: `func(r, **kwargs)`
- **val_arr** (`numpy.ndarray`) – Radii defining the annuli.
- **p** (`float`, optional) – The potency defining the p-mean. Default: `2.0`
- **ann_dim** (`float`, optional) – The dimension of the annuli. Default: `2.0`
- **arg_dict** (`dict` or `None`, optional) – Keyword-arguments given as a dictionary that are forwarded to the function given in `func`. Will be merged with `**kwargs`. This is designed for overlapping keywords in `annular_pmean` and `func`. Default: `None`
- ****kwargs** – Keyword-arguments that are forwarded to the function given in `func`. Will be merged with `arg_dict`

Returns Array with all calculated p-means

Return type `numpy.ndarray`

Raises

- `ValueError` – If *func* is not callable.
- `ValueError` – If `val_arr` has less than 2 values.
- `ValueError` – If `val_arr` is not sorted in incresing order.

Notes

If the last value in val_arr is "inf", the given function should provide a value for "inf" as input: `func(float("inf"))`

anaflow.tools.special

Anaflow subpackage providing special functions.

The following functions are provided

Shaper([time, rad, struc_grid])	A class to reshape radius-time input-output in a good way.
step_f(rad, r_part, f_part)	Callalbe step function.
sph_surf(dim)	Surface of the d-dimensional sphere.
specialrange(val_min, val_max, steps[, typ])	Calculation of special point ranges.
specialrange_cut(val_min, val_max, steps[, ...])	Calculation of special point ranges.
aniso(e)	The anisotropy function.
well_solution(time, rad, storage, transmissivity)	The classical Theis solution.
grf_solution(time, rad, storage, conductivity)	The general radial flow (GRF) model for a pumping test.
inc_gamma(s, x)	The (upper) incomplete gamma function.
tpl_hyp(rad, dim, hurst, corr, prop)	Hyp_2F1 for the TPL CG model.
neuman2004_trans(rad, trans_gmean, var, ...)	The apparent transmissivity from Neuman 2004.

class Shaper (*time=0, rad=0, struc_grid=True*)

Bases: `object`

A class to reshape radius-time input-output in a good way.

Parameters

- **time** (`numpy.ndarray` or `float`, optional) – Array with all time-points where the function should be evaluated. Default: 0
- **rad** (`numpy.ndarray` or `float`, optional) – Array with all radii where the function should be evaluated. Default: 0
- **struc_grid** (`bool`, optional) – If this is set to `False`, the *rad* and *time* array will be merged and interpreted as single, r-t points. In this case they need to have the same shapes. Otherwise a structured t-r grid is created. Default: `True`

Methods

reshape(result)	Reshape a time-rad result according to the input shape.

reshape (*result*)

Reshape a time-rad result according to the input shape.

step_f (*rad, r_part, f_part*)

Callalbe step function.

sph_surf (*dim*)

Surface of the d-dimensional sphere.

specialrange (*val_min, val_max, steps, typ='exp'*)

Calculation of special point ranges.

Parameters

- **val_min** (`float`) – Starting value.
- **val_max** (`float`) – Ending value
- **steps** (`int`) – Number of steps.

- **typ** (`str` or `float`, optional) – Setting the kind of range-distribution. One can choose between
  - `"exp"`: for exponential behavior
  - `"log"`: for logarithmic behavior
  - `"geo"`: for geometric behavior
  - `"lin"`: for linear behavior
  - `"quad"`: for quadratic behavior
  - `"cub"`: for cubic behavior
  - `float`: here you can specifi any exponent ("quad" would be equivalent to 2)

  Default: `"exp"`

Returns Array containing the special range

Return type `numpy.ndarray`

Examples

```
>>> specialrange(1,10,4)
array([ 1.        ,  2.53034834,  5.23167968, 10.        ])
```

specialrange_cut (*val_min*, *val_max*, *steps*, *val_cut=None*, *typ='exp'*)

Calculation of special point ranges.

Calculation of special point ranges with a cut-off value.

Parameters

- **val_min** (`float`) – Starting value.
- **val_max** (`float`) – Ending value
- **steps** (`int`) – Number of steps.
- **val_cut** (`float`, optional) – Cutting value, if val_max is bigger than this value, the last interval will be between val_cut and val_max. Default: 100.0
- **typ** (`str` or `float`, optional) – Setting the kind of range-distribution. One can choose between
  - `"exp"`: for exponential behavior
  - `"log"`: for logarithmic behavior
  - `"geo"`: for geometric behavior
  - `"lin"`: for linear behavior
  - `"quad"`: for quadratic behavior
  - `"cub"`: for cubic behavior
  - `float`: here you can specifi any exponent ("quad" would be equivalent to 2)

  Default: `"exp"`

Returns Array containing the special range

Return type `numpy.ndarray`

Examples

```
>>> specialrange_cut(1,10,4)
array([ 1.        ,  2.53034834,  5.23167968, 10.        ])
```

aniso(*e*)

The anisotropy function.

Known from ''Dagan (1989)"[R2].

Parameters **e** (float) – Anisotropy-ratio of the vertical and horizontal corralation-lengths

Returns **aniso** – Value of the anisotropy function for the given value.

Return type float

Raises ValueError – If the Anisotropy-ratio e is not within 0 and 1.

References

Examples

```
>>> aniso(0.5)
0.2363998587187151
```

well_solution(*time*, *rad*, *storage*, *transmissivity*, *rate=-0.0001*, *h_bound=0.0*, *struc_grid=True*)

The classical Theis solution.

The classical Theis solution for transient flow under a pumping condition in a confined and homogeneous aquifer.

This solution was presented in ''Theis 1935"[R9].

Parameters

- **time** (numpy.ndarray) – Array with all time-points where the function should be evaluated.
- **rad** (numpy.ndarray) – Array with all radii where the function should be evaluated.
- **storage** (float) – Storage of the aquifer.
- **transmissivity** (float) – Transmissivity of the aquifer.
- **rate** (float, optional) – Pumpingrate at the well. Default: -1e-4
- **h_bound** (float, optional) – Reference head at the outer boundary at infinity. Default: 0.0
- **struc_grid** (bool, optional) – If this is set to "False", the "rad" and "time" array will be merged and interpreted as single, r-t points. In this case they need to have the same shapes. Otherwise a structured r-t grid is created. Default: True

Returns **head** – Array with all heads at the given radii and time-points.

Return type numpy.ndarray

Raises

- ValueError – If rad is not positiv.
- ValueError – If time is negative.
- ValueError – If shape of rad and time differ in case of struc_grid is True.
- ValueError – If transmissivity is not positiv.
- ValueError – If storage is not positiv.

References

Notes

The parameters `rad`, `T` and `S` will be checked for positivity. If you want to use cartesian coordiantes, just use the formula `r = sqrt(x**2 + y**2)`

Examples

```
>>> well_solution([10,100], [1,2,3], 0.001, 0.001, -0.001)
array([[-0.24959541, -0.14506368, -0.08971485],
       [-0.43105106, -0.32132823, -0.25778313]])
```

grf_solution(*time, rad, storage, conductivity, dim=2, lat_ext=1.0, rate=-0.0001, h_bound=0.0, struc_grid=True*)

The general radial flow (GRF) model for a pumping test.

Parameters

- **time** (`numpy.ndarray`) – Array with all time-points where the function should be evaluated.
- **rad** (`numpy.ndarray`) – Array with all radii where the function should be evaluated.
- **storage** (`float`) – Storage coefficient of the aquifer.
- **conductivity** (`float`) – Conductivity of the aquifer.
- **dim** (`float`, optional) – Fractional dimension of the aquifer. Default: `2.0`
- **lat_ext** (`float`, optional) – Lateral extend of the aquifer. Default: `1.0`
- **rate** (`float`, optional) – Pumpingrate at the well. Default: -1e-4
- **h_bound** (`float`, optional) – Reference head at the outer boundary at infinity. Default: `0.0`
- **struc_grid** (`bool`, optional) – If this is set to “False”, the “rad” and “time” array will be merged and interpreted as single, r-t points. In this case they need to have the same shapes. Otherwise a structured r-t grid is created. Default: `True`

Returns **head** – Array with all heads at the given radii and time-points.

Return type `numpy.ndarray`

Raises

- `ValueError` – If `rad` is not positiv.
- `ValueError` – If `time` is negative.
- `ValueError` – If shape of `rad` and `time` differ in case of `struc_grid` is `True`.
- `ValueError` – If `conductivity` is not positiv.
- `ValueError` – If `storage` is not positiv.

inc_gamma(*s, x*)

The (upper) incomplete gamma function.

Given by: $\Gamma(s,x) = \int_x^\infty t^{s-1}\,e^{-t}\,\mathrm{d}t$

Parameters

- **s** (`float`) – exponent in the integral
- **x** (`numpy.ndarray`) – input values

tpl_hyp (*rad*, *dim*, *hurst*, *corr*, *prop*)
Hyp_2F1 for the TPL CG model.

neuman2004_trans (*rad*, *trans_gmean*, *var*, *len_scale*)
The apparent transmissivity from Neuman 2004.

Parameters

- **rad** (numpy.ndarray) – Array with all radii where the function should be evaluated
- **trans_gmean** (float) – Geometric-mean transmissivity.
- **var** (float) – Variance of log-transmissivity.
- **len_scale** (float) – Correlation-length of log-transmissivity.

anaflow.tools.coarse_graining

Anaflow subpackage providing helper functions related to coarse graining.

The following functions are provided

T_CG(rad, trans_gmean, var, len_scale[, ...])	The coarse-graining Transmissivity.
T_CG_inverse(T, trans_gmean, var, len_scale)	The inverse coarse-graining Transmissivity.
T_CG_error(err, trans_gmean, var, len_scale)	Calculating the radial-point for given error.
K_CG(rad, cond_gmean, var, len_scale, anis)	The coarse-graining conductivity.
K_CG_inverse(K, cond_gmean, var, len_scale, anis)	The inverse coarse-graining conductivity.
K_CG_error(err, cond_gmean, var, len_scale, anis)	Calculating the radial-point for given error.
TPL_CG(rad, cond_gmean, len_scale, hurst[, ...])	The gaussian truncated power-law coarse-graining conductivity.
TPL_CG_error(err, cond_gmean, len_scale, hurst)	Calculating the radial-point for given error.

T_CG(*rad*, *trans_gmean*, *var*, *len_scale*, *T_well=None*, *prop=1.6*)

The coarse-graining Transmissivity.

This solution was presented in ''Schneider & Attinger 2008''[R3].

This functions gives an effective transmissivity for 2D pumpingtests in heterogenous aquifers, where the transmissivity is following a log-normal distribution and a gaussian correlation function.

Parameters

- **rad** (`numpy.ndarray`) – Array with all radii where the function should be evaluated
- **trans_gmean** (`float`) – Geometric-mean transmissivity.
- **var** (`float`) – Variance of log-transmissivity.
- **len_scale** (`float`) – Correlation-length of log-transmissivity.
- **T_well** (`float`, optional) – Explicit transmissivity value at the well. Harmonic mean by default.
- **prop** (`float`, optional) – Proportionality factor used within the upscaling procedure. Default: `1.6`

Returns **T_CG** – Array containing the effective transmissivity values.

Return type `numpy.ndarray`

References

Examples

```
>>> T_CG([1,2,3], 0.001, 1, 10, 2)
array([0.00061831, 0.00064984, 0.00069236])
```

T_CG_inverse(*T*, *trans_gmean*, *var*, *len_scale*, *T_well=None*, *prop=1.6*)

The inverse coarse-graining Transmissivity.

See: *T_CG()*

Parameters

- **T** (`numpy.ndarray`) – Array with all transmissivity values where the function should be evaluated
- **trans_gmean** (`float`) – Geometric-mean transmissivity.
- **var** (`float`) – Variance of log-transmissivity.

- **len_scale** (float) – Correlation-length of log-transmissivity.
- **T_well** (float, optional) – Explicit transmissivity value at the well. Harmonic mean by default.
- **prop** (float, optional) – Proportionality factor used within the upscaling procedure. Default: 1.6

Returns **rad** – Array containing the radii belonging to the given transmissivity values

Return type numpy.ndarray

Examples

```
>>> T_CG_inverse([7e-4,8e-4,9e-4], 0.001, 1, 10, 2)
array([3.16952925, 5.56935826, 9.67679026])
```

T_CG_error (*err*, *trans_gmean*, *var*, *len_scale*, *T_well=None*, *prop=1.6*)

Calculating the radial-point for given error.

Calculating the radial-point where the relative error of the farfield value is less than the given tollerance. See: *T_CG()*

Parameters

- **err** (float) – Given relative error for the farfield transmissivity
- **trans_gmean** (float) – Geometric-mean transmissivity.
- **var** (float) – Variance of log-transmissivity.
- **len_scale** (float) – Correlation-length of log-transmissivity.
- **T_well** (float, optional) – Explicit transmissivity value at the well. Harmonic mean by default.
- **prop** (float, optional) – Proportionality factor used within the upscaling procedure. Default: 1.6

Returns **rad** – Radial point, where the relative error is less than the given one.

Return type float

Examples

```
>>> T_CG_error(0.01, 0.001, 1, 10, 2)
34.91045016779039
```

K_CG (*rad*, *cond_gmean*, *var*, *len_scale*, *anis*, *K_well='KH'*, *prop=1.6*)

The coarse-graining conductivity.

This solution was presented in ''Zech 2013''[R8].

This functions gives an effective conductivity for 3D pumpingtests in heterogenous aquifers, where the conductivity is following a log-normal distribution and a gaussian correlation function and taking vertical anisotropy into account.

Parameters

- **rad** (numpy.ndarray) – Array with all radii where the function should be evaluated
- **cond_gmean** (float) – Geometric-mean conductivity.
- **var** (float) – Variance of the log-conductivity.
- **len_scale** (float) – Corralation-length of log-conductivity.

- **anis** (float) – Anisotropy-ratio of the vertical and horizontal corralation-lengths.
- **K_well** (*string/float, optional*) – Explicit conductivity value at the well. One can choose between the harmonic mean ("KH"), the arithmetic mean ("KA") or an arbitrary float value. Default: "KH"
- **prop** (float, optional) – Proportionality factor used within the upscaling procedure. Default: 1.6

Returns **K_CG** – Array containing the effective conductivity values.

Return type numpy.ndarray

References

Examples

```
>>> K_CG([1,2,3], 0.001, 1, 10, 1, 2)
array([0.00063008, 0.00069285, 0.00077595])
```

K_CG_inverse (*K, cond_gmean, var, len_scale, anis, K_well='KH', prop=1.6*)

The inverse coarse-graining conductivity.

See: *K_CG()*

Parameters

- **K** (numpy.ndarray) – Array with all conductivity values where the function should be evaluated
- **cond_gmean** (float) – Geometric-mean conductivity.
- **var** (float) – Variance of the log-conductivity.
- **len_scale** (float) – Corralation-length of log-conductivity.
- **anis** (float) – Anisotropy-ratio of the vertical and horizontal corralation-lengths.
- **K_well** (*string/float, optional*) – Explicit conductivity value at the well. One can choose between the harmonic mean ("KH"), the arithmetic mean ("KA") or an arbitrary float value. Default: "KH"
- **prop** (float, optional) – Proportionality factor used within the upscaling procedure. Default: 1.6

Returns **rad** – Array containing the radii belonging to the given conductivity values

Return type numpy.ndarray

Examples

```
>>> K_CG_inverse([7e-4,8e-4,9e-4], 0.001, 1, 10, 1, 2)
array([2.09236867, 3.27914996, 4.52143956])
```

K_CG_error (*err, cond_gmean, var, len_scale, anis, K_well='KH', prop=1.6*)

Calculating the radial-point for given error.

Calculating the radial-point where the relative error of the farfield value is less than the given tollerance. See: *K_CG()*

Parameters

- **err** (float) – Given relative error for the farfield conductivity
- **cond_gmean** (float) – Geometric-mean conductivity.

- **var** (float) – Variance of the log-conductivity.
- **len_scale** (float) – Corralation-length of log-conductivity.
- **anis** (float) – Anisotropy-ratio of the vertical and horizontal corralation-lengths.
- **K_well** (*string/float, optional*) – Explicit conductivity value at the well. One can choose between the harmonic mean ("KH"), the arithmetic mean ("KA") or an arbitrary float value. Default: "KH"
- **prop** (float, optional) – Proportionality factor used within the upscaling procedure. Default: 1.6

Returns **rad** – Radial point, where the relative error is less than the given one.

Return type float

Examples

```
>>> K_CG_error(0.01, 0.001, 1, 10, 1, 2)
19.612796453639845
```

TPL_CG (*rad, cond_gmean, len_scale, hurst, var=None, c=1.0, anis=1, dim=2.0, K_well='KH', prop=1.6*)

The gaussian truncated power-law coarse-graining conductivity.

Parameters

- **rad** (numpy.ndarray) – Array with all radii where the function should be evaluated
- **cond_gmean** (float) – Geometric-mean conductivity
- **len_scale** (float) – upper bound of the corralation-length of conductivity-distribution
- **hurst** (float) – Hurst coefficient of the TPL model. Should be in (0, 1).
- **var** (float or None, optional) – Variance of log-conductivity If given, c will be calculated accordingly. Default: None
- **c** (float, optional) – Intensity of variation in the TPL model. Is overwritten if var is given. Default: 1.0
- **anis** (float, optional) – Anisotropy-ratio of the vertical and horizontal corralation-lengths. This is only applied in 3 dimensions. Default: 1.0
- **dim** (float, optional) – Dimension of space. Default: 2.0
- **K_well** (str or float, optional) – Explicit conductivity value at the well. One can choose between the harmonic mean ("KH"), the arithmetic mean ("KA") or an arbitrary float value. Default: "KH"
- **prop** (float, optional) – Proportionality factor used within the upscaling procedure. Default: 1.6

Returns **TPL_CG** – Array containing the effective conductivity values.

Return type numpy.ndarray

TPL_CG_error (*err, cond_gmean, len_scale, hurst, var=None, c=1.0, anis=1, dim=2.0, K_well='KH', prop=1.6*)

Calculating the radial-point for given error.

Calculating the radial-point where the relative error of the farfield value is less than the given tollerance. See: *TPL_CG()*

Parameters

- **err** (float) – Given relative error for the farfield conductivity

- **cond_gmean** (float) – Geometric-mean conductivity
- **len_scale** (float) – upper bound of the corralation-length of conductivity-distribution
- **hurst** (float) – Hurst coefficient of the TPL model. Should be in (0, 1).
- **var** (float or None, optional) – Variance of log-conductivity If given, c will be calculated accordingly. Default: None
- **c** (float, optional) – Intensity of variation in the TPL model. Is overwritten if var is given. Default: 1.0
- **anis** (float, optional) – Anisotropy-ratio of the vertical and horizontal corralation-lengths. This is only applied in 3 dimensions. Default: 1.0
- **dim** (float, optional) – Dimension of space. Default: 2.0
- **K_well** (str or float, optional) – Explicit conductivity value at the well. One can choose between the harmonic mean ("KH"), the arithmetic mean ("KA") or an arbitrary float value. Default: "KH"
- **prop** (float, optional) – Proportionality factor used within the upscaling procedure. Default: 1.6

Returns **rad** – Radial point, where the relative error is less than the given one.

Return type float

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