GeoStatTools Documentation

Release 1.3.1

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CONTENTS

CHAPTER 1

GSTOOLS QUICKSTART

Get in Touch!

1.1 Purpose

GeoStatTools provides geostatistical tools for various purposes:

- random field generation
- simple, ordinary, universal and external drift kriging
- conditioned field generation
- incompressible random vector field generation
- (automated) variogram estimation and fitting
- directional variogram estimation and modelling
- data normalization and transformation
- many readily provided and even user-defined covariance models
- metric spatio-temporal modelling
- plotting and exporting routines

1.2 Installation

conda

GSTools can be installed via conda on Linux, Mac, and Windows. Install the package by typing the following command in a command terminal:

```
conda install gstools
```

In case conda forge is not set up for your system yet, see the easy to follow instructions on conda forge. Using conda, the parallelized version of GSTools should be installed.

pip

GSTools can be installed via pip on Linux, Mac, and Windows. On Windows you can install WinPython to get Python and pip running. Install the package by typing the following into command in a command terminal:

```
pip install gstools
```

To get the latest development version you can install it directly from GitHub:

```
pip install git+git://github.com/GeoStat-Framework/GSTools.git@develop
```

If something went wrong during installation, try the `-I` flag from pip.

To enable the OpenMP support, you have to provide a C compiler and OpenMP. Parallel support is controlled by an environment variable `GSTOOLS_BUILD_PARALLEL`, that can be `0` or `1` (interpreted as `0` if not present). GSTools then needs to be installed from source:

```
export GSTOOLS_BUILD_PARALLEL=1
pip install --no-binary=gstools gstools
```

Note, that the `--no-binary=gstools` option forces pip to not use a wheel for GSTools.

For the development version, you can do almost the same:

```
export GSTOOLS_BUILD_PARALLEL=1
pip install git+git://github.com/GeoStat-Framework/GSTools.git@develop
```

1.3 Citation

At the moment you can cite the Zenodo code publication of GSTools:

Sebastian Müller & Lennart Schüler. GeoStat-Framework/GSTools. Zenodo.
https://doi.org/10.5281/zenodo.1313628

If you want to cite a specific version, have a look at the Zenodo site.

A publication for the GeoStat-Framework is in preperation.

1.4 Tutorials and Examples

The documentation also includes some tutorials, showing the most important use cases of GSTools, which are

- Random Field Generation
- The Covariance Model
- Variogram Estimation
- Random Vector Field Generation
- Kriging
- Conditioned random field generation
- Field transformations
- Geographic Coordinates
- Spatio-Temporal Modelling
- Normalizing Data
- Miscellaneous examples

1.5 Spatial Random Field Generation

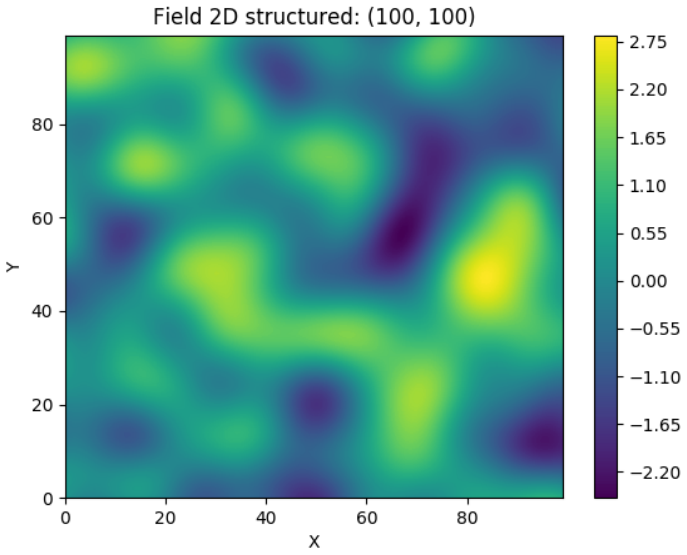
The core of this library is the generation of spatial random fields. These fields are generated using the randomisation method, described by Heße et al. 2014.

Examples

Gaussian Covariance Model

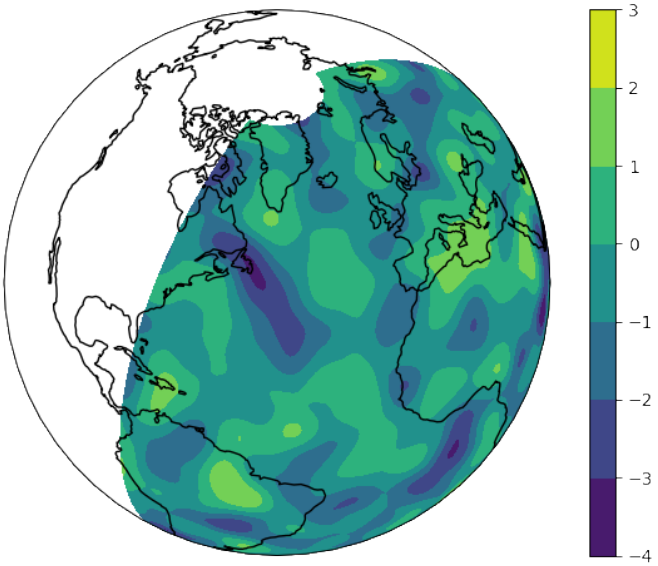
This is an example of how to generate a 2 dimensional spatial random field (*SRF*) with a *Gaussian* covariance model.

```
import gstools as gs
# structured field with a size 100x100 and a grid-size of 1x1
x = y = range(100)
model = gs.Gaussian(dim=2, var=1, len_scale=10)
srf = gs.SRF(model)
srf((x, y), mesh_type='structured')
srf.plot()
```



GSTools also provides support for geographic coordinates. This works perfectly well with cartopy.

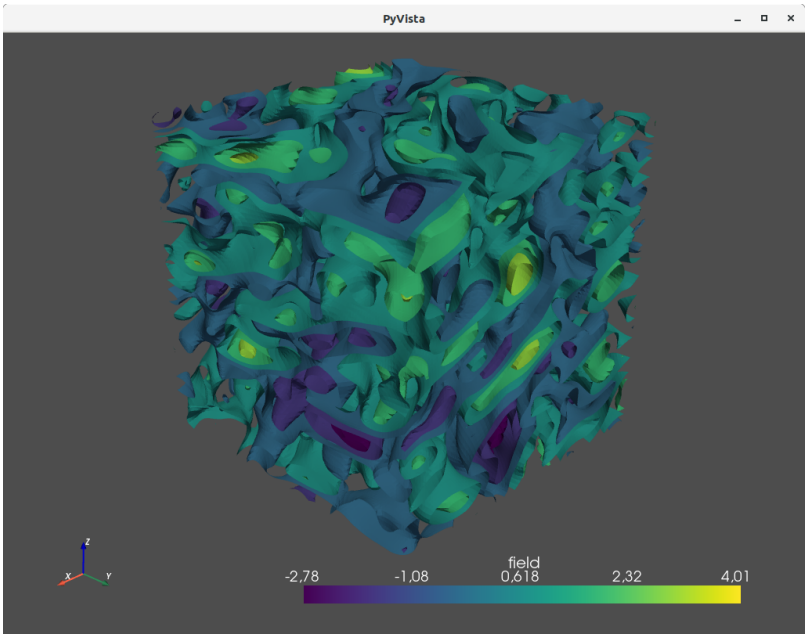
```
import matplotlib.pyplot as plt
import cartopy.crs as ccrs
import gstools as gs
# define a structured field by latitude and longitude
lat = lon = range(-80, 81)
model = gs.Gaussian(latlon=True, len_scale=777, rescale=gs.EARTH_RADIUS)
srf = gs.SRF(model, seed=12345)
field = srf.structured((lat, lon))
# Orthographic plotting with cartopy
ax = plt.subplot(projection=ccrs.Orthographic(-45, 45))
cont = ax.contourf(lon, lat, field, transform=ccrs.PlateCarree())
ax.coastlines()
ax.set_global()
plt.colorbar(cont)
```



A similar example but for a three dimensional field is exported to a VTK file, which can be visualized with ParaView or PyVista in Python:

```
import gstools as gs
# structured field with a size 100x100x100 and a grid-size of 1x1x1
x = y = z = range(100)
model = gs.Gaussian(dim=3, len_scale=[16, 8, 4], angles=(0.8, 0.4, 0.2))
srf = gs.SRF(model)
srf((x, y, z), mesh_type='structured')
srf.vtk_export('3d_field') # Save to a VTK file for ParaView

mesh = srf.to_pyvista() # Create a PyVista mesh for plotting in Python
mesh.contour(isosurfaces=8).plot()
```



1.6 Estimating and fitting variograms

The spatial structure of a field can be analyzed with the variogram, which contains the same information as the covariance function.

All covariance models can be used to fit given variogram data by a simple interface.

Examples

This is an example of how to estimate the variogram of a 2 dimensional unstructured field and estimate the parameters of the covariance model again.

```
import numpy as np
import gstools as gs
# generate a synthetic field with an exponential model
x = np.random.RandomState(19970221).rand(1000) * 100.
y = np.random.RandomState(20011012).rand(1000) * 100.
model = gs.Exponential(dim=2, var=2, len_scale=8)
srf = gs.SRF(model, mean=0, seed=19970221)
field = srf((x, y))
# estimate the variogram of the field
bin_center, gamma = gs.vario_estimate((x, y), field)
# fit the variogram with a stable model. (no nugget fitted)
```

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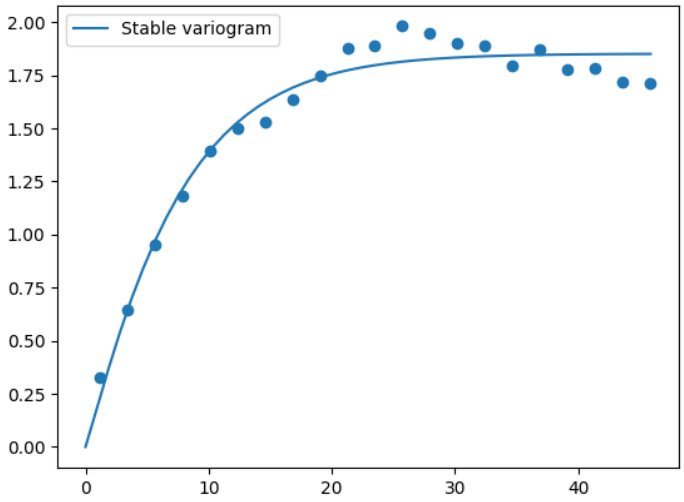
```
fit_model = gs.Stable(dim=2)
fit_model.fit_variogram(bin_center, gamma, nugget=False)
# output
ax = fit_model.plot(x_max=max(bin_center))
ax.scatter(bin_center, gamma)
print(fit_model)
```

Which gives:

```
Stable(dim=2, var=1.85, len_scale=7.42, nugget=0.0, anis=[1.0], angles=[0.0], alpha=1.
↪09)
```



1.7 Kriging and Conditioned Random Fields

An important part of geostatistics is Kriging and conditioning spatial random fields to measurements. With conditioned random fields, an ensemble of field realizations with their variability depending on the proximity of the measurements can be generated.

Example

For better visualization, we will condition a 1d field to a few "measurements", generate 100 realizations and plot them:

```
import numpy as np
import matplotlib.pyplot as plt
import gstools as gs

# conditions
cond_pos = [0.3, 1.9, 1.1, 3.3, 4.7]
cond_val = [0.47, 0.56, 0.74, 1.47, 1.74]

gridx = np.linspace(0.0, 15.0, 151)

# conditioned spatial random field class
```

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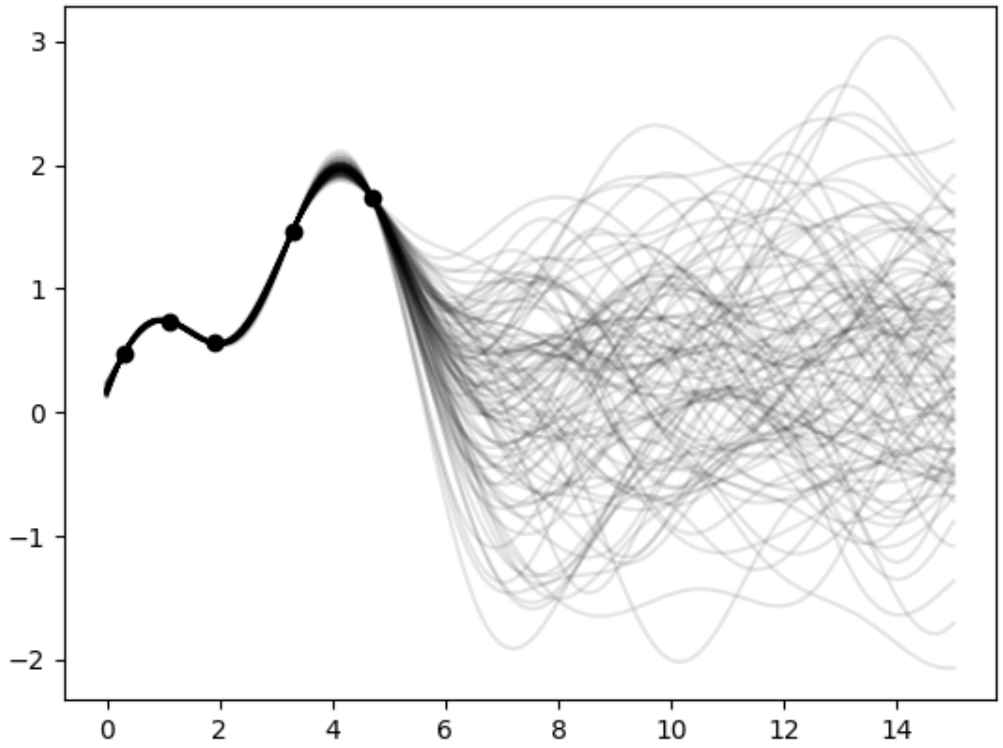
```
model = gs.Gaussian(dim=1, var=0.5, len_scale=2)
krige = gs.krige.Ordinary(model, cond_pos, cond_val)
cond_srf = gs.CondSRF(krige)

# generate the ensemble of field realizations
fields = []
for i in range(100):
    fields.append(cond_srf(gridx, seed=i))
    plt.plot(gridx, fields[i], color="k", alpha=0.1)
plt.scatter(cond_pos, cond_val, color="k")
plt.show()
```



1.8 User defined covariance models

One of the core-features of GSTools is the powerful *CovModel* class, which allows to easy define covariance models by the user.

Example

Here we re-implement the Gaussian covariance model by defining just the correlation function, which takes a non-dimensional distance `h = r/l`

```
import numpy as np
import gstools as gs
# use CovModel as the base-class
class Gau(gs.CovModel):
    def cor(self, h):
        return np.exp(-h**2)
```

And that's it! With `Gau` you now have a fully working covariance model, which you could use for field generation or variogram fitting as shown above.

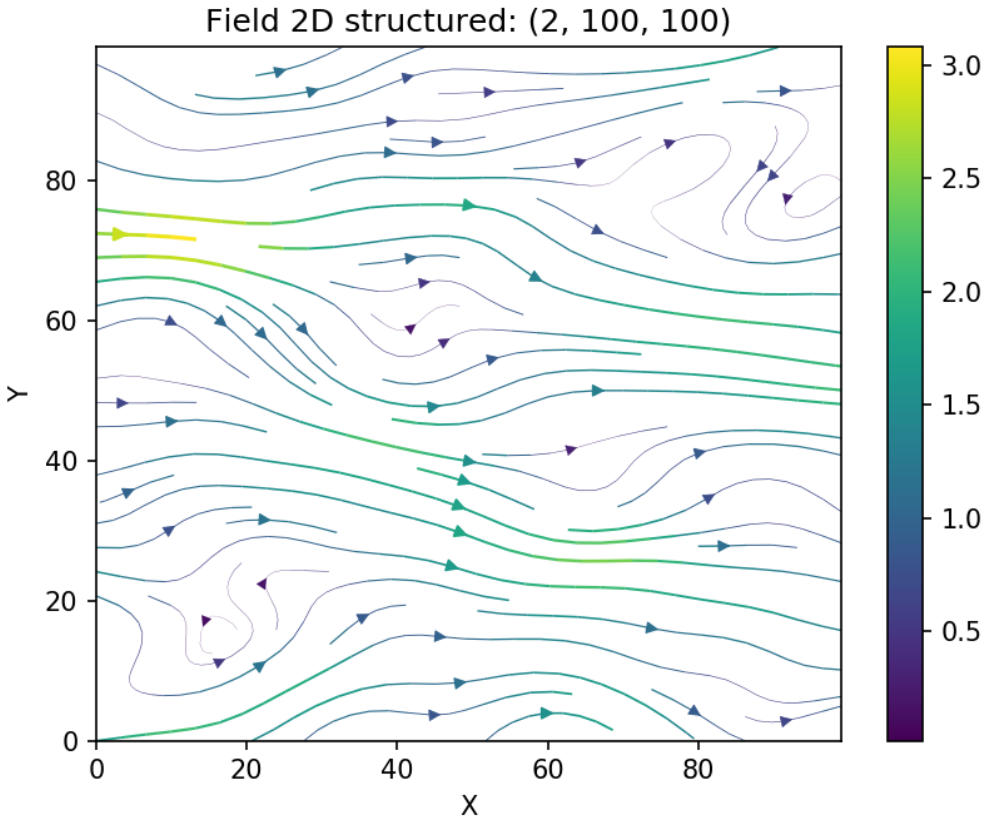
1.9 Incompressible Vector Field Generation

Using the original Kraichnan method, incompressible random spatial vector fields can be generated.

Example

```
import numpy as np
import gstools as gs
x = np.arange(100)
y = np.arange(100)
model = gs.Gaussian(dim=2, var=1, len_scale=10)
srf = gs.SRF(model, generator='VectorField', seed=19841203)
srf((x, y), mesh_type='structured')
srf.plot()
```

yielding

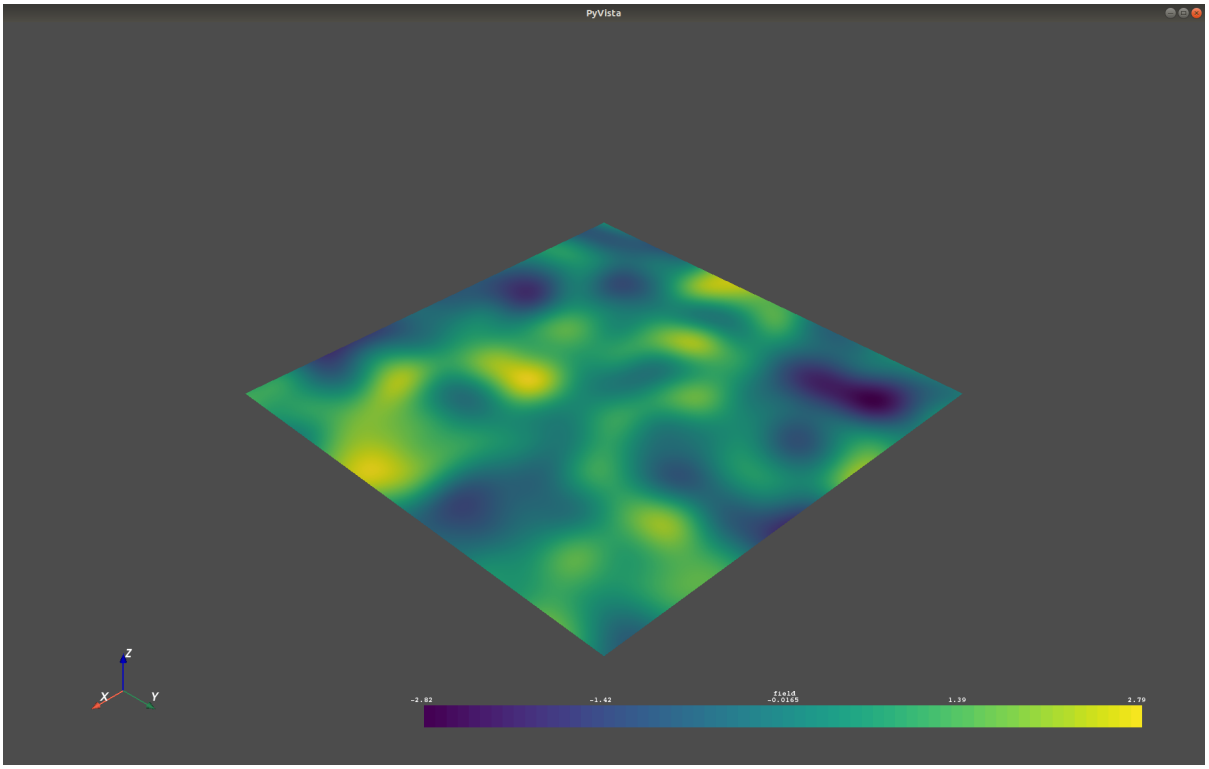


1.10 VTK/PyVista Export

After you have created a field, you may want to save it to file, so we provide a handy VTK export routine using the `vtk_export()` or you could create a VTK/PyVista dataset for use in Python with to `to_pyvista()` method:

```
import gstools as gs
x = y = range(100)
model = gs.Gaussian(dim=2, var=1, len_scale=10)
srf = gs.SRF(model)
srf((x, y), mesh_type='structured')
srf.vtk_export("field") # Saves to a VTK file
mesh = srf.to_pyvista() # Create a VTK/PyVista dataset in memory
mesh.plot()
```

Which gives a RectilinearGrid VTK file `field.vtr` or creates a PyVista mesh in memory for immediate 3D plotting in Python.



1.11 Requirements

- Numpy >= 1.14.5
- SciPy >= 1.1.0
- hankel >= 1.0.2
- emcee >= 3.0.0
- pyevtk >= 1.1.1
- meshio>=4.0.3, <5.0

Optional

- matplotlib
- pyvista

Contact

You can contact us via info@geostat-framework.org.

1.12 License

LGPLv3

CHAPTER 2

GSTOOLS TUTORIALS

In the following you will find several Tutorials on how to use GSTools to explore its whole beauty and power.

2.1 Random Field Generation

The main feature of GSTools is the spatial random field generator *SRF*, which can generate random fields following a given covariance model. The generator provides a lot of nice features, which will be explained in the following

GSTools generates spatial random fields with a given covariance model or semi-variogram. This is done by using the so-called randomization method. The spatial random field is represented by a stochastic Fourier integral and its discretised modes are evaluated at random frequencies.

GSTools supports arbitrary and non-isotropic covariance models.

Examples

A Very Simple Example

We are going to start with a very simple example of a spatial random field with an isotropic Gaussian covariance model and following parameters:

- variance $\sigma^2 = 1$
- correlation length $\lambda = 10$

First, we set things up and create the axes for the field. We are going to need the *SRF* class for the actual generation of the spatial random field. But *SRF* also needs a covariance model and we will simply take the *Gaussian* model.

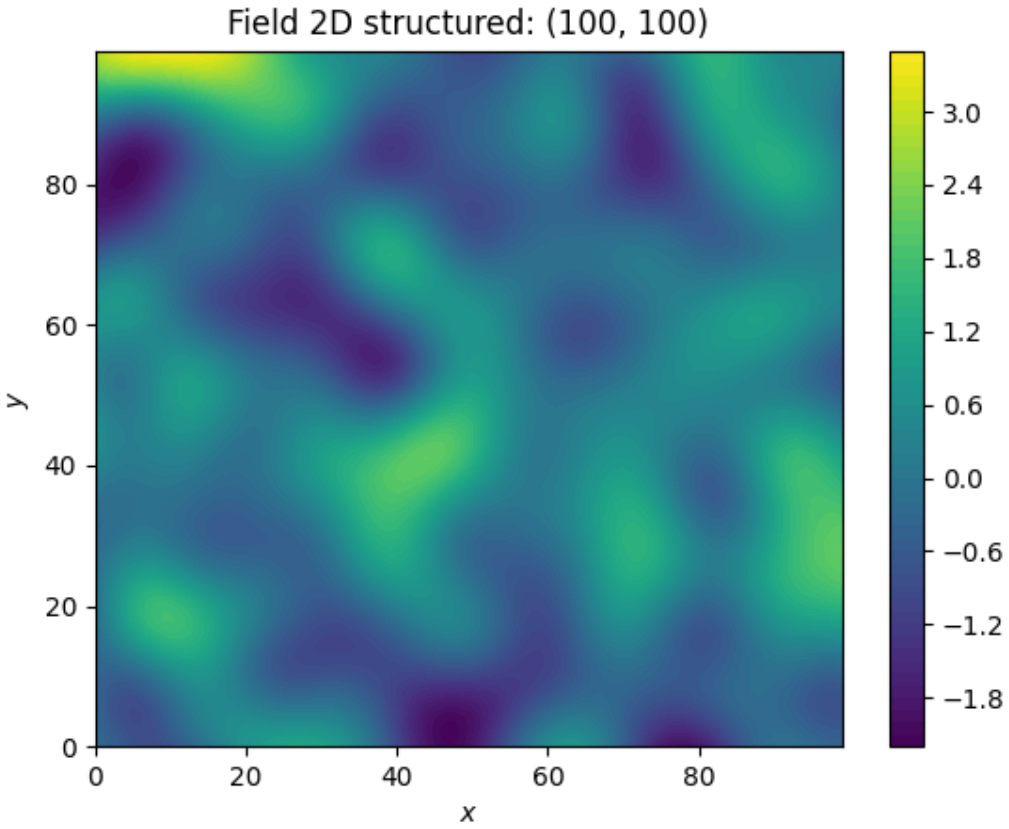
```
import gstools as gs

x = y = range(100)
```

Now we create the covariance model with the parameters σ^2 and λ and hand it over to *SRF*. By specifying a seed, we make sure to create reproducible results:

```
model = gs.Gaussian(dim=2, var=1, len_scale=10)
srf = gs.SRF(model, seed=20170519)
```

With these simple steps, everything is ready to create our first random field. We will create the field on a structured grid (as you might have guessed from the x and y), which makes it easier to plot.

```
field = srf.structured([x, y])
srf.plot()
```



Wow, that was pretty easy!

Total running time of the script: (0 minutes 0.738 seconds)

Creating an Ensemble of Fields

Creating an ensemble of random fields would also be a great idea. Let's reuse most of the previous code.

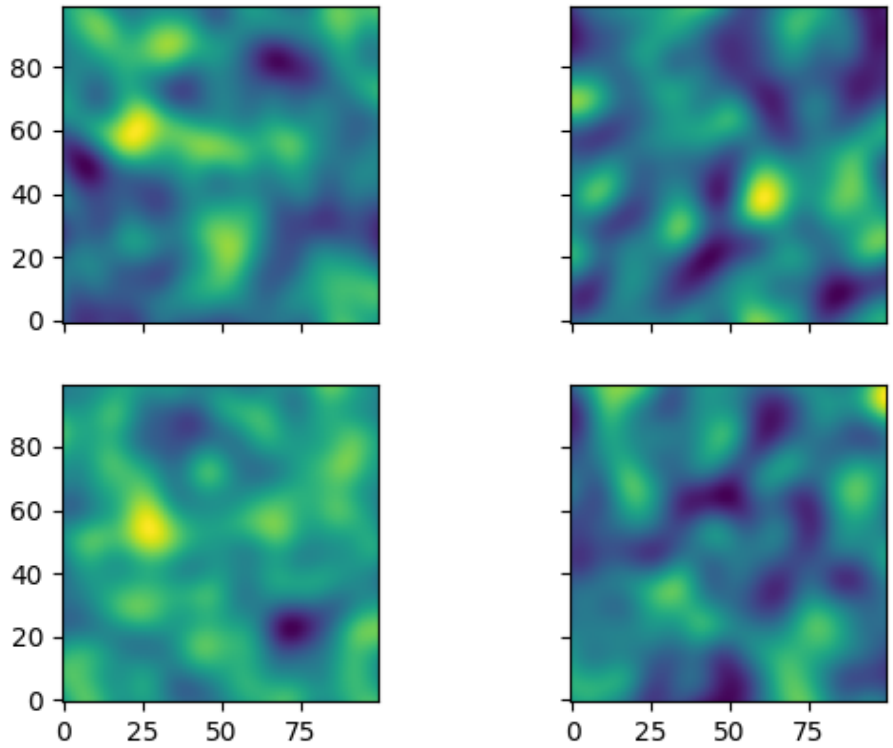
```
import numpy as np
import matplotlib.pyplot as pt
import gstools as gs

x = y = np.arange(100)

model = gs.Gaussian(dim=2, var=1, len_scale=10)
srf = gs.SRF(model)
```

This time, we did not provide a seed to *SRF*, as the seeds will used during the actual computation of the fields. We will create four ensemble members, for better visualisation and save them in a list and in a first step, we will be using the loop counter as the seeds.

```
ens_no = 4
field = []
for i in range(ens_no):
    field.append(srf.structured([x, y], seed=i))
```

Now let's have a look at the results:

```
fig, ax = pt.subplots(2, 2, sharex=True, sharey=True)
ax = ax.flatten()
for i in range(ens_no):
    ax[i].imshow(field[i].T, origin="lower")
pt.show()
```



Using better Seeds

It is not always a good idea to use incrementing seeds. Therefore GSTools provides a seed generator *MasterRNG*. The loop, in which the fields are generated would then look like

```
from gstools.random import MasterRNG

seed = MasterRNG(20170519)
for i in range(ens_no):
    field.append(srf.structured([x, y], seed=seed()))
```

Total running time of the script: (0 minutes 4.057 seconds)

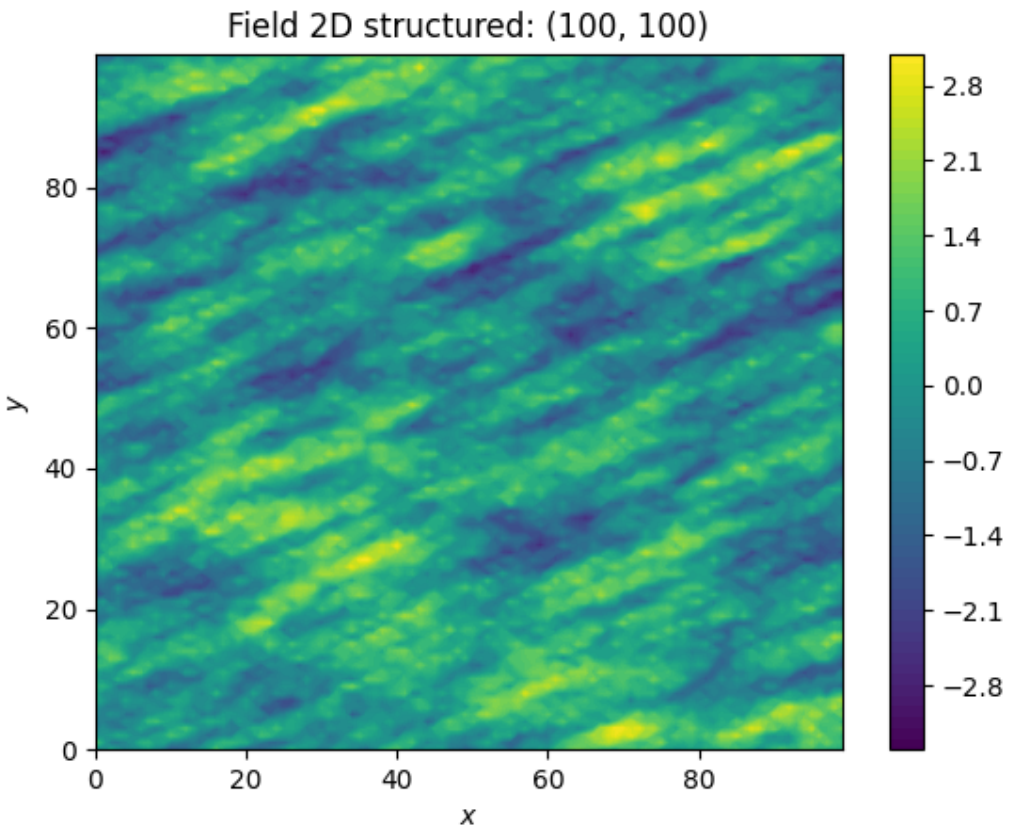
Creating Fancier Fields

Only using Gaussian covariance fields gets boring. Now we are going to create much rougher random fields by using an exponential covariance model and we are going to make them anisotropic.

The code is very similar to the previous examples, but with a different covariance model class *Exponential*. As model parameters we a using following

- variance $\sigma^2 = 1$
- correlation length $\lambda = (12, 3)^T$
- rotation angle $\theta = \pi/8$

```
import numpy as np
import gstools as gs

x = y = np.arange(100)
model = gs.Exponential(dim=2, var=1, len_scale=[12.0, 3.0], angles=np.pi / 8)
srf = gs.SRF(model, seed=20170519)
srf.structured([x, y])
srf.plot()
```



The anisotropy ratio could also have been set with

```
model = gs.Exponential(dim=2, var=1, len_scale=12, anis=0.25, angles=np.pi / 8)
```

Total running time of the script: (0 minutes 1.054 seconds)

Using an Unstructured Grid

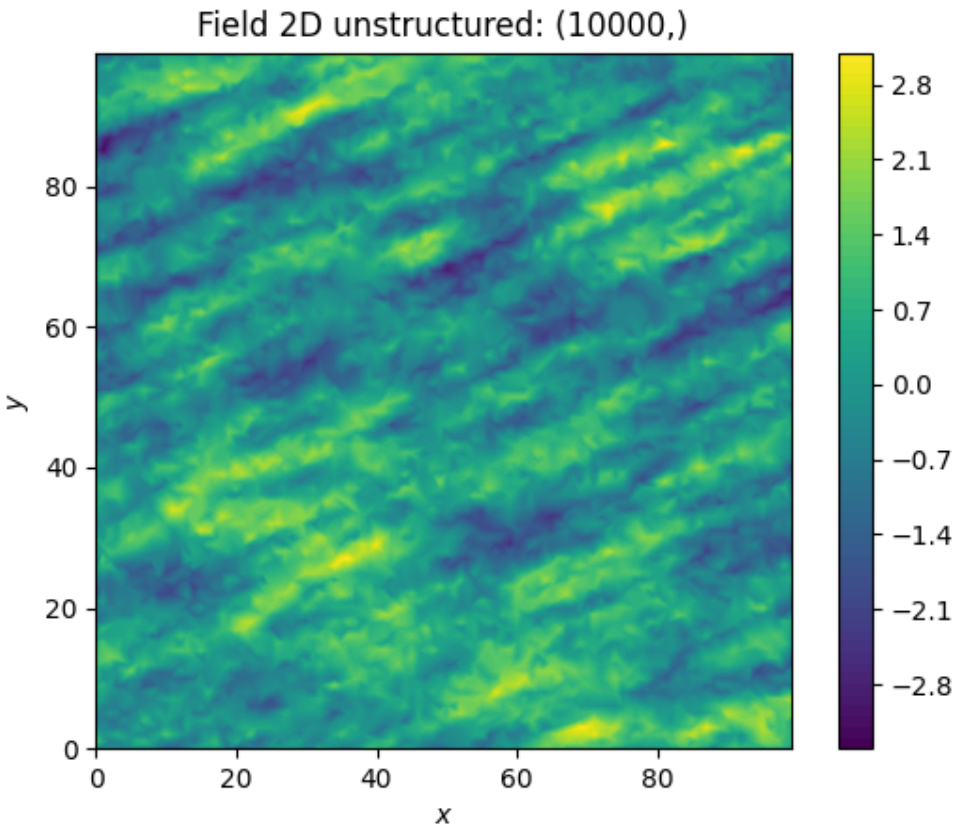
For many applications, the random fields are needed on an unstructured grid. Normally, such a grid would be read in, but we can simply generate one and then create a random field at those coordinates.

```
import numpy as np
import gstools as gs
```

Creating our own unstructured grid

```
seed = gs.random.MasterRNG(19970221)
rng = np.random.RandomState(seed())
x = rng.randint(0, 100, size=10000)
y = rng.randint(0, 100, size=10000)

model = gs.Exponential(dim=2, var=1, len_scale=[12, 3], angles=np.pi / 8)
srf = gs.SRF(model, seed=20170519)
field = srf((x, y))
srf.vtk_export("field")
# Or create a PyVista dataset
# mesh = srf.to_pyvista()
```

```
ax = srf.plot()
ax.set_aspect("equal")
```



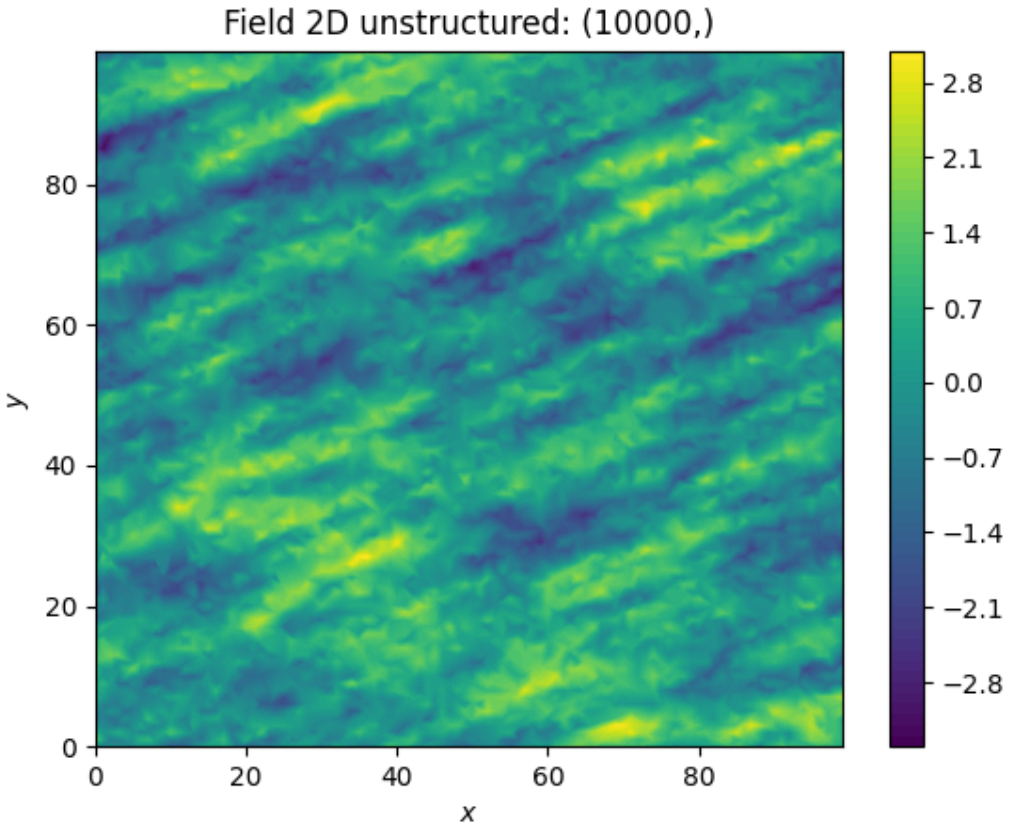
Comparing this image to the previous one, you can see that be using the same seed, the same field can be computed on different grids.

Total running time of the script: (0 minutes 1.115 seconds)

Merging two Fields

We can even generate the same field realisation on different grids. Let's try to merge two unstructured rectangular fields.

```
import numpy as np
import gstools as gs

# creating our own unstructured grid
seed = gs.random.MasterRNG(19970221)
rng = np.random.RandomState(seed())
x = rng.randint(0, 100, size=10000)
y = rng.randint(0, 100, size=10000)

model = gs.Exponential(dim=2, var=1, len_scale=[12, 3], angles=np.pi / 8)
srf = gs.SRF(model, seed=20170519)
field1 = srf((x, y))
srf.plot()
```



But now we extend the field on the right hand side by creating a new unstructured grid and calculating a field with the same parameters and the same seed on it:

```
# new grid
seed = gs.random.MasterRNG(20011012)
rng = np.random.RandomState(seed())
x2 = rng.randint(99, 150, size=10000)
y2 = rng.randint(20, 80, size=10000)

field2 = srf((x2, y2))
```

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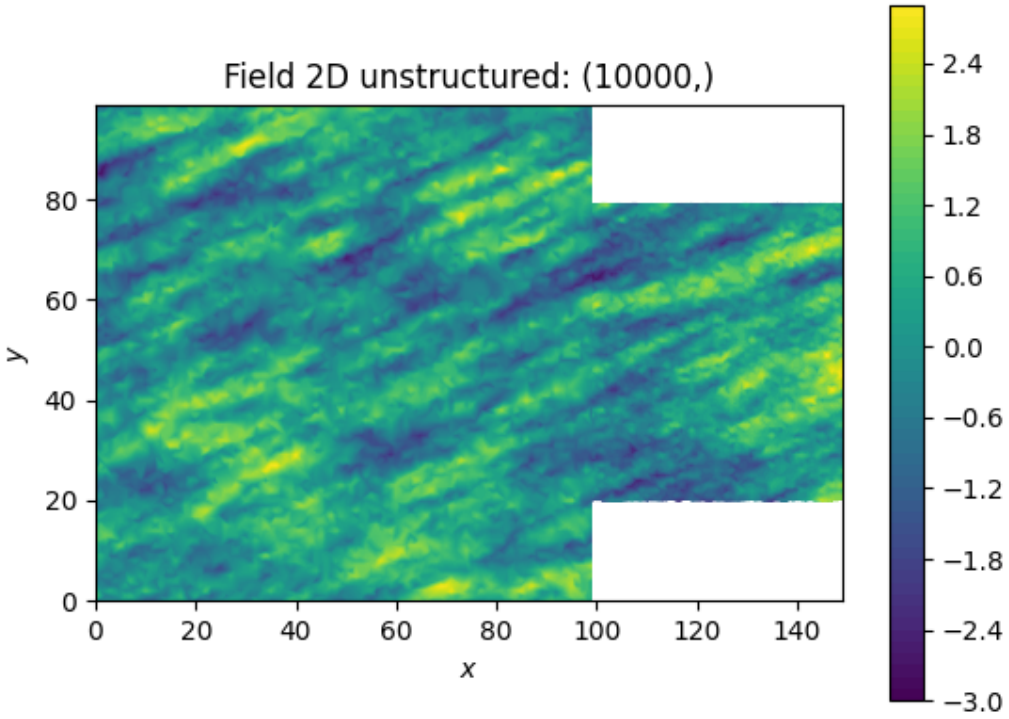
```
ax = srf.plot()
ax.tricontourf(x, y, field1.T, levels=256)
ax.set_aspect("equal")
```



The slight mismatch where the two fields were merged is merely due to interpolation problems of the plotting routine. You can convince yourself be increasing the resolution of the grids by a factor of 10.

Of course, this merging could also have been done by appending the grid point `(x2, y2)` to the original grid `(x, y)` before generating the field. But one application scenario would be to generate hugh fields, which would not fit into memory anymore.

Total running time of the script: (0 minutes 2.426 seconds)

Generating Fields on Meshes

GSTools provides an interface for meshes, to support meshio and ogs5py meshes.

When using *meshio*, the generated fields will be stored immediately in the mesh container.

There are two options to generate a field on a given mesh:

- *points="points"* will generate a field on the mesh points
- *points="centroids"* will generate a field on the cell centroids

In this example, we will generate a simple mesh with the aid of meshzoo.

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.tri as tri
```

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```
import meshzoo
import meshio
import gstools as gs

# generate a triangulated hexagon with meshzoo
points, cells = meshzoo.ngon(6, 4)
mesh = meshio.Mesh(points, {"triangle": cells})
```

Now we prepare the SRF class as always. We will generate an ensemble of fields on the generated mesh.

```
# number of fields
fields_no = 12
# model setup
model = gs.Gaussian(dim=2, len_scale=0.5)
srf = gs.SRF(model, mean=1)
```

To generate fields on a mesh, we provide a separate method: *SRF.mesh*. First we generate fields on the mesh-centroids controlled by a seed. You can specify the field name by the keyword *name*.

```
for i in range(fields_no):
    srf.mesh(mesh, points="centroids", name="c-field-{}".format(i), seed=i)
```

Now we generate fields on the mesh-points again controlled by a seed.

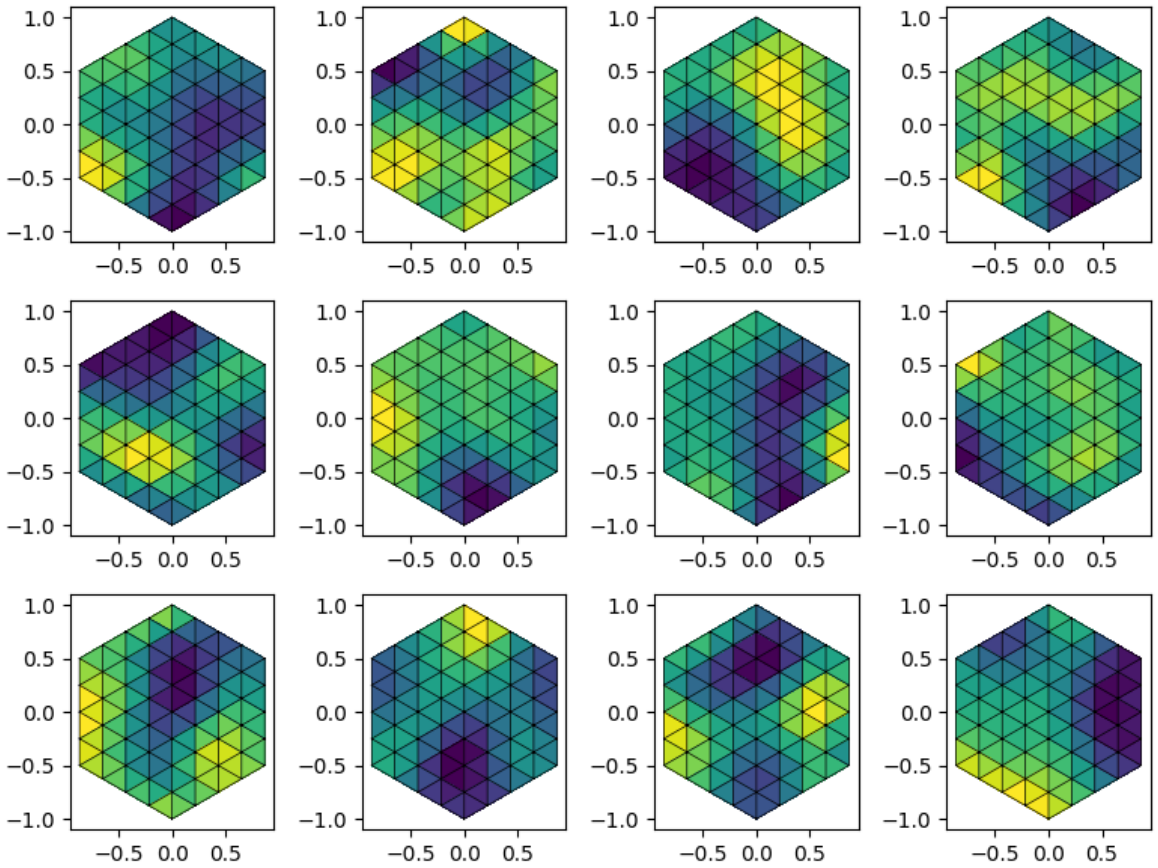
```
for i in range(fields_no):
    srf.mesh(mesh, points="points", name="p-field-{}".format(i), seed=i)
```

To get an impression we now want to plot the generated fields. Luckily, matplotlib supports triangular meshes.

```
triangulation = tri.Triangulation(points[:, 0], points[:, 1], cells)
# figure setup
cols = 4
rows = int(np.ceil(fields_no / cols))
```

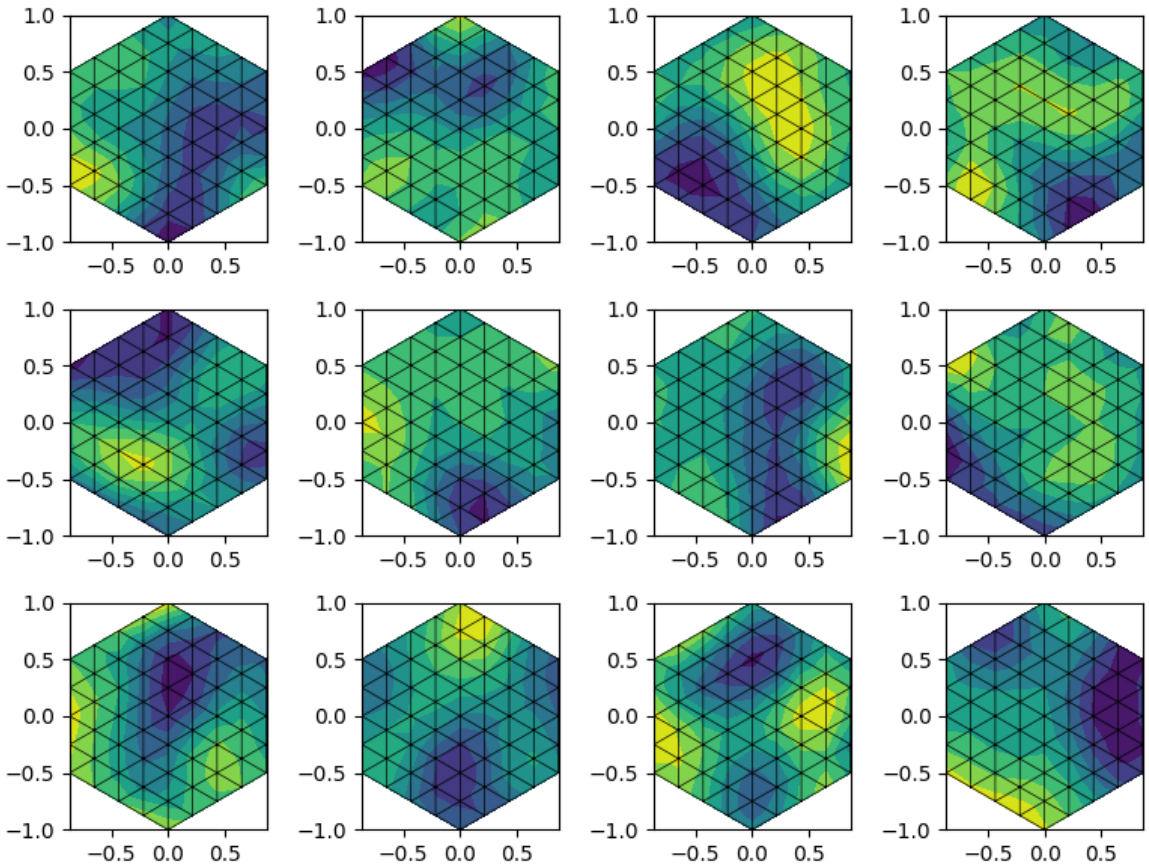
Cell data can be easily visualized with matplotlibs *tripcolor*. To highlight the cell structure, we use *triplot*.

```
fig = plt.figure(figsize=[2 * cols, 2 * rows])
for i, field in enumerate(mesh.cell_data, 1):
    ax = fig.add_subplot(rows, cols, i)
    ax.tripcolor(triangulation, mesh.cell_data[field][0])
    ax.triplot(triangulation, linewidth=0.5, color="k")
    ax.set_aspect("equal")
fig.tight_layout()
```



Point data is plotted via *tricontourf*.

```
fig = plt.figure(figsize=[2 * cols, 2 * rows])
for i, field in enumerate(mesh.point_data, 1):
    ax = fig.add_subplot(rows, cols, i)
    ax.tricontourf(triangulation, mesh.point_data[field])
    ax.triplot(triangulation, linewidth=0.5, color="k")
    ax.set_aspect("equal")
fig.tight_layout()
plt.show()
```



Last but not least, *meshio* can be used for what is does best: Exporting. Tada!

```
mesh.write("mesh_ensemble.vtk")
```

Out:

```
WARNING:root:VTK requires 3D points, but 2D points given. Appending 0 third component.
```

Total running time of the script: (0 minutes 1.731 seconds)

Using PyVista meshes

PyVista is a helper module for the Visualization Toolkit (VTK) that takes a different approach on interfacing with VTK through NumPy and direct array access.

It provides mesh data structures and filtering methods for spatial datasets, makes 3D plotting simple and is built for large/complex data geometries.

The *Field.mesh* method enables easy field creation on PyVista meshes used by the *SRF* or *Krige* class.

```
import pyvista as pv
import gstools as gs
```

We create a structured grid with PyVista containing 50 segments on all three axes each with a length of 2 (whatever unit).

```
dim, spacing = (50, 50, 50), (2, 2, 2)
grid = pv.UniformGrid(dim, spacing)
```

Now we set up the SRF class as always. We'll use an anisotropic model.

```
model = gs.Gaussian(dim=3, len_scale=[16, 8, 4], angles=(0.8, 0.4, 0.2))
srf = gs.SRF(model, seed=19970221)
```

The PyVista mesh can now be directly passed to the *SRF.mesh* method. When dealing with meshes, one can choose if the field should be generated on the mesh-points (*"points"*) or the cell-centroids (*"centroids"*).

In addition we can set a name, under which the resulting field is stored in the mesh.

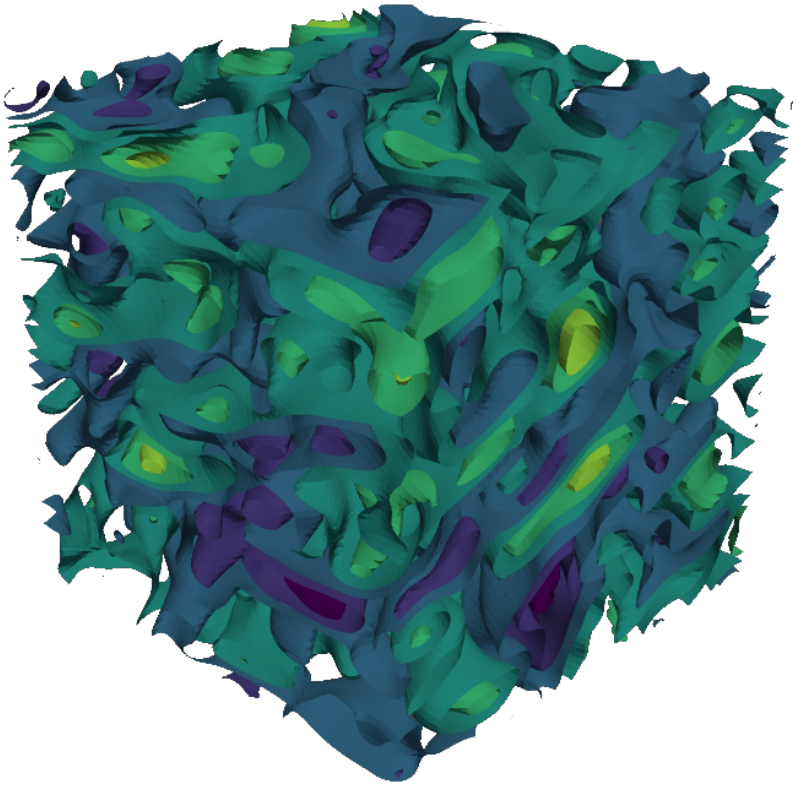
```
srf.mesh(grid, points="points", name="random-field")
```

Now we have access to PyVista's abundancy of methods to explore the field.

Note: PyVista is not working on readthedocs, but you can try it out yourself by uncommenting the following line of code.

```
# grid.contour(isosurfaces=8).plot()
```

The result should look like this:



Total running time of the script: (0 minutes 7.177 seconds)

Higher Dimensions

GSTools provides experimental support for higher dimensions.

Anisotropy is the same as in lower dimensions:

- in *n* dimensions we need *(n-1)* anisotropy ratios

Rotation on the other hand is a bit more complex. With increasing dimensions more and more rotation angles are added in order to properply describe the rotated axes of anisotropy.

By design the first rotation angles coincide with the lower ones:

- 2D (rotation in x-y plane) -> 3D: first angle describes xy-plane rotation
- 3D (Tait-Bryan angles) -> 4D: first 3 angles coincide with Tait-Bryan angles

By increasing the dimension from *n* to *(n+1)*, *n* angles are added:

- 2D (1 angle) -> 3D: 3 angles (2 added)
- 3D (3 angles) -> 4D: 6 angles (3 added)

the following list of rotation-planes are described by the list of angles in the model:

1. x-y plane
2. x-z plane
3. y-z plane
4. x-v plane
5. y-v plane
6. z-v plane
7. ...

The rotation direction in these planes have alternating signs in order to match Tait-Bryan in 3D.

Let's have a look at a 4D example, where we naively add a 4th dimension.

```
import matplotlib.pyplot as plt
import gstools as gs

dim = 4
size = 20
pos = [range(size)] * dim
model = gs.Exponential(dim=dim, len_scale=5)
srf = gs.SRF(model, seed=20170519)
field = srf.structured(pos)
```

In order to "prove" correctness, we can calculate an empirical variogram of the generated field and fit our model to it.

```
bin_center, vario = gs.vario_estimate(
    pos, field, sampling_size=2000, mesh_type="structured"
)
model.fit_variogram(bin_center, vario)
print(model)
```

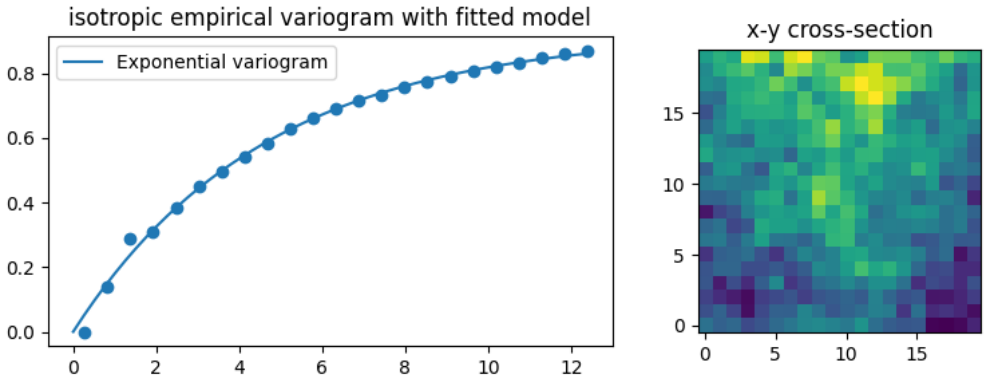
Out:

```
Exponential(dim=4, var=0.924, len_scale=4.62, nugget=4.13e-22)
```

As you can see, the estimated variance and length scale match our input quite well.

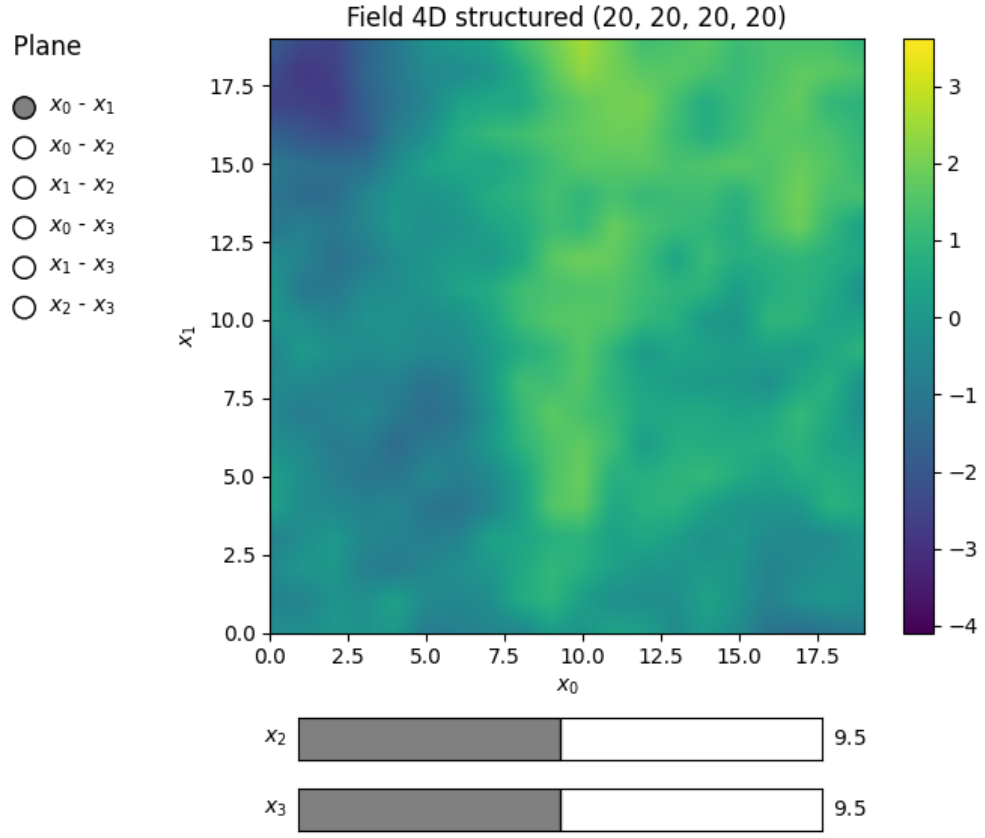
Let's have a look at the fit and a x-y cross-section of the 4D field:

```
f, a = plt.subplots(1, 2, gridspec_kw={"width_ratios": [2, 1]}, figsize=[9, 3])
model.plot(x_max=max(bin_center), ax=a[0])
a[0].scatter(bin_center, vario)
a[1].imshow(field[:, :, 0, 0].T, origin="lower")
a[0].set_title("isotropic empirical variogram with fitted model")
a[1].set_title("x-y cross-section")
f.show()
```



GSTools also provides plotting routines for higher dimensions. Fields are shown by 2D cross-sections, where other dimensions can be controlled via sliders.

```
srf.plot()
```



Total running time of the script: (0 minutes 10.880 seconds)

2.2 The Covariance Model

One of the core-features of GSTools is the powerful `CovModel` class, which allows you to easily define arbitrary covariance models by yourself. The resulting models provide a bunch of nice features to explore the covariance models.

A covariance model is used to characterize the semi-variogram, denoted by γ, of a spatial random field. In GSTools, we use the following form for an isotropic and stationary field:

$$\gamma(r) = \sigma^2 \cdot \left(1 - \mathrm{cor}\left(s \cdot \frac{r}{\ell}\right)\right) + n$$

Where:

- r is the lag distance
- ℓ is the main correlation length
- s is a scaling factor for unit conversion or normalization
- σ^2 is the variance
- n is the nugget (subscale variance)
- $\mathrm{cor}(h)$ is the normalized correlation function depending on the non-dimensional distance $h = s \cdot \frac{r}{\ell}$

Depending on the normalized correlation function, all covariance models in GSTools are providing the following functions:

- $\rho(r) = \mathrm{cor}\left(s \cdot \frac{r}{\ell}\right)$ is the so called correlation function
- $C(r) = \sigma^2 \cdot \rho(r)$ is the so called covariance function, which gives the name for our GSTools class

---

Note: We are not limited to isotropic models. GSTools supports anisotropy ratios for length scales in orthogonal transversal directions like:

- x_0 (main direction)
- x_1 (1. transversal direction)
- x_2 (2. transversal direction)
- ...

These main directions can also be rotated. Just have a look at the corresponding examples.

Provided Covariance Models

The following standard covariance models are provided by GSTools

`Gaussian`([dim, var, len_scale, nugget, ...])	The Gaussian covariance model.
`Exponential`([dim, var, len_scale, nugget, ...])	The Exponential covariance model.
`Matern`([dim, var, len_scale, nugget, anis, ...])	The Matérn covariance model.
`Stable`([dim, var, len_scale, nugget, anis, ...])	The stable covariance model.
`Rational`([dim, var, len_scale, nugget, ...])	The rational quadratic covariance model.
`Cubic`([dim, var, len_scale, nugget, anis, ...])	The Cubic covariance model.
`Linear`([dim, var, len_scale, nugget, anis, ...])	The bounded linear covariance model.
`Circular`([dim, var, len_scale, nugget, ...])	The circular covariance model.
`Spherical`([dim, var, len_scale, nugget, ...])	The Spherical covariance model.
`HyperSpherical`([dim, var, len_scale, ...])	The Hyper-Spherical covariance model.
`SuperSpherical`([dim, var, len_scale, ...])	The Super-Spherical covariance model.
`JBessel`([dim, var, len_scale, nugget, anis, ...])	The J-Bessel hole model.
`TPLSimple`([dim, var, len_scale, nugget, ...])	The simply truncated power law model.

As a special feature, we also provide truncated power law (TPL) covariance models

TPLGaussian([dim, var, len_scale, nugget, ...])	Truncated-Power-Law with Gaussian modes.
TPLExponential([dim, var, len_scale, ...])	Truncated-Power-Law with Exponential modes.
TPLStable([dim, var, len_scale, nugget, ...])	Truncated-Power-Law with Stable modes.

These models provide a lower and upper length scale truncation for superpositioned models.

Examples

Introductory example

Let us start with a short example of a self defined model (Of course, we provide a lot of predefined models [See: *gstools.covmodel*], but they all work the same way). Therefore we reimplement the Gaussian covariance model by defining just the "normalized" correlation function:

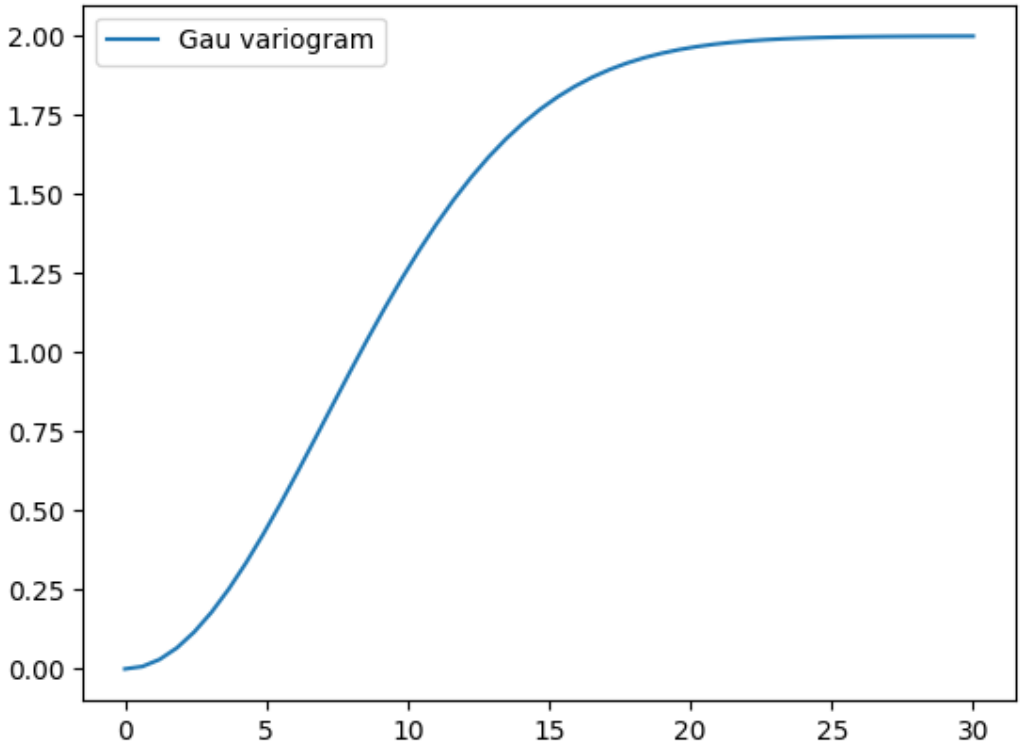
```
import numpy as np
import gstools as gs


# use CovModel as the base-class
class Gau(gs.CovModel):
    def cor(self, h):
        return np.exp(-(h ** 2))
```

Here the parameter `h` stands for the normalized range `r / len_scale`. Now we can instantiate this model:

```
model = Gau(dim=2, var=2.0, len_scale=10)
```

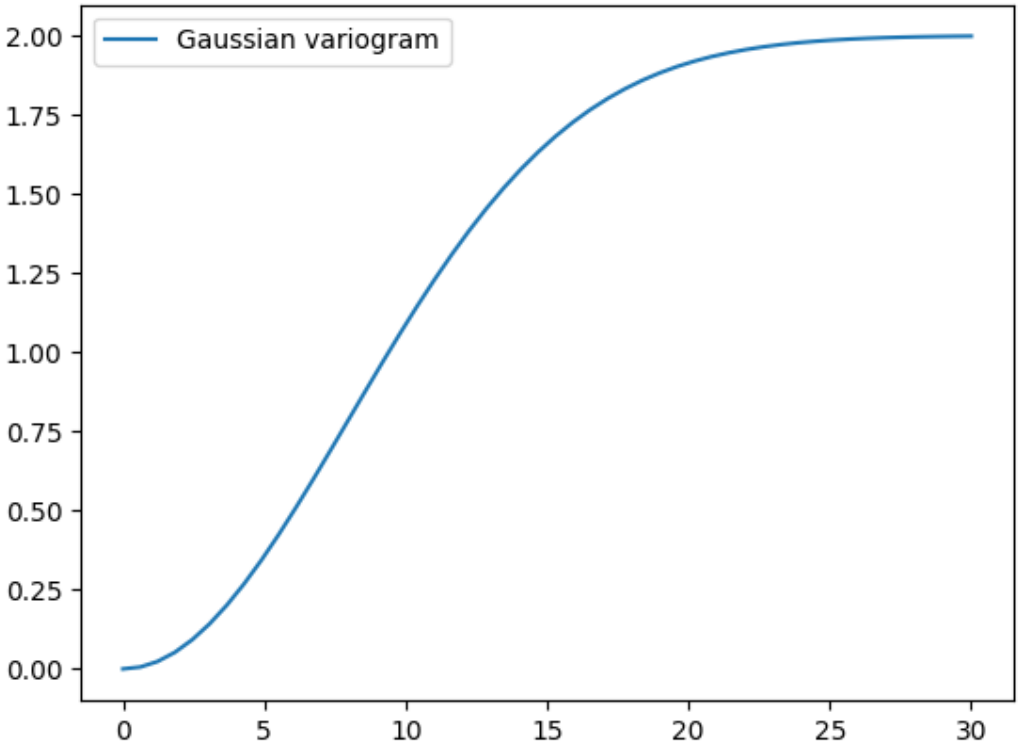
To have a look at the variogram, let's plot it:

```
model.plot()
```



This is almost identical to the already provided *Gaussian* model. There, a scaling factor is implemented so the len_scale coincides with the integral scale:

```
gau_model = gs.Gaussian(dim=2, var=2.0, len_scale=10)
gau_model.plot()
```



Parameters

We already used some parameters, which every covariance models has. The basic ones are:

- **dim** : dimension of the model
- **var** : variance of the model (on top of the subscale variance)
- **len_scale** : length scale of the model
- **nugget** : nugget (subscale variance) of the model

These are the common parameters used to characterize a covariance model and are therefore used by every model in GSTools. You can also access and reset them:

```
print("old model:", model)
model.dim = 3
model.var = 1
model.len_scale = 15
model.nugget = 0.1
print("new model:", model)
```

Out:

```
old model: Gau(dim=2, var=2.0, len_scale=10.0, nugget=0.0)
new model: Gau(dim=3, var=1.0, len_scale=15.0, nugget=0.1)
```

Note:

- The sill of the variogram is calculated by `sill = variance + nugget` So we treat the variance as everything **above** the nugget, which is sometimes called **partial sill**.
- A covariance model can also have additional parameters.

Total running time of the script: (0 minutes 0.191 seconds)

Basic Methods

The covariance model class *CovModel* of GSTools provides a set of handy methods.

One of the following functions defines the main characterization of the variogram:

- `CovModel.variogram` : The variogram of the model given by

$$\gamma(r) = \sigma^2 \cdot (1 - \rho(r)) + n$$

- `CovModel.covariance` : The (auto-)covariance of the model given by

$$C(r) = \sigma^2 \cdot \rho(r)$$

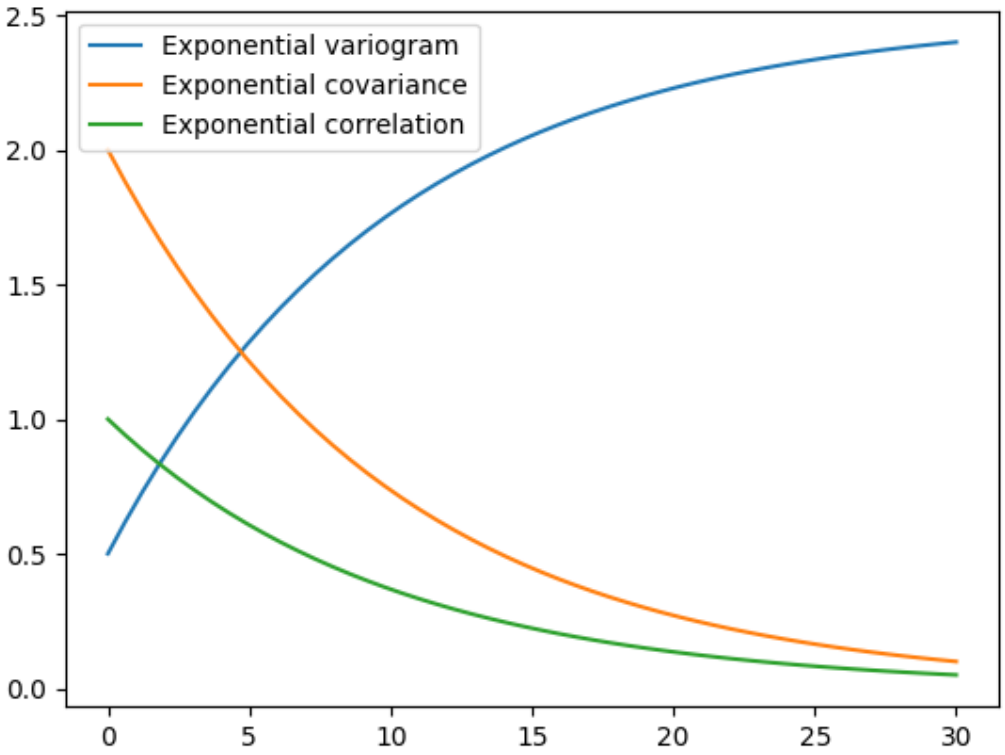
- `CovModel.correlation` : The (auto-)correlation (or normalized covariance) of the model given by

$$\rho(r)$$

- `CovModel.cor` : The normalized correlation taking a normalized range given by:

$$\mathrm{cor}\left(\frac{r}{\ell}\right) = \rho(r)$$

As you can see, it is the easiest way to define a covariance model by giving a correlation function as demonstrated in the introductory example. If one of the above functions is given, the others will be determined:



```
import gstools as gs

model = gs.Exponential(dim=3, var=2.0, len_scale=10, nugget=0.5)
ax = model.plot("variogram")
model.plot("covariance", ax=ax)
model.plot("correlation", ax=ax)
```

Total running time of the script: (0 minutes 0.099 seconds)

Anisotropy and Rotation

The internally used (semi-) variogram represents the isotropic case for the model. Nevertheless, you can provide anisotropy ratios by:

```
import gstools as gs

model = gs.Gaussian(dim=3, var=2.0, len_scale=10, anis=0.5)
print(model.anis)
print(model.len_scale_vec)
```

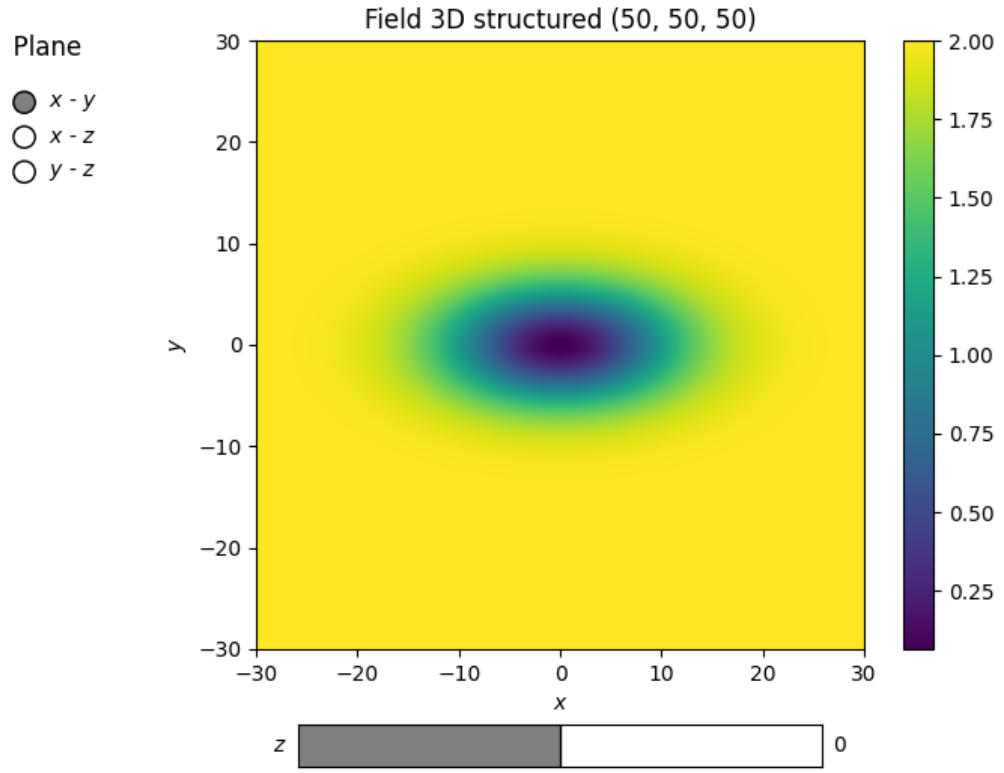
Out:

```
[1.  0.5]
[10. 10.  5.]
```

As you can see, we defined just one anisotropy-ratio and the second transversal direction was filled up with `1.`. You can get the length-scales in each direction by the attribute *CovModel.len_scale_vec*. For full control you can set a list of anistropy ratios: `anis=[0.5, 0.4]`.

Alternatively you can provide a list of length-scales:

```
model = gs.Gaussian(dim=3, var=2.0, len_scale=[10, 5, 4])
model.plot("vario_spatial")
print("Anisotropy representations:")
print("Anis. ratios:", model.anis)
print("Main length scale", model.len_scale)
print("All length scales", model.len_scale_vec)
```



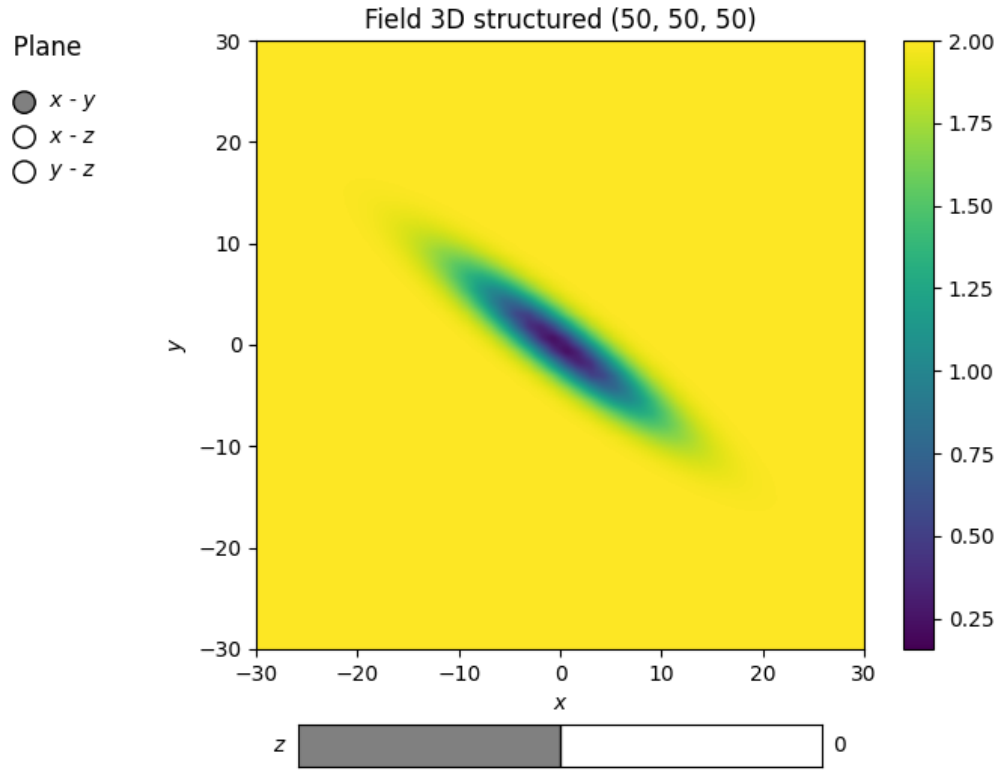
Out:

```
Anisotropy representations:
Anis. ratios: [0.5 0.4]
Main length scale 10.0
All length scales [10.  5.  4.]
```

Rotation Angles

The main directions of the field don't have to coincide with the spatial directions x, y and z. Therefore you can provide rotation angles for the model:

```
model = gs.Gaussian(dim=3, var=2.0, len_scale=[10, 2], angles=2.5)
model.plot("vario_spatial")
print("Rotation angles", model.angles)
```



Out:

```
Rotation angles [2.5 0.  0. ]
```

Again, the angles were filled up with `0.` to match the dimension and you could also provide a list of angles. The number of angles depends on the given dimension:

- in 1D: no rotation performable
- in 2D: given as rotation around z-axis
- in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)
- in nD: See the random field example about higher dimensions

Total running time of the script: (0 minutes 0.929 seconds)

Spectral methods

The spectrum of a covariance model is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(\|\mathbf{r}\|) e^{ib\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Since the covariance function $C(r)$ is radially symmetric, we can calculate this by the hankel-transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{(bk)^{n/2-1}} \int_0^\infty r^{n/2-1} C(r) J_{n/2-1}(bkr) r dr$$

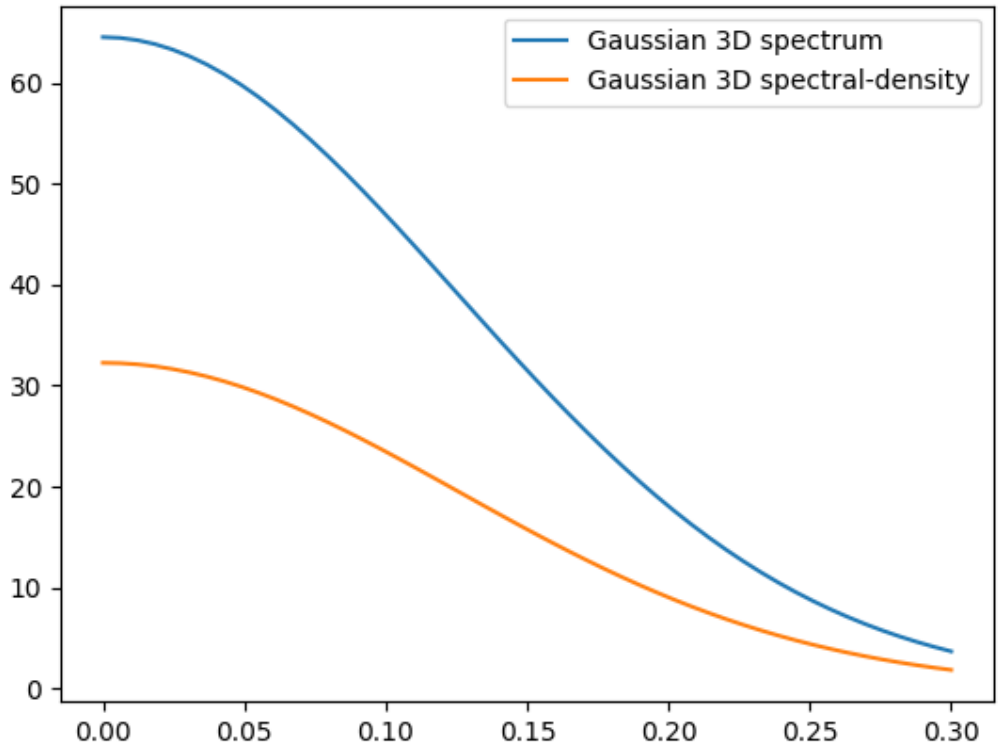
Where $k = \|\mathbf{k}\|$.

Depending on the spectrum, the spectral-density is defined by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

You can access these methods by:

```
import gstools as gs

model = gs.Gaussian(dim=3, var=2.0, len_scale=10)
ax = model.plot("spectrum")
model.plot("spectral_density", ax=ax)
```



Note: The spectral-density is given by the radius of the input phase. But it is **not** a probability density function for the radius of the phase. To obtain the pdf for the phase-radius, you can use the methods *CovModel.spectral_rad_pdf* or *CovModel.ln_spectral_rad_pdf* for the logarithm.

The user can also provide a cdf (cumulative distribution function) by defining a method called `spectral_rad_cdf` and/or a ppf (percent-point function) by `spectral_rad_ppf`.

The attributes *CovModel.has_cdf* and *CovModel.has_ppf* will check for that.

Total running time of the script: (0 minutes 0.094 seconds)

Different scales

Besides the length-scale, there are many other ways of characterizing a certain scale of a covariance model. We provide two common scales with the covariance model.

Integral scale

The integral scale of a covariance model is calculated by:

$$I = \int_0^\infty \rho(r) dr$$

You can access it by:

```
import gstools as gs

model = gs.Stable(dim=3, var=2.0, len_scale=10)
print("Main integral scale:", model.integral_scale)
print("All integral scales:", model.integral_scale_vec)
```

Out:

```
Main integral scale: 9.027452929509336
All integral scales: [9.02745293 9.02745293 9.02745293]
```

You can also specify integral length scales like the ordinary length scale, and len_scale/anis will be recalculated:

```
model = gs.Stable(dim=3, var=2.0, integral_scale=[10, 4, 2])
print("Anisotropy ratios:", model.anis)
print("Main length scale:", model.len_scale)
print("All length scales:", model.len_scale_vec)
print("Main integral scale:", model.integral_scale)
print("All integral scales:", model.integral_scale_vec)
```

Out:

```
Anisotropy ratios: [0.4 0.2]
Main length scale: 11.077321674324725
All length scales: [11.07732167  4.43092867  2.21546433]
Main integral scale: 10.0
All integral scales: [10.  4.  2.]
```

Percentile scale

Another scale characterizing the covariance model, is the percentile scale. It is the distance, where the normalized variogram reaches a certain percentage of its sill.

```
model = gs.Stable(dim=3, var=2.0, len_scale=10)
per_scale = model.percentile_scale(0.9)
int_scale = model.integral_scale
len_scale = model.len_scale
print("90% Percentile scale:", per_scale)
print("Integral scale:", int_scale)
print("Length scale:", len_scale)
```

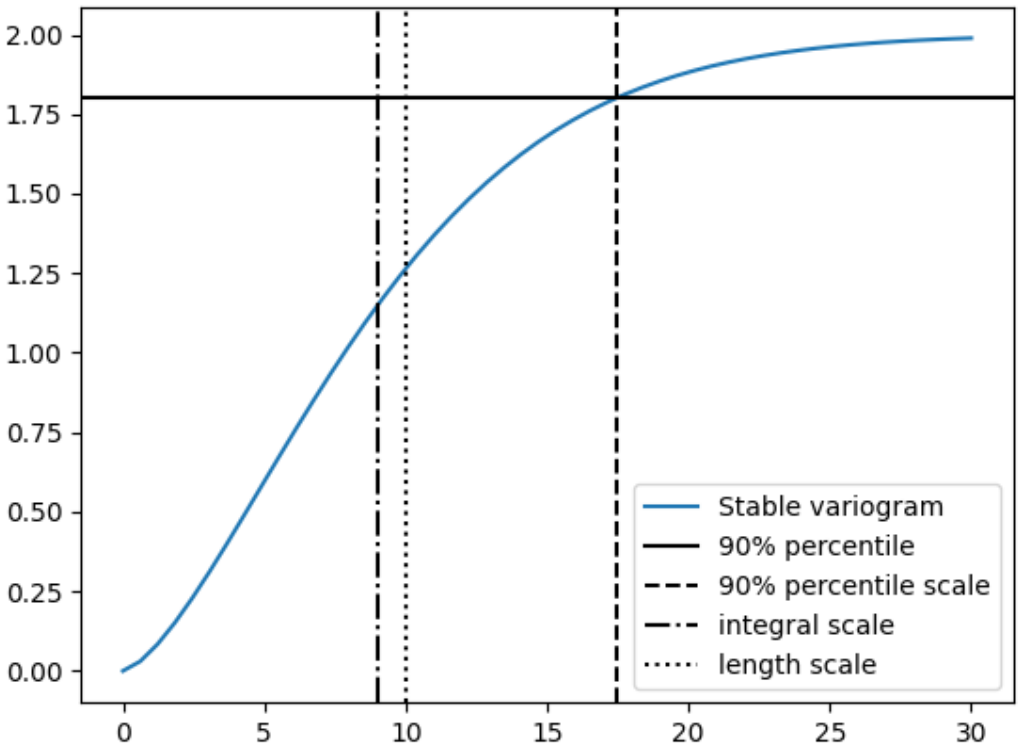
Out:

```
90% Percentile scale: 17.437215135964117
Integral scale: 9.027452929509336
Length scale: 10.0
```

Note: The nugget is neglected by the percentile scale.

Comparison

```
ax = model.plot()
ax.axhline(1.8, color="k", label=r"90% percentile")
ax.axvline(per_scale, color="k", linestyle="--", label=r"90% percentile scale")
ax.axvline(int_scale, color="k", linestyle="-.", label=r"integral scale")
ax.axvline(len_scale, color="k", linestyle=":", label=r"length scale")
ax.legend()
```



Total running time of the script: (0 minutes 0.164 seconds)

Additional Parameters

Let's pimp our self-defined model `Gau` from the introductory example by setting the exponent as an additional parameter:

$$\rho(r) := \exp\left(-\left(\frac{r}{\ell}\right)^{\alpha}\right)$$

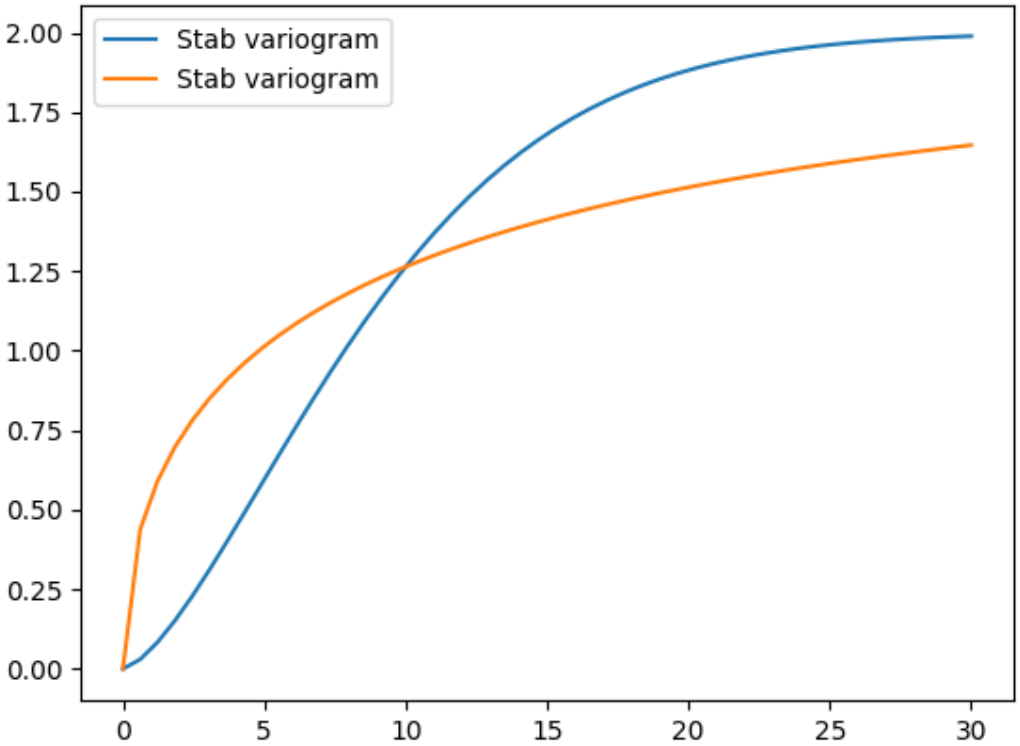
This leads to the so called **stable** covariance model and we can define it by

```
import numpy as np
import gstools as gs


class Stab(gs.CovModel):
    def default_opt_arg(self):
        return {"alpha": 1.5}

    def cor(self, h):
        return np.exp(-(h ** self.alpha))
```

As you can see, we override the method *CovModel.default_opt_arg* to provide a standard value for the optional argument `alpha`. We can access it in the correlation function by `self.alpha`

Now we can instantiate this model by either setting alpha implicitly with the default value or explicitly:

```
model1 = Stab(dim=2, var=2.0, len_scale=10)
model2 = Stab(dim=2, var=2.0, len_scale=10, alpha=0.5)
ax = model1.plot()
model2.plot(ax=ax)
```



Apparently, the parameter alpha controls the slope of the variogram and consequently the roughness of a generated random field.

Note: You don't have to override the *CovModel.default_opt_arg*, but you will get a ValueError if you don't set it on creation.

Total running time of the script: (0 minutes 0.106 seconds)

Fitting variogram data

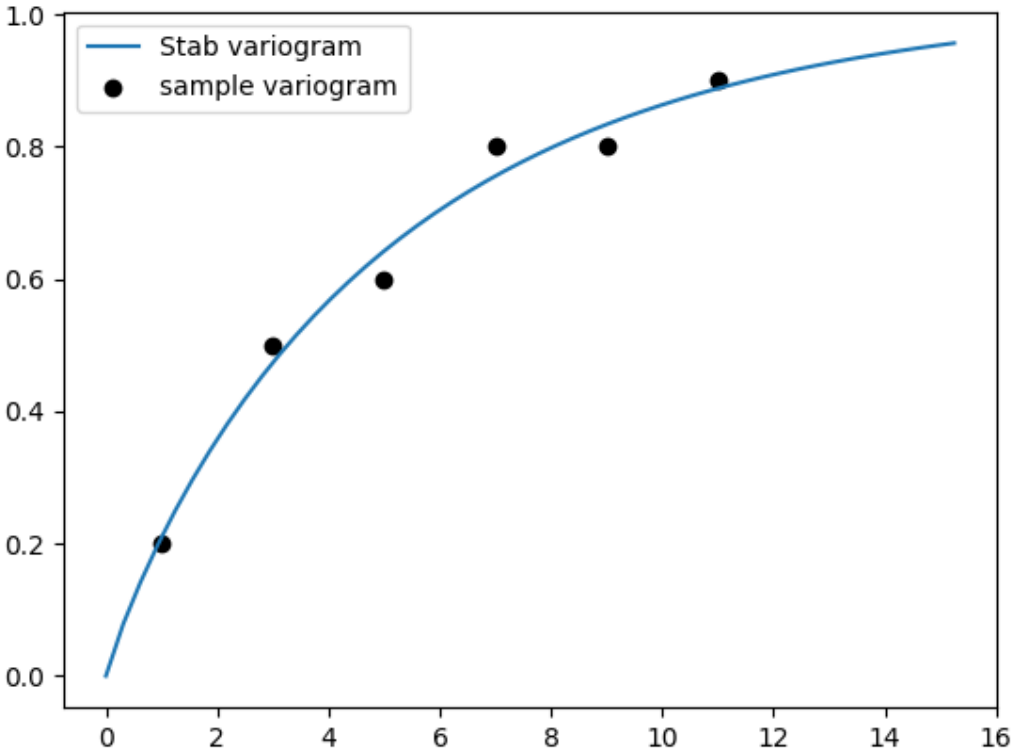
The model class comes with a routine to fit the model-parameters to given variogram data. In the following we will use the self defined stable model from a previous example.

```
import numpy as np
import gstools as gs


class Stab(gs.CovModel):
    def default_opt_arg(self):
        return {"alpha": 1.5}

    def cor(self, h):
        return np.exp(-(h ** self.alpha))


# Exemplary variogram data (e.g. estimated from field observations)
bins = [1.0, 3.0, 5.0, 7.0, 9.0, 11.0]
est_vario = [0.2, 0.5, 0.6, 0.8, 0.8, 0.9]
# fitting model
model = Stab(dim=2)
# we have to provide boundaries for the parameters
model.set_arg_bounds(alpha=[0, 3])
results, pcov = model.fit_variogram(bins, est_vario, nugget=False)
print("Results:", results)
```

Out:

```
Results: {'var': 1.0245739533522298, 'len_scale': 5.081591843438552, 'nugget': 0.0,
↪'alpha': 0.9067041165465699}
```

```
ax = model.plot()
ax.scatter(bins, est_vario, color="k", label="sample variogram")
ax.legend()
```



As you can see, we have to provide boundaries for the parameters. As a default, the following bounds are set:

- additional parameters: `[-np.inf, np.inf]`
- variance: `[0.0, np.inf]`
- len_scale: `[0.0, np.inf]`
- nugget: `[0.0, np.inf]`

Also, you can deselect parameters from fitting, so their predefined values will be kept. In our case, we fixed a `nugget` of `0.0`, which was set by default. You can deselect any standard or optional argument of the covariance model. The second return value `pcov` is the estimated covariance of `popt` from the used scipy routine `scipy.optimize.curve_fit`.

You can use the following methods to manipulate the used bounds:

CovModel.default_opt_arg_bounds()	Provide default boundaries for optional arguments.
CovModel.default_arg_bounds()	Provide default boundaries for arguments.
CovModel.set_arg_bounds([check_args])	Set bounds for the parameters of the model.
CovModel.check_arg_bounds()	Check arguments to be within their given bounds.

You can override the *CovModel.default_opt_arg_bounds* to provide standard bounds for your additional parameters.

To access the bounds you can use:

CovModel.var_bounds	Bounds for the variance.
CovModel.len_scale_bounds	Bounds for the lenght scale.
CovModel.nugget_bounds	Bounds for the nugget.
CovModel.opt_arg_bounds	Bounds for the optional arguments.

continues on next page

Table 4 – continued from previous page

CovModel.arg_bounds	Bounds for all parameters.

Total running time of the script: (0 minutes 0.126 seconds)

2.3 Variogram Estimation

Estimating the spatial correlations is an important part of geostatistics. These spatial correlations can be expressed by the variogram, which can be estimated with the subpackage *gstools.variogram*. The variograms can be estimated on structured and unstructured grids.

The same (semi-)variogram as *The Covariance Model* is being used by this subpackage.

Examples

Fit Variogram

```
import numpy as np
import gstools as gs
```

Generate a synthetic field with an exponential model.

```
x = np.random.RandomState(19970221).rand(1000) * 100.0
y = np.random.RandomState(20011012).rand(1000) * 100.0
model = gs.Exponential(dim=2, var=2, len_scale=8)
srf = gs.SRF(model, mean=0, seed=19970221)
field = srf((x, y))
```

Estimate the variogram of the field with 40 bins.

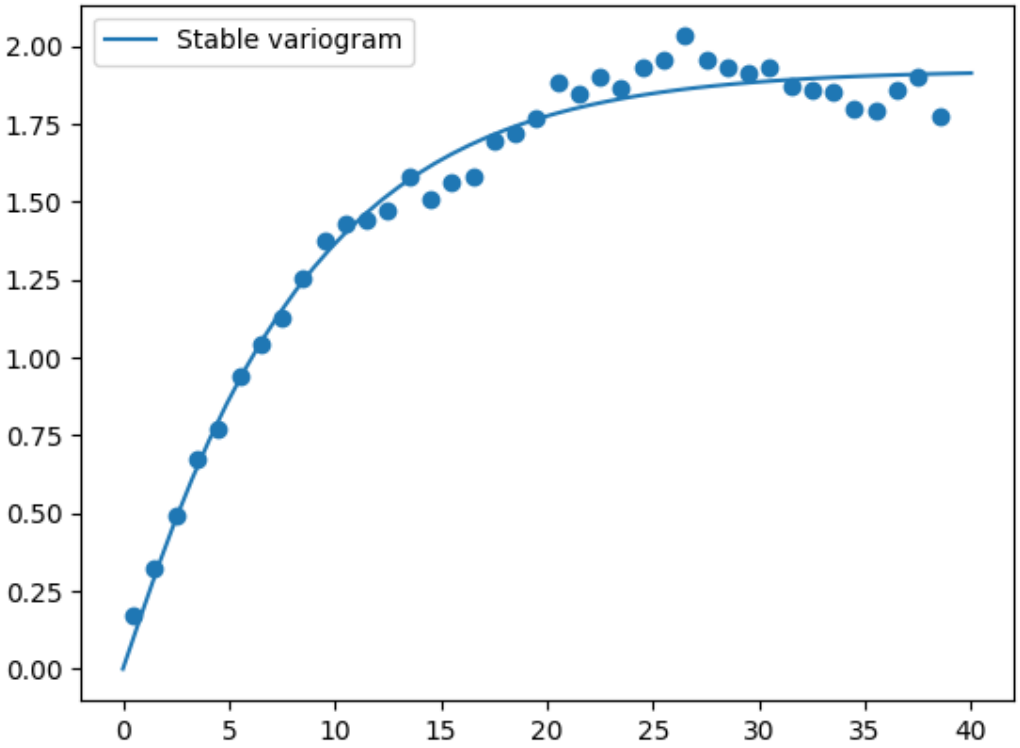
```
bins = np.arange(40)
bin_center, gamma = gs.vario_estimate((x, y), field, bins)
```

Fit the variogram with a stable model (no nugget fitted).

```
fit_model = gs.Stable(dim=2)
fit_model.fit_variogram(bin_center, gamma, nugget=False)
```

Plot the fitting result.

```
ax = fit_model.plot(x_max=40)
ax.scatter(bin_center, gamma)
print(fit_model)
```



Out:

```
Stable(dim=2, var=1.92, len_scale=8.15, nugget=0.0, alpha=1.05)
```

Total running time of the script: (0 minutes 0.489 seconds)

Finding the best fitting variogram model

```
import numpy as np
import gstools as gs
from matplotlib import pyplot as plt
```

Generate a synthetic field with an exponential model.

```
x = np.random.RandomState(19970221).rand(1000) * 100.0
y = np.random.RandomState(20011012).rand(1000) * 100.0
model = gs.Exponential(dim=2, var=2, len_scale=8)
srf = gs.SRF(model, mean=0, seed=19970221)
field = srf((x, y))
```

Estimate the variogram of the field with 40 bins and plot the result.

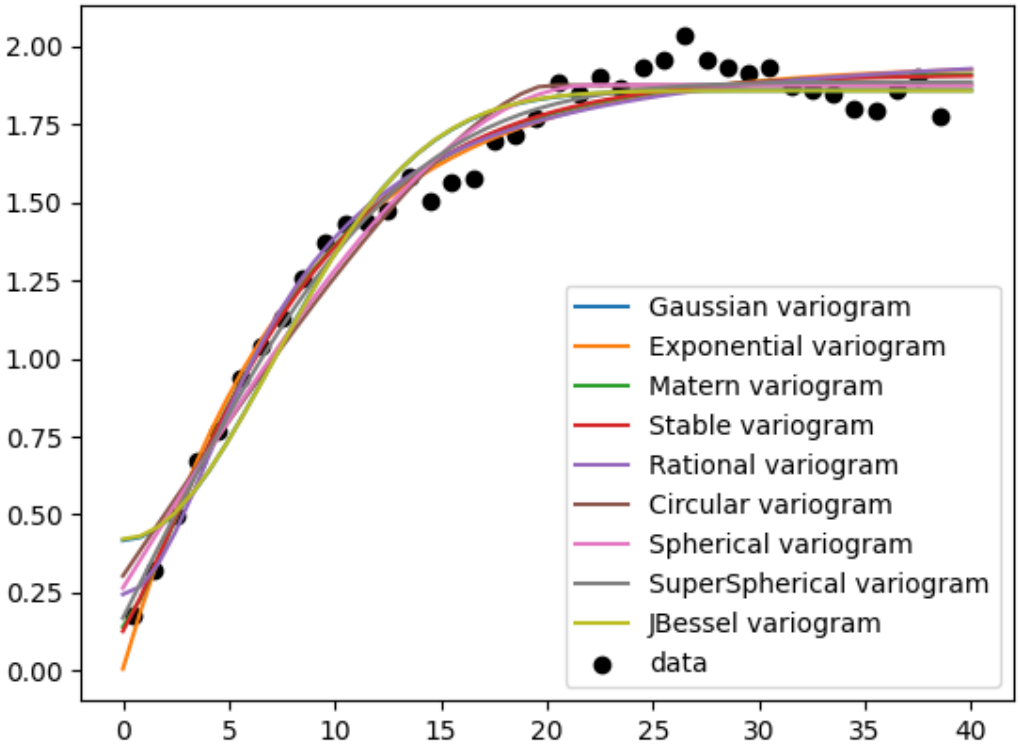
```
bins = np.arange(40)
bin_center, gamma = gs.vario_estimate((x, y), field, bins)
```

Define a set of models to test.

```
models = {
    "Gaussian": gs.Gaussian,
    "Exponential": gs.Exponential,
    "Matern": gs.Matern,
    "Stable": gs.Stable,
    "Rational": gs.Rational,
    "Circular": gs.Circular,
    "Spherical": gs.Spherical,
    "SuperSpherical": gs.SuperSpherical,
    "JBessel": gs.JBessel,
}
scores = {}
```

Iterate over all models, fit their variogram and calculate the r2 score.

```
# plot the estimated variogram
plt.scatter(bin_center, gamma, color="k", label="data")
ax = plt.gca()

# fit all models to the estimated variogram
for model in models:
    fit_model = models[model](dim=2)
    para, pcov, r2 = fit_model.fit_variogram(bin_center, gamma, return_r2=True)
    fit_model.plot(x_max=40, ax=ax)
    scores[model] = r2
```



Create a ranking based on the score and determine the best models

```
ranking = [
    (k, v)
    for k, v in sorted(scores.items(), key=lambda item: item[1], reverse=True)
]
print("RANKING")
for i, (model, score) in enumerate(ranking, 1):
    print(i, model, score)

plt.show()
```

Out:

```
RANKING
1 Stable 0.9821836193000343
2 Matern 0.9817602690672453
3 SuperSpherical 0.9814051618626767
4 Exponential 0.980407470735337
5 Rational 0.9771067080321653
6 Spherical 0.9733371670897375
7 Circular 0.9672526098783125
8 Gaussian 0.9592818084007272
9 JBessel 0.9583119496108051
```

Total running time of the script: (0 minutes 1.002 seconds)

Multi-field variogram estimation

In this example, we demonstrate how to estimate a variogram from multiple fields on the same point-set that should have the same statistical properties.

```
import numpy as np
import gstools as gs
import matplotlib.pyplot as plt


x = np.random.RandomState(19970221).rand(1000) * 100.0
y = np.random.RandomState(20011012).rand(1000) * 100.0
model = gs.Exponential(dim=2, var=2, len_scale=8)
srf = gs.SRF(model, mean=0)
```

Generate two synthetic fields with an exponential model.

```
field1 = srf((x, y), seed=19970221)
field2 = srf((x, y), seed=20011012)
fields = [field1, field2]
```

Now we estimate the variograms for both fields individually and then again simultaneously with only one call.

```
bins = np.arange(40)
bin_center, gamma1 = gs.vario_estimate((x, y), field1, bins)
bin_center, gamma2 = gs.vario_estimate((x, y), field2, bins)
bin_center, gamma = gs.vario_estimate((x, y), fields, bins)
```

Now we demonstrate that the mean variogram from both fields coincides with the joined estimated one.

```
plt.plot(bin_center, gamma1, label="field 1")
plt.plot(bin_center, gamma2, label="field 2")
```

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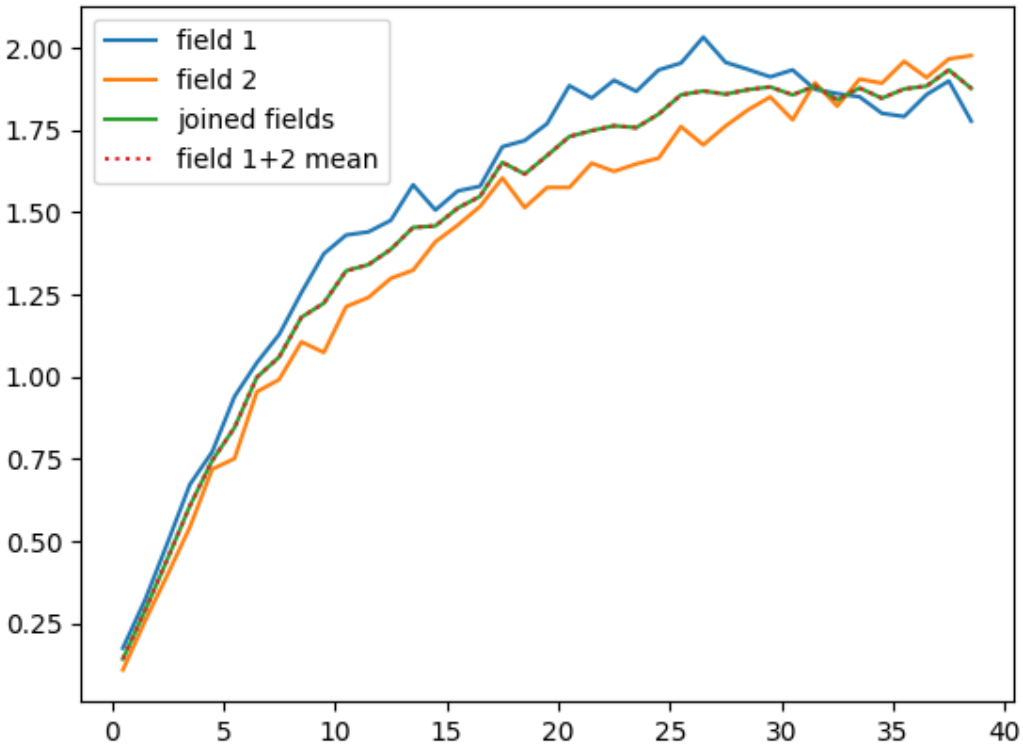
```
plt.plot(bin_center, gamma, label="joined fields")
plt.plot(bin_center, 0.5 * (gamma1 + gamma2), ":", label="field 1+2 mean")
plt.legend()
plt.show()
```



Total running time of the script: (0 minutes 1.112 seconds)

Directional variogram estimation and fitting in 2D

In this example, we demonstrate how to estimate a directional variogram by setting the direction angles in 2D.

Afterwards we will fit a model to this estimated variogram and show the result.

```
import numpy as np
import gstools as gs
from matplotlib import pyplot as plt
```

Generating synthetic field with anisotropy and a rotation of 22.5 degree.

```
angle = np.pi / 8
model = gs.Exponential(dim=2, len_scale=[10, 5], angles=angle)
x = y = range(101)
srf = gs.SRF(model, seed=123456)
field = srf((x, y), mesh_type="structured")
```

Now we are going to estimate a directional variogram with an angular tolerance of 11.25 degree and a bandwith of 8.

```
bins = range(0, 40, 2)
bin_center, dir_vario, counts = gs.vario_estimate(
    *((x, y), field, bins),
    direction=gs.rotated_main_axes(dim=2, angles=angle),
    angles_tol=np.pi / 16,
    bandwidth=8,
    mesh_type="structured",
    return_counts=True,
)
```

Afterwards we can use the estimated variogram to fit a model to it:

```
print("Original:")
print(model)
model.fit_variogram(bin_center, dir_vario)
print("Fitted:")
print(model)
```

Out:

```
Original:
Exponential(dim=2, var=1.0, len_scale=10.0, nugget=0.0, anis=[0.5], angles=[0.393])
Fitted:
Exponential(dim=2, var=0.942, len_scale=9.14, nugget=1.1e-17, anis=[0.529], angles=[0.
↪393])
```

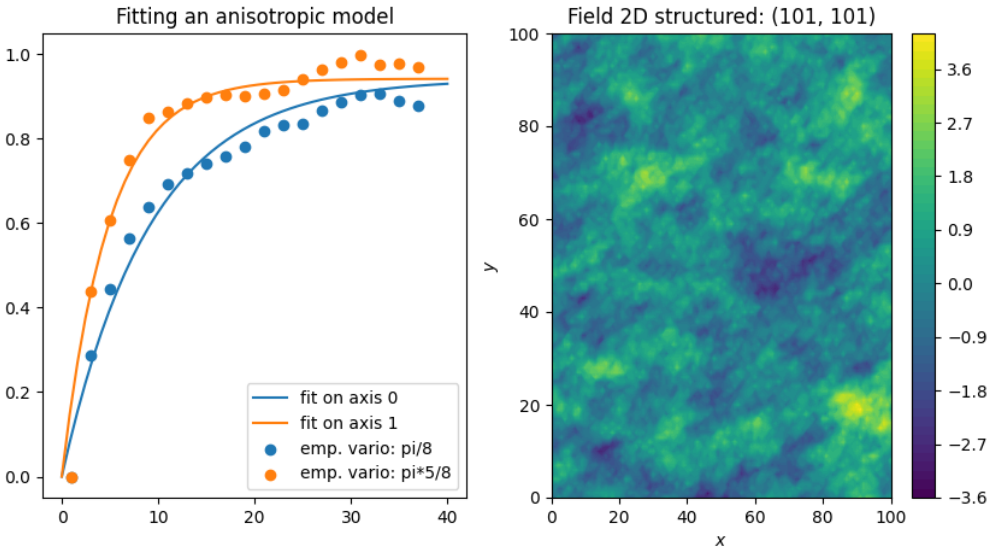
Plotting.

```
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=[10, 5])

ax1.scatter(bin_center, dir_vario[0], label="emp. vario: pi/8")
ax1.scatter(bin_center, dir_vario[1], label="emp. vario: pi*5/8")
ax1.legend(loc="lower right")

model.plot("vario_axis", axis=0, ax=ax1, x_max=40, label="fit on axis 0")
model.plot("vario_axis", axis=1, ax=ax1, x_max=40, label="fit on axis 1")
ax1.set_title("Fitting an anisotropic model")

srf.plot(ax=ax2)
plt.show()
```



Without fitting a model, we see that the correlation length in the main direction is greater than the transversal one.

Total running time of the script: (0 minutes 8.312 seconds)

Directional variogram estimation and fitting in 3D

In this example, we demonstrate how to estimate a directional variogram by setting the estimation directions in 3D.

Afterwards we will fit a model to this estimated variogram and show the result.

```
import numpy as np
import gstools as gs
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
```

Generating synthetic field with anisotropy and rotation by Tait-Bryan angles.

```
dim = 3
# rotation around z, y, x
angles = [np.deg2rad(90), np.deg2rad(45), np.deg2rad(22.5)]
model = gs.Gaussian(dim=3, len_scale=[16, 8, 4], angles=angles)
x = y = z = range(50)
pos = (x, y, z)
srf = gs.SRF(model, seed=1001)
field = srf.structured(pos)
```

Here we generate the axes of the rotated coordinate system to get an impression what the rotation angles do.

```
# All 3 axes of the rotated coordinate-system
main_axes = gs.rotated_main_axes(dim, angles)
axis1, axis2, axis3 = main_axes
```

Now we estimate the variogram along the main axes. When the main axes are unknown, one would need to sample multiple directions and look for the one with the longest correlation length (flattest gradient). Then check the transversal directions and so on.

```
bin_center, dir_vario, counts = gs.vario_estimate(
    pos,
    field,
```

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```
    direction=main_axes,
    bandwidth=10,
    sampling_size=2000,
    sampling_seed=1001,
    mesh_type="structured",
    return_counts=True,
)
```

Afterwards we can use the estimated variogram to fit a model to it. Note, that the rotation angles need to be set beforehand.

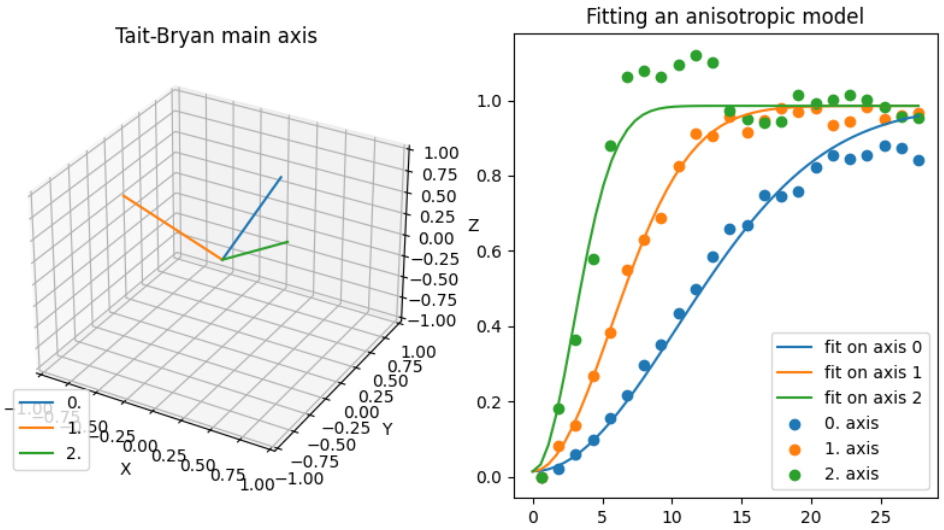
```
print("Original:")
print(model)
model.fit_variogram(bin_center, dir_vario)
print("Fitted:")
print(model)
```

Out:

```
Original:
Gaussian(dim=3, var=1.0, len_scale=16.0, nugget=0.0, anis=[0.5, 0.25], angles=[1.57,␣
↪0.785, 0.393])
Fitted:
Gaussian(dim=3, var=0.972, len_scale=13.0, nugget=0.0138, anis=[0.542, 0.281],␣
↪angles=[1.57, 0.785, 0.393])
```

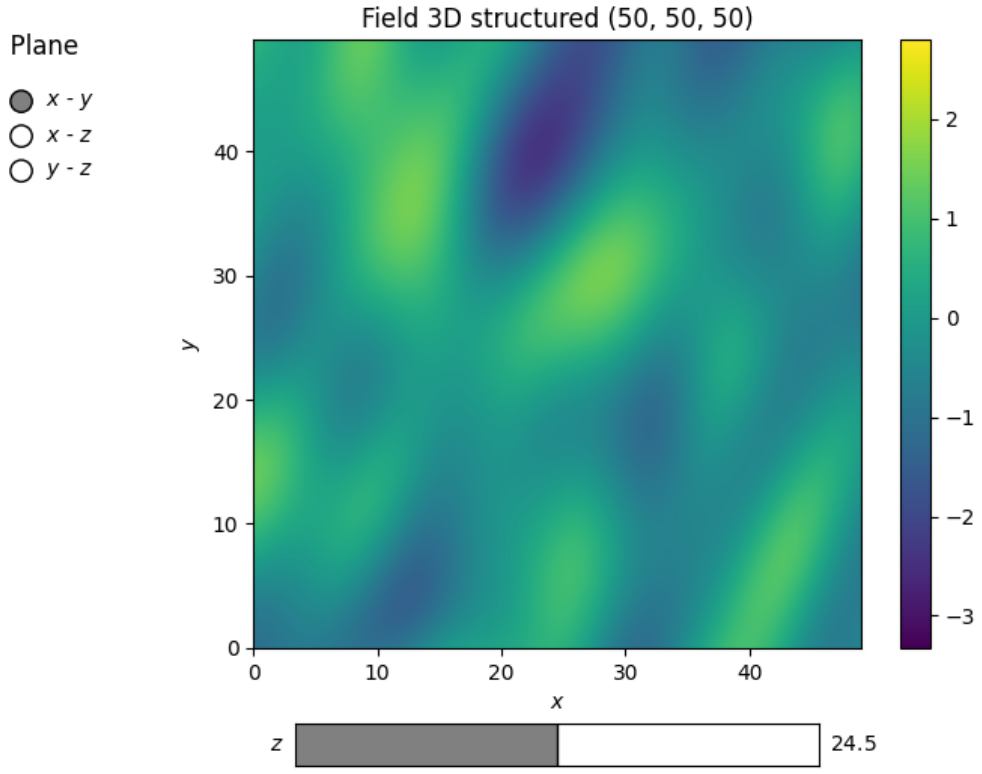
Plotting main axes and the fitted directional variogram.

```
fig = plt.figure(figsize=[10, 5])
ax1 = fig.add_subplot(121, projection=Axes3D.name)
ax2 = fig.add_subplot(122)

ax1.plot([0, axis1[0]], [0, axis1[1]], [0, axis1[2]], label="0.")
ax1.plot([0, axis2[0]], [0, axis2[1]], [0, axis2[2]], label="1.")
ax1.plot([0, axis3[0]], [0, axis3[1]], [0, axis3[2]], label="2.")
ax1.set_xlim(-1, 1)
ax1.set_ylim(-1, 1)
ax1.set_zlim(-1, 1)
ax1.set_xlabel("X")
ax1.set_ylabel("Y")
ax1.set_zlabel("Z")
ax1.set_title("Tait-Bryan main axis")
ax1.legend(loc="lower left")

x_max = max(bin_center)
ax2.scatter(bin_center, dir_vario[0], label="0. axis")
ax2.scatter(bin_center, dir_vario[1], label="1. axis")
ax2.scatter(bin_center, dir_vario[2], label="2. axis")
model.plot("vario_axis", axis=0, ax=ax2, x_max=x_max, label="fit on axis 0")
model.plot("vario_axis", axis=1, ax=ax2, x_max=x_max, label="fit on axis 1")
model.plot("vario_axis", axis=2, ax=ax2, x_max=x_max, label="fit on axis 2")
ax2.set_title("Fitting an anisotropic model")
ax2.legend()

plt.show()
```



Also, let's have a look at the field.

```
srf.plot()
```



Total running time of the script: (0 minutes 8.544 seconds)

Fit Variogram with automatic binning

```
import numpy as np
import gstools as gs
```

Generate a synthetic field with an exponential model.

```
x = np.random.RandomState(19970221).rand(1000) * 100.0
y = np.random.RandomState(20011012).rand(1000) * 100.0
model = gs.Exponential(dim=2, var=2, len_scale=8)
srf = gs.SRF(model, mean=0, seed=19970221)
field = srf((x, y))
print(field.var())
```

Out:

```
1.6791948750716688
```

Estimate the variogram of the field with automatic binning.

```
bin_center, gamma = gs.vario_estimate((x, y), field)
print("estimated bin number:", len(bin_center))
print("maximal bin distance:", max(bin_center))
```

Out:

```
estimated bin number: 21
maximal bin distance: 45.88516574202333
```

Fit the variogram with a stable model (no nugget fitted).

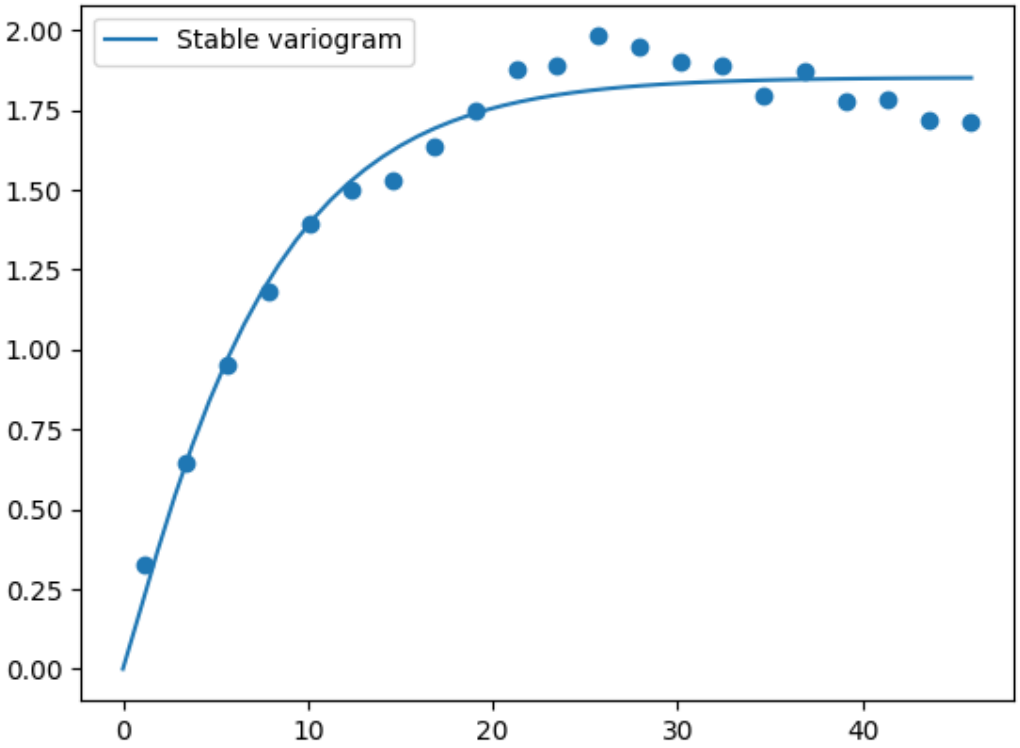
```
fit_model = gs.Stable(dim=2)
fit_model.fit_variogram(bin_center, gamma, nugget=False)
print(fit_model)
```

Out:

```
Stable(dim=2, var=1.85, len_scale=7.42, nugget=0.0, alpha=1.09)
```

Plot the fitting result.

```
ax = fit_model.plot(x_max=max(bin_center))
ax.scatter(bin_center, gamma)
```



Total running time of the script: (0 minutes 0.360 seconds)

Automatic binning with lat-lon data

In this example we demonstrate automatic binning for a tiny data set containing temperature records from Germany (See the detailed DWD example for more information on the data).

We use a data set from 20 meteo-stations choosen randomly.

```
import numpy as np
import gstools as gs

# lat, lon, temperature
data = np.array(
    [
        [52.9336, 8.237, 15.7],
        [48.6159, 13.0506, 13.9],
        [52.4853, 7.9126, 15.1],
        [50.7446, 9.345, 17.0],
        [52.9437, 12.8518, 21.9],
        [53.8633, 8.1275, 11.9],
        [47.8342, 10.8667, 11.4],
        [51.0881, 12.9326, 17.2],
        [48.406, 11.3117, 12.9],
        [49.7273, 8.1164, 17.2],
        [49.4691, 11.8546, 13.4],
        [48.0197, 12.2925, 13.9],
        [50.4237, 7.4202, 18.1],
```

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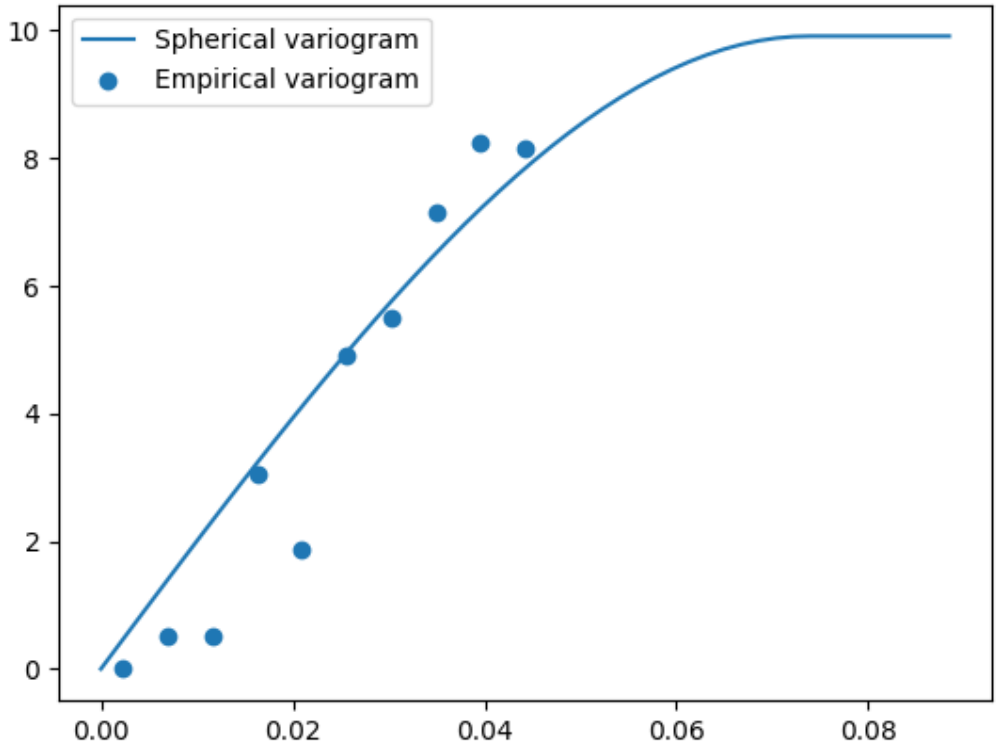
```
        [53.0316, 13.9908, 21.3],
        [53.8412, 13.6846, 21.3],
        [54.6792, 13.4343, 17.4],
        [49.9694, 9.9114, 18.6],
        [51.3745, 11.292, 20.2],
        [47.8774, 11.3643, 12.7],
        [50.5908, 12.7139, 15.8],
    ]
)
pos = data.T[:2]  # lat, lon
field = data.T[2]  # temperature
```

Since the overall range of these meteo-stations is too low, we can use the data-variance as additional information during the fit of the variogram.

```
emp_v = gs.vario_estimate(pos, field, latlon=True)
sph = gs.Spherical(latlon=True, rescale=gs.EARTH_RADIUS)
sph.fit_variogram(*emp_v, sill=np.var(field))
ax = sph.plot(x_max=2 * np.max(emp_v[0]))
ax.scatter(*emp_v, label="Empirical variogram")
ax.legend()
print(sph)
```



Out:

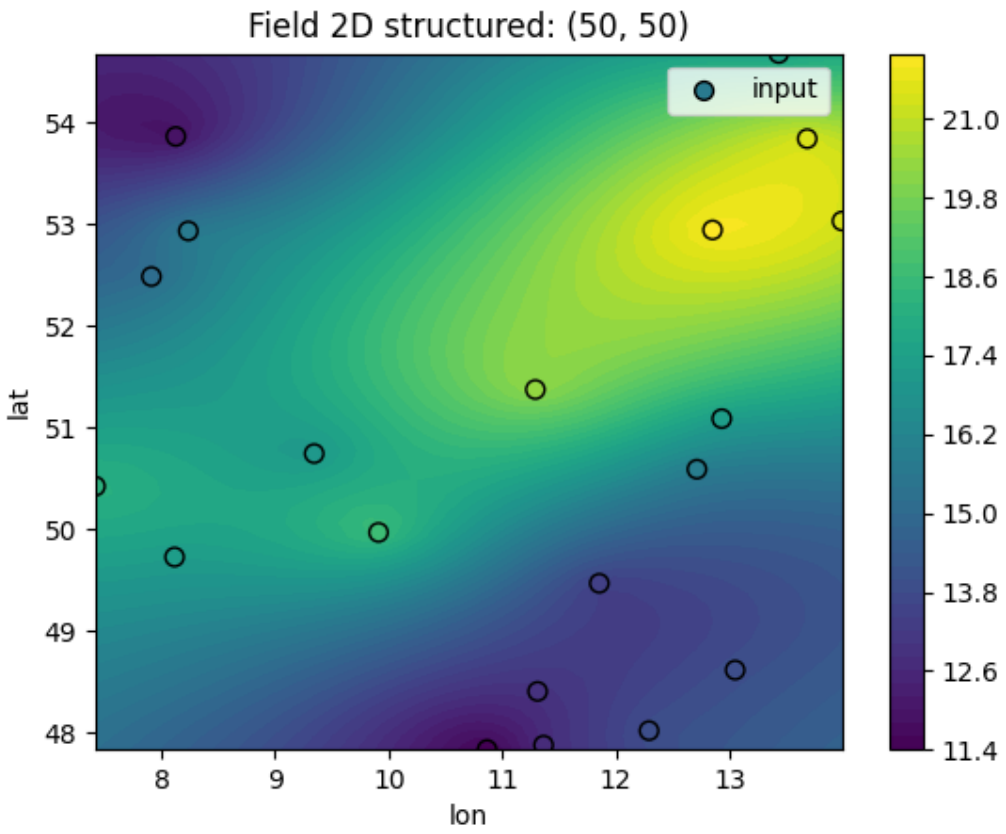
```
Spherical(latlon=True, var=9.91, len_scale=4.7e+02, nugget=1.78e-15, rescale=6.37e+03)
```

As we can see, the variogram fitting was successful and providing the data variance helped finding the right length-

scale.

Now, we'll use this covariance model to interpolate the given data with ordinary kriging.

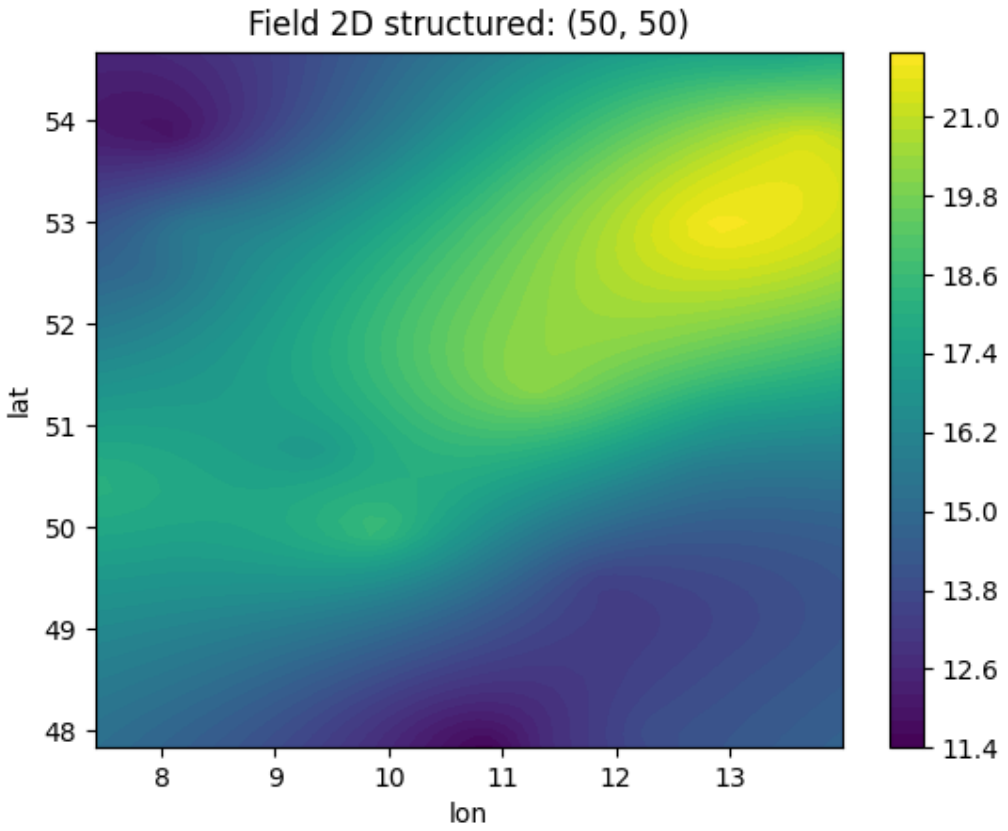
```
# enclosing box for data points
grid_lat = np.linspace(np.min(pos[0]), np.max(pos[0]))
grid_lon = np.linspace(np.min(pos[1]), np.max(pos[1]))
# ordinary kriging
krige = gs.krige.Ordinary(sph, pos, field)
krige((grid_lat, grid_lon), mesh_type="structured")
ax = krige.plot()
# plotting lat on y-axis and lon on x-axis
ax.scatter(pos[1], pos[0], 50, c=field, edgecolors="k", label="input")
ax.legend()
```



Looks good, doesn't it?

This workflow is also implemented in the *Krige* class, by setting `fit_variogram=True`. Then the whole procedure shortens:

```
krige = gs.krige.Ordinary(sph, pos, field, fit_variogram=True)
krige.structured((grid_lat, grid_lon))

# plot the result
krige.plot()
# show the fitting results
print(krige.model)
```



Out:

```
Spherical(latlon=True, var=9.91, len_scale=4.7e+02, nugget=1.78e-15, rescale=6.37e+03)
```

This example shows, that setting up variogram estimation and kriging routines is straight forward with GSTools!

Total running time of the script: (0 minutes 0.581 seconds)

2.4 Random Vector Field Generation

In 1970, Kraichnan was the first to suggest a randomization method. For studying the diffusion of single particles in a random incompressible velocity field, he came up with a randomization method which includes a projector which ensures the incompressibility of the vector field.

Without loss of generality we assume that the mean velocity $\bar{U}$ is oriented towards the direction of the first basis vector $\mathbf{e}_1$. Our goal is now to generate random fluctuations with a given covariance model around this mean velocity. And at the same time, making sure that the velocity field remains incompressible or in other words, ensure $\nabla \cdot \mathbf{U} = 0$. This can be done by using the randomization method we already know, but adding a projector to every mode being summed:

$$\mathbf{U}(\mathbf{x}) = \bar{U}\mathbf{e}_1 - \sqrt{\frac{\sigma^2}{N}} \sum_{i=1}^{N} \mathbf{p}(\mathbf{k}_i) \left[Z_{1,i} \cos\left(\langle \mathbf{k}_i, \mathbf{x} \rangle\right) + \sin\left(\langle \mathbf{k}_i, \mathbf{x} \rangle\right) \right]$$

with the projector

$$\mathbf{p}(\mathbf{k}_i) = \mathbf{e}_1 - \frac{\mathbf{k}_i k_1}{k^2} \ .$$

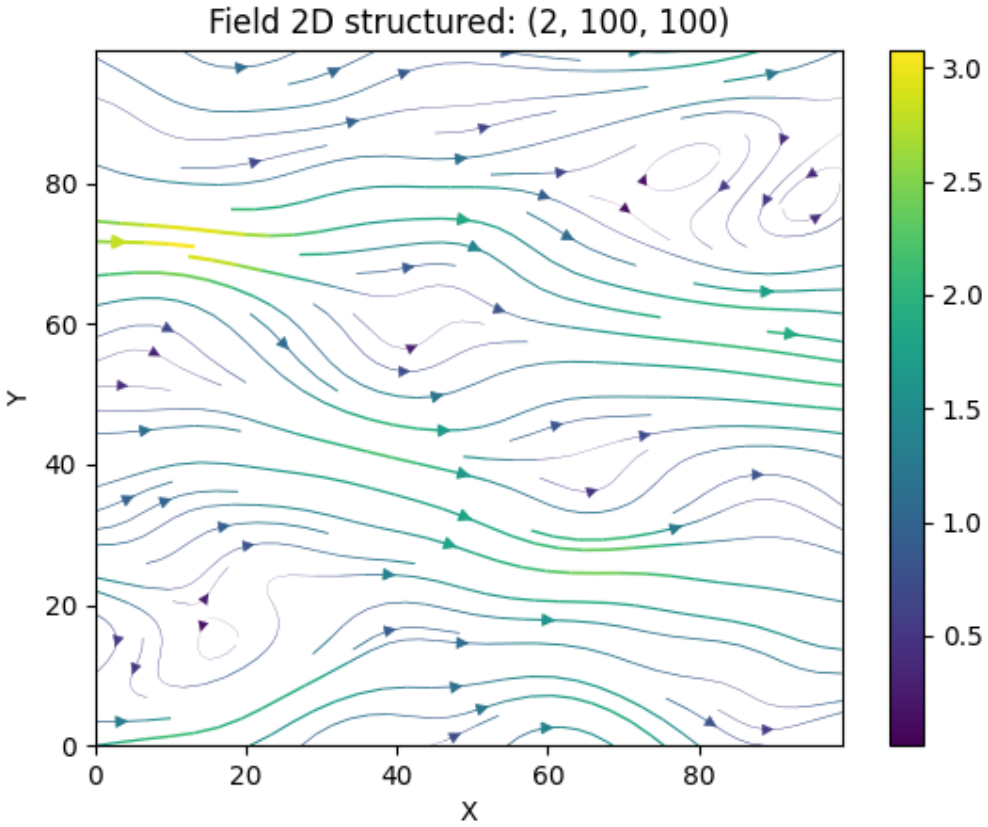
By calculating $\nabla \cdot \mathbf{U} = 0$, it can be verified, that the resulting field is indeed incompressible.

Examples

Generating a Random 2D Vector Field

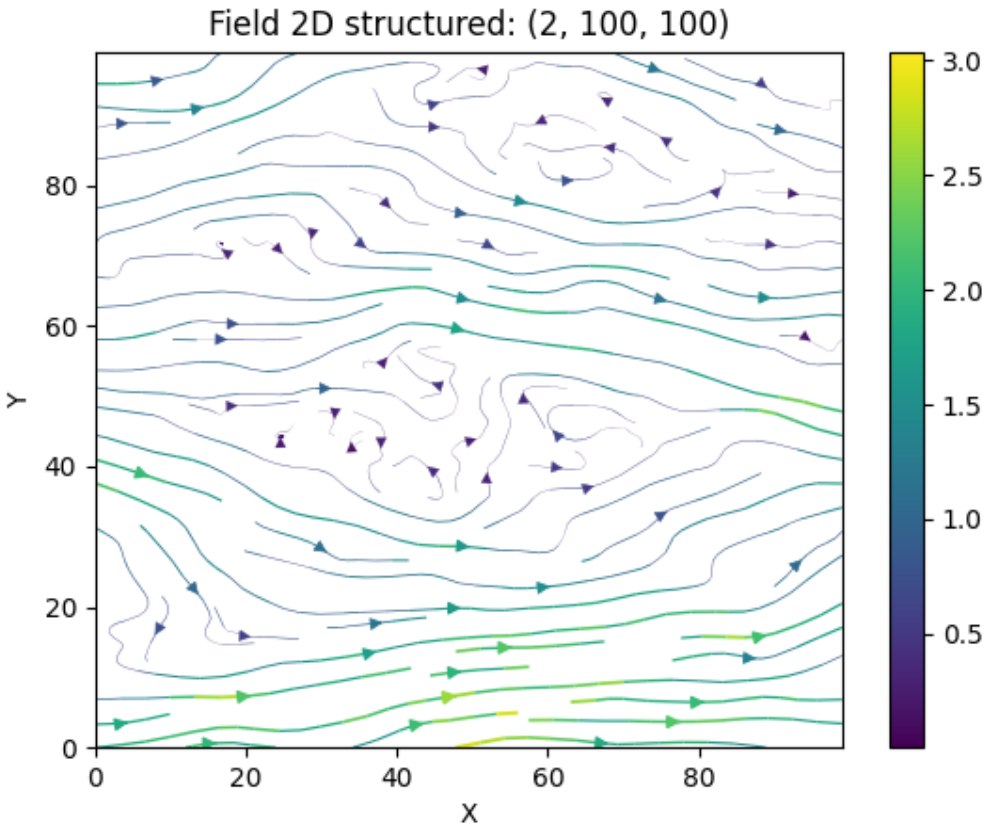
As a first example we are going to generate a 2d vector field with a Gaussian covariance model on a structured grid:

```
import numpy as np
import gstools as gs

# the grid
x = np.arange(100)
y = np.arange(100)

# a smooth Gaussian covariance model
model = gs.Gaussian(dim=2, var=1, len_scale=10)
srf = gs.SRF(model, generator="VectorField", seed=19841203)
srf((x, y), mesh_type="structured")
srf.plot()
```



Let us have a look at the influence of the covariance model. Choosing the exponential model and keeping all other parameters the same

```
# a rougher exponential covariance model
model2 = gs.Exponential(dim=2, var=1, len_scale=10)
srf.model = model2
srf((x, y), mesh_type="structured", seed=19841203)
srf.plot()
```



and we see, that the wiggles are much “rougher” than the smooth Gaussian ones.

Applications

One great advantage of the Kraichnan method is, that after some initializations, one can compute the velocity field at arbitrary points, online, with hardly any overhead. This means, that for a Lagrangian transport simulation for example, the velocity can be evaluated at each particle position very efficiently and without any interpolation. These field interpolations are a common problem for Lagrangian methods.

Total running time of the script: (0 minutes 2.974 seconds)

Generating a Random 3D Vector Field

In this example we are going to generate a random 3D vector field with a Gaussian covariance model. The mesh on which we generate the field will be externally defined and it will be generated by PyVista.

```
import gstools as gs
import pyvista as pv

# mainly for setting a white background
pv.set_plot_theme("document")
```

create a uniform grid with PyVista

```
dim, spacing, origin = (40, 30, 10), (1, 1, 1), (-10, 0, 0)
mesh = pv.UniformGrid(dim, spacing, origin)
```

create an incompressible random 3d velocity field on the given mesh with added mean velocity in x-direction

```
model = gs.Gaussian(dim=3, var=3, len_scale=1.5)
srf = gs.SRF(model, mean=(0.5, 0, 0), generator="VectorField", seed=198412031)
srf.mesh(mesh, points="points", name="Velocity")
```

Now, we can do the plotting

```
streamlines = mesh.streamlines(
    "Velocity",
    terminal_speed=0.0,
    n_points=800,
    source_radius=2.5,
)

# set a fancy camera position
cpos = [(25, 23, 17), (0, 10, 0), (0, 0, 1)]

p = pv.Plotter()
# adding an outline might help navigating in 3D space
# p.add_mesh(mesh.outline(), color="k")
p.add_mesh(
    streamlines.tube(radius=0.005),
    show_scalar_bar=False,
    diffuse=0.5,
    ambient=0.5,
)
```

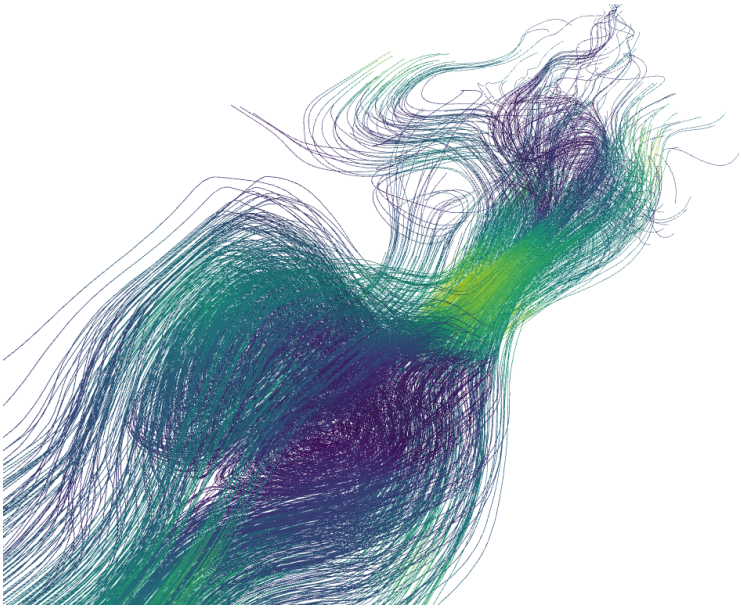
Out:

```
/home/docs/checkouts/readthedocs.org/user_builds/gstools/envs/v1.3.1/lib/python3.7/
↪site-packages/pyvista/plotting/plotting.py:97: UserWarning:
This system does not appear to be running an xserver.
PyVista will likely segfault when rendering.

Try starting a virtual frame buffer with xvfb, or using
  ``pyvista.start_xvfb()``

  warnings.warn('\n'
```

Note: PyVista is not working on readthedocs, but you can try it out yourself by uncommenting the following line of code.

```
# p.show(cpos=cpos)
```

The result should look like this:



Total running time of the script: (0 minutes 5.047 seconds)

2.5 Kriging

The subpackage `gstools.krige` provides routines for Gaussian process regression, also known as kriging. Kriging is a method of data interpolation based on predefined covariance models.

The aim of kriging is to derive the value of a field at some point x_0, when there are fixed observed values $z(x_1) \dots z(x_n)$ at given points x_i.

The resluting value z_0 at x_0 is calculated as a weighted mean:

$$z_0 = \sum_{i=1}^{n} w_i \cdot z_i$$

The weights $W = (w_1, \dots, w_n)$ depent on the given covariance model and the location of the target point.

The different kriging approaches provide different ways of calculating W.

The `Krige` class provides everything in one place and you can switch on/off the features you want:

- *unbiased*: the weights have to sum up to *1*. If true, this results in `Ordinary` kriging, where the mean is estimated, otherwise it will result in `Simple` kriging, where the mean has to be given.
- *drift_functions*: you can give a polynomial order or a list of self defined functions representing the internal drift of the given values. This drift will be fitted internally during the kriging interpolation. This results in `Universal` kriging.
- *ext_drift*: You can also give an external drift per point to the routine. In contrast to the internal drift, that is evaluated at the desired points with the given functions, the external drift has to given for each point form an "external" source. This results in `ExtDrift` kriging.
- *trend*, *mean*, *normalizer*: These are used to pre- and post-process data. If you already have fitted a trend model that is provided as a callable function, you can give it to the kriging routine. Normalizer are power-transformations to gain normality. *mean* behaves similar to *trend* but is applied at another position:
  1. conditioning data is de-trended (substracting trend)
  2. detrended conditioning data is then normalized (in order to follow a normal distribution)
  3. normalized conditioning data is set to zero mean (subtracting mean)

Cosequently, when there is no normalizer given, trend and mean are the same thing and only one should be used. *Detrended* kriging is a shortcut to provide only a trend and simple kriging with normal data.

- *exact* and *cond_err*: To incorporate the nugget effect and/or measurement errors, one can set *exact* to *False* and provide either individual measurement errors for each point or set the nugget as a constant measurement error everywhere.
- *pseudo_inv*: Sometimes the inversion of the kriging matrix can be numerically unstable. This occurs for examples in cases of redundant input values. In this case we provide a switch to use the pseudo-inverse of the matrix. Then redundant conditional values will automatically be averaged.

Note: All mentioned features can be combined within the *Krige* class. All other kriging classes are just shortcuts to this class with a limited list of input parameters.

The routines for kriging are almost identical to the routines for spatial random fields, with regard to their handling. First you define a covariance model, as described in *The Covariance Model*, then you initialize the kriging class with this model:

```
import gstools as gs
# condtions
cond_pos = [...]
cond_val = [...]
model = gs.Gaussian(dim=1, var=0.5, len_scale=2)
krig = gs.krige.Simple(model, cond_pos=cond_pos, cond_val=cond_val, mean=1)
```

The resulting field instance `krig` has the same methods as the *SRF* class. You can call it to evaluate the kriged field at different points, you can plot the latest field or you can export the field and so on.

Provided Kriging Methods

The following kriging methods are provided within the submodule *gstools.krige*.

Krige(model, cond_pos, cond_val[, ...])	A Swiss Army knife for kriging.
Simple(model, cond_pos, cond_val[, mean, ...])	Simple kriging.
Ordinary(model, cond_pos, cond_val[, ...])	Ordinary kriging.
Universal(model, cond_pos, cond_val, ...[, ...])	Universal kriging.
ExtDrift(model, cond_pos, cond_val, ext_drift)	External drift kriging (EDK).
Detrended(model, cond_pos, cond_val, trend)	Detrended simple kriging.

Examples

Simple Kriging

Simple kriging assumes a known mean of the data. For simplicity we assume a mean of 0, which can be achieved by subtracting the mean from the observed values and subsequently adding it to the resulting data.

The resulting equation system for W is given by:

$$W = \begin{pmatrix} c(x_1, x_1) & \cdots & c(x_1, x_n) \\ \vdots & \ddots & \vdots \\ c(x_n, x_1) & \cdots & c(x_n, x_n) \end{pmatrix}^{-1} \begin{pmatrix} c(x_1, x_0) \\ \vdots \\ c(x_n, x_0) \end{pmatrix}$$

Thereby $c(x_i, x_j)$ is the covariance of the given observations.

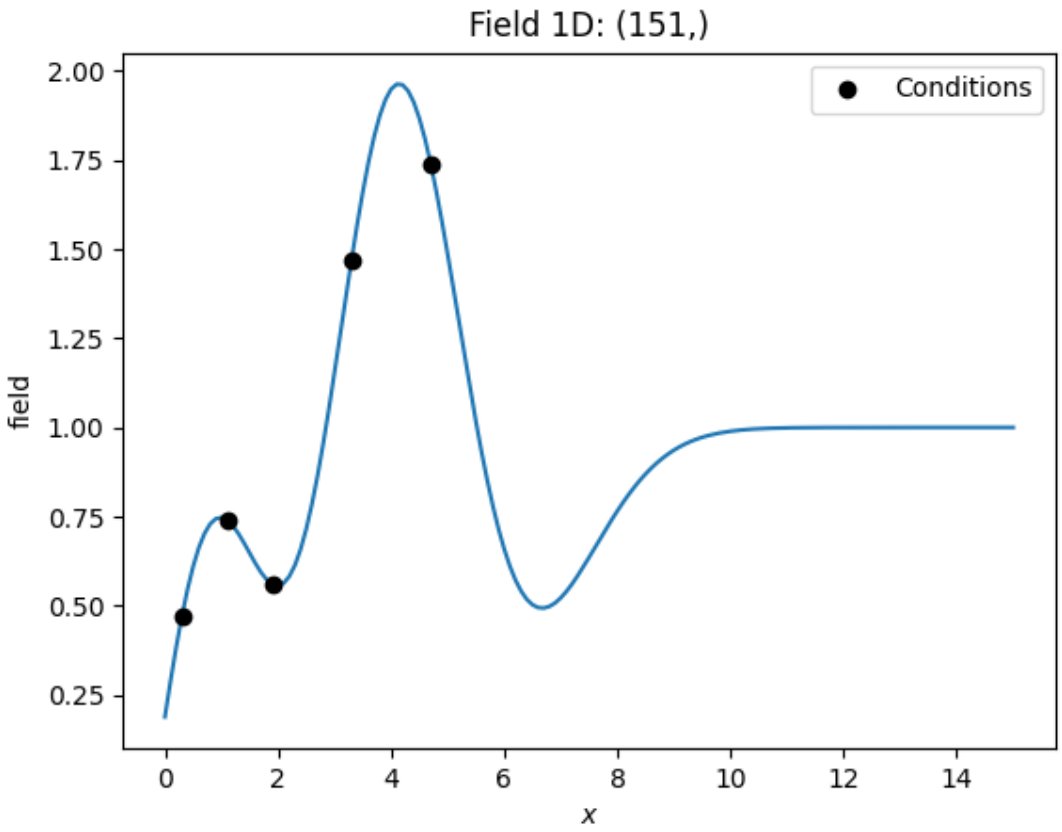
Example

Here we use simple kriging in 1D (for plotting reasons) with 5 given observations/conditions. The mean of the field has to be given beforehand.

```
import numpy as np
from gstools import Gaussian, krige

# condtions
cond_pos = [0.3, 1.9, 1.1, 3.3, 4.7]
cond_val = [0.47, 0.56, 0.74, 1.47, 1.74]
# resulting grid
gridx = np.linspace(0.0, 15.0, 151)
# spatial random field class
model = Gaussian(dim=1, var=0.5, len_scale=2)
```

```
krig = krige.Simple(model, mean=1, cond_pos=cond_pos, cond_val=cond_val)
krig(gridx)
```

```
ax = krig.plot()
ax.scatter(cond_pos, cond_val, color="k", zorder=10, label="Conditions")
ax.legend()
```



Total running time of the script: (0 minutes 0.110 seconds)

Ordinary Kriging

Ordinary kriging will estimate an appropriate mean of the field, based on the given observations/conditions and the covariance model used.

The resulting system of equations for W is given by:

$$\begin{pmatrix} W \\ \mu \end{pmatrix} = \begin{pmatrix} c(x_1, x_1) & \cdots & c(x_1, x_n) & 1 \\ \vdots & \ddots & \vdots & \vdots \\ c(x_n, x_1) & \cdots & c(x_n, x_n) & 1 \\ 1 & \cdots & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} c(x_1, x_0) \\ \vdots \\ c(x_n, x_0) \\ 1 \end{pmatrix}$$

Thereby $c(x_i, x_j)$ is the covariance of the given observations and μ is a Lagrange multiplier to minimize the kriging error and estimate the mean.

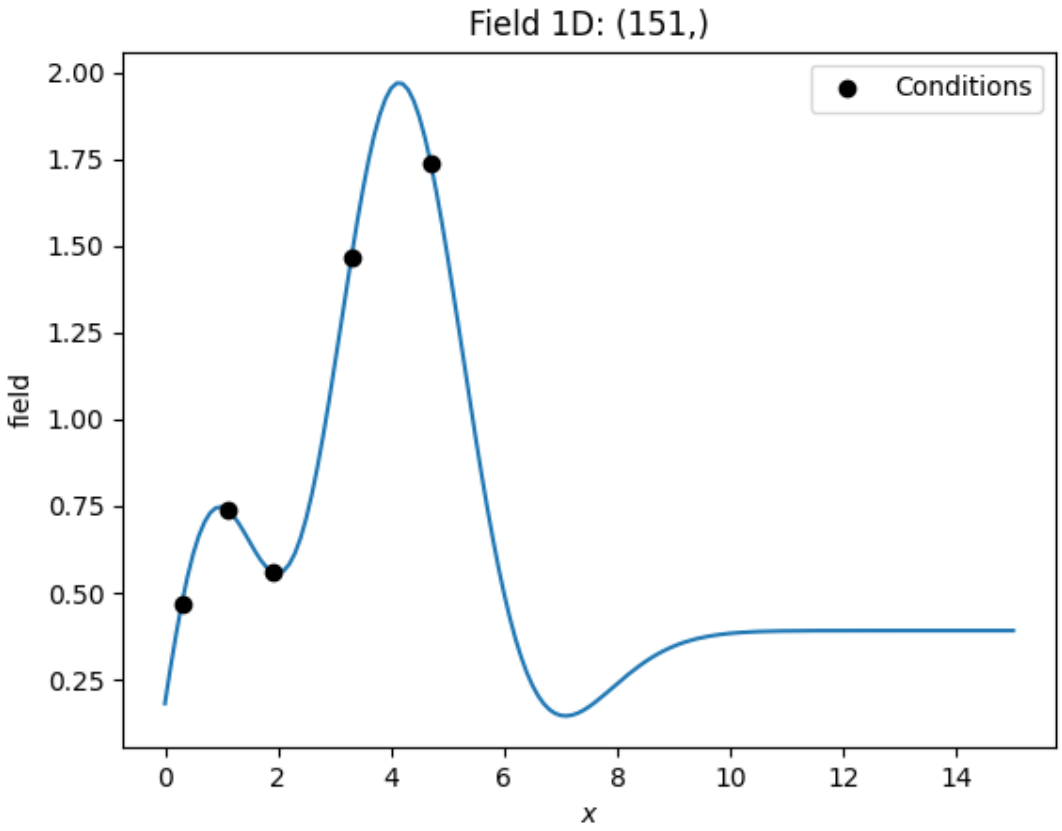
Example

Here we use ordinary kriging in 1D (for plotting reasons) with 5 given observations/conditions. The estimated mean can be accessed by `krig.mean`.

```
import numpy as np
from gstools import Gaussian, krige

# condtions
cond_pos = [0.3, 1.9, 1.1, 3.3, 4.7]
cond_val = [0.47, 0.56, 0.74, 1.47, 1.74]
# resulting grid
gridx = np.linspace(0.0, 15.0, 151)
# spatial random field class
model = Gaussian(dim=1, var=0.5, len_scale=2)
```

```
krig = krige.Ordinary(model, cond_pos=cond_pos, cond_val=cond_val)
krig(gridx)
```

```
ax = krig.plot()
ax.scatter(cond_pos, cond_val, color="k", zorder=10, label="Conditions")
ax.legend()
```



Total running time of the script: (0 minutes 0.107 seconds)

Interface to PyKrige

To use fancier methods like regression kriging, we provide an interface to PyKrige (>v1.5), which means you can pass a GSTools covariance model to the kriging routines of PyKrige.

To demonstrate the general workflow, we compare ordinary kriging of PyKrige with the corresponding GSTools routine in 2D:

```
import numpy as np
import gstools as gs
from pykrige.ok import OrdinaryKriging
from matplotlib import pyplot as plt

# conditioning data
cond_x = [0.3, 1.9, 1.1, 3.3, 4.7]
cond_y = [1.2, 0.6, 3.2, 4.4, 3.8]
cond_val = [0.47, 0.56, 0.74, 1.47, 1.74]

# grid definition for output field
gridx = np.arange(0.0, 5.5, 0.1)
gridy = np.arange(0.0, 6.5, 0.1)
```

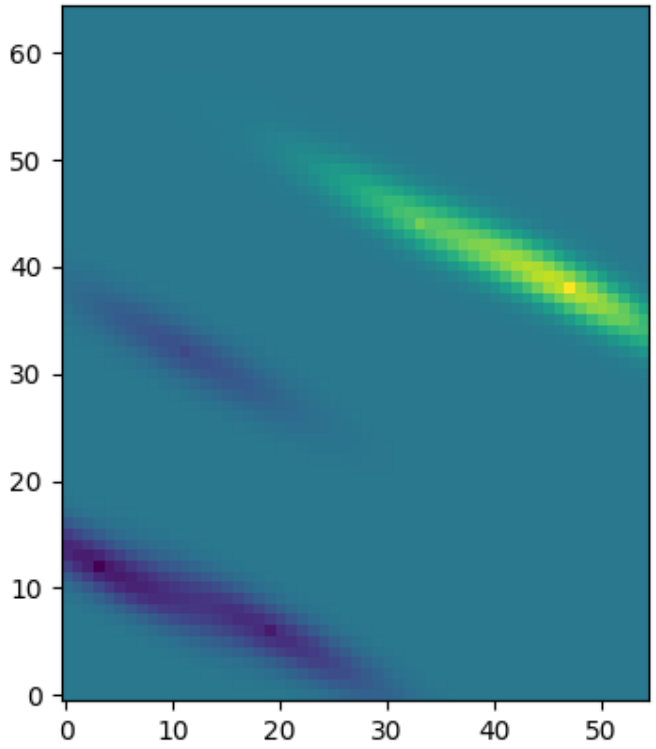
A GSTools based *Gaussian* covariance model:

```
model = gs.Gaussian(
    dim=2, len_scale=1, anis=0.2, angles=-0.5, var=0.5, nugget=0.1
)
```

Ordinary Kriging with PyKrige

One can pass the defined GSTools model as variogram model, which will *not* be fitted to the given data. By providing the GSTools model, rotation and anisotropy are also automatically defined:

```
OK1 = OrdinaryKriging(cond_x, cond_y, cond_val, variogram_model=model)
z1, ss1 = OK1.execute("grid", gridx, gridy)
plt.imshow(z1, origin="lower")
plt.show()
```

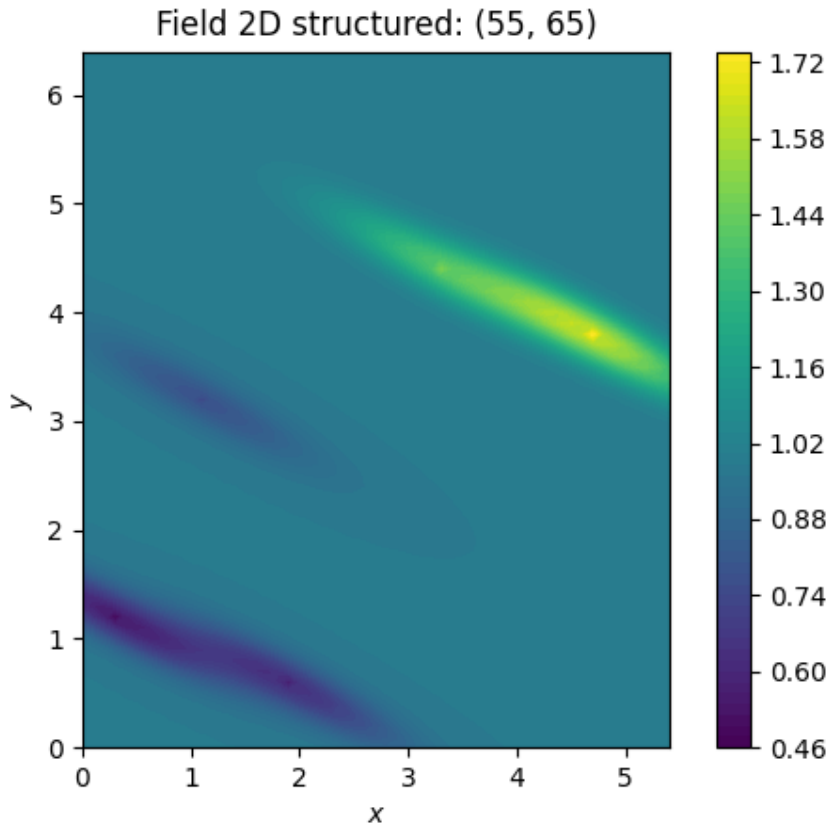


Ordinary Kriging with GSTools

The *Ordinary* kriging class is provided by GSTools as a shortcut to define ordinary kriging with the general *Krige* class.

PyKrige's routines are using exact kriging by default (when given a nugget). To reproduce this behavior in GSTools, we have to set `exact=True`.

```
OK2 = gs.krige.Ordinary(model, [cond_x, cond_y], cond_val, exact=True)
OK2.structured([gridx, gridy])
ax = OK2.plot()
ax.set_aspect("equal")
```



Total running time of the script: (0 minutes 0.319 seconds)

Compare Kriging

```
import numpy as np
from gstools import Gaussian, krige
import matplotlib.pyplot as plt

# condtions
cond_pos = [0.3, 1.9, 1.1, 3.3, 4.7]
cond_val = [0.47, 0.56, 0.74, 1.47, 1.74]
# resulting grid
gridx = np.linspace(0.0, 15.0, 151)
```

A gaussian variogram model.

```
model = Gaussian(dim=1, var=0.5, len_scale=2)
```

Two kriged fields. One with simple and one with ordinary kriging.

```
kr1 = krige.Simple(model=model, mean=1, cond_pos=cond_pos, cond_val=cond_val)
kr2 = krige.Ordinary(model=model, cond_pos=cond_pos, cond_val=cond_val)
kr1(gridx)
kr2(gridx)
```

```
plt.plot(gridx, kr1.field, label="simple kriged field")
plt.plot(gridx, kr2.field, label="ordinary kriged field")
```

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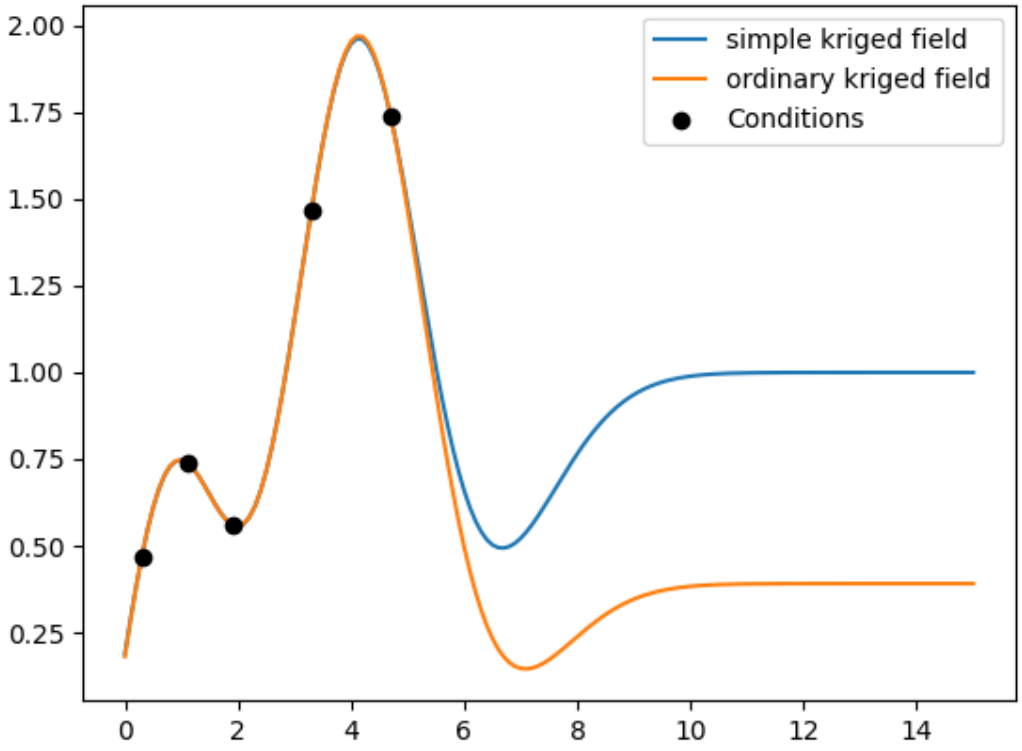
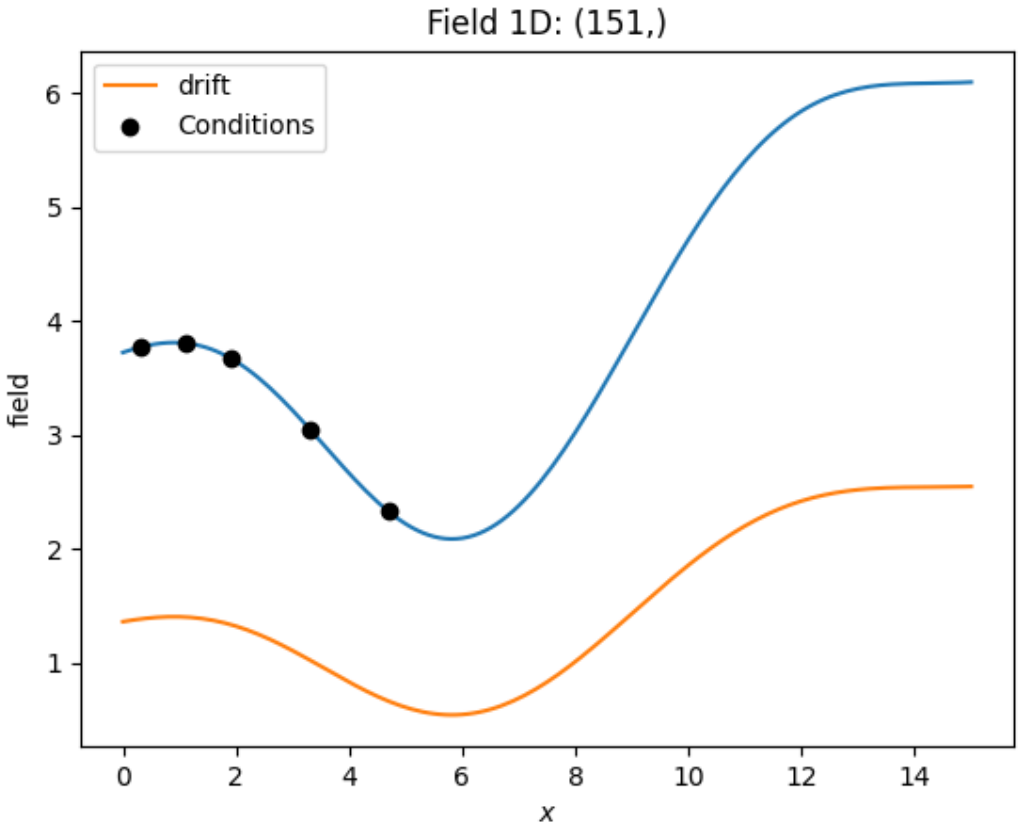
```
plt.scatter(cond_pos, cond_val, color="k", zorder=10, label="Conditions")
plt.legend()
plt.show()
```



Total running time of the script: (0 minutes 0.113 seconds)

External Drift Kriging




```
import numpy as np
from gstools import SRF, Gaussian, krige

# synthetic condtions with a drift
drift_model = Gaussian(dim=1, len_scale=4)
drift = SRF(drift_model, seed=1010)
cond_pos = [0.3, 1.9, 1.1, 3.3, 4.7]
ext_drift = drift(cond_pos)
cond_val = ext_drift * 2 + 1
# resulting grid
gridx = np.linspace(0.0, 15.0, 151)
grid_drift = drift(gridx)
# kriging
model = Gaussian(dim=1, var=2, len_scale=4)
krig = krige.ExtDrift(model, cond_pos, cond_val, ext_drift)
krig(gridx, ext_drift=grid_drift)
ax = krig.plot()
ax.scatter(cond_pos, cond_val, color="k", zorder=10, label="Conditions")
ax.plot(gridx, grid_drift, label="drift")
ax.legend()
```

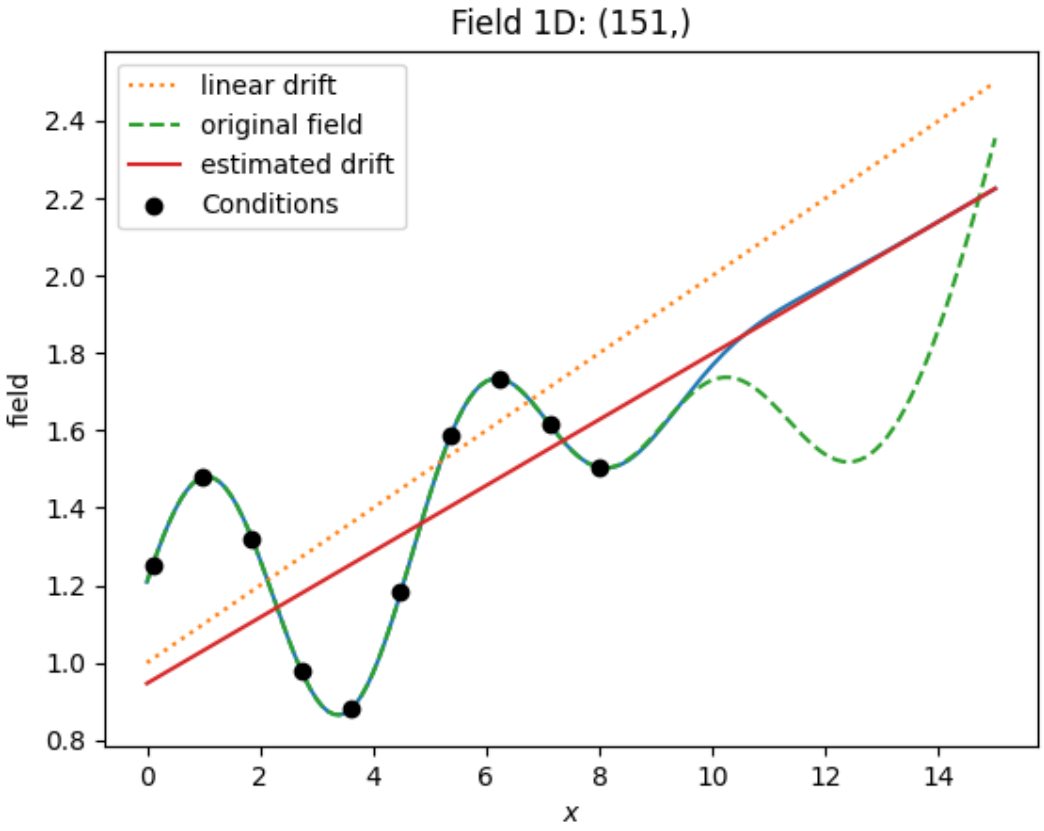
Total running time of the script: (0 minutes 0.117 seconds)

Universal Kriging

You can give a polynomial order or a list of self defined functions representing the internal drift of the given values. This drift will be fitted internally during the kriging interpolation.

In the following we are creating artificial data, where a linear drift was added. The resulting samples are then used as input for Universal kriging.

The "linear" drift is then estimated during the interpolation. To access only the estimated mean/drift, we provide a switch *only_mean* in the call routine.




```
import numpy as np
from gstools import SRF, Gaussian, krige

# synthetic condtions with a drift
drift_model = Gaussian(dim=1, var=0.1, len_scale=2)
drift = SRF(drift_model, seed=101)
cond_pos = np.linspace(0.1, 8, 10)
cond_val = drift(cond_pos) + cond_pos * 0.1 + 1
# resulting grid
gridx = np.linspace(0.0, 15.0, 151)
drift_field = drift(gridx) + gridx * 0.1 + 1
# kriging
model = Gaussian(dim=1, var=0.1, len_scale=2)
krig = krige.Universal(model, cond_pos, cond_val, "linear")
krig(gridx)
ax = krig.plot()
ax.scatter(cond_pos, cond_val, color="k", zorder=10, label="Conditions")
ax.plot(gridx, gridx * 0.1 + 1, ":", label="linear drift")
```

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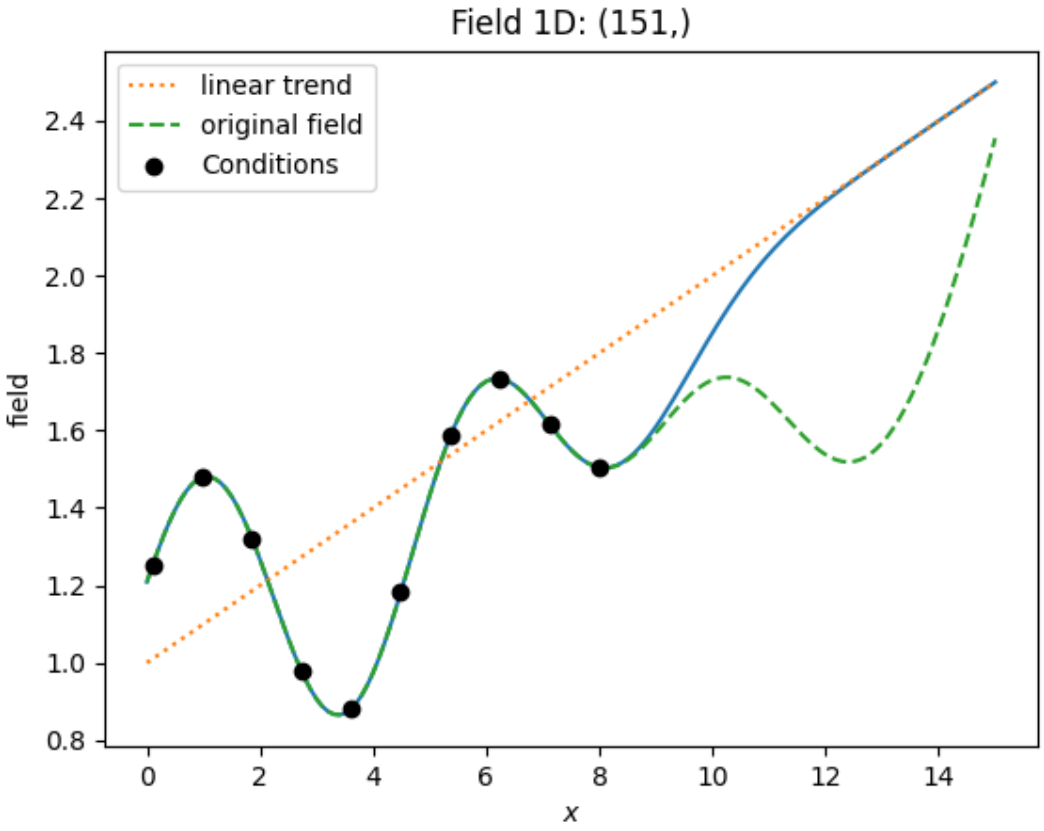
```
ax.plot(gridx, drift_field, "--", label="original field")

mean = krig(gridx, only_mean=True)
ax.plot(gridx, mean, label="estimated drift")

ax.legend()
```

Total running time of the script: (0 minutes 0.136 seconds)

Detrended Kriging




```
import numpy as np
from gstools import SRF, Gaussian, krige


def trend(x):
    """Example for a simple linear trend."""
    return x * 0.1 + 1


# synthetic condtions with trend/drift
drift_model = Gaussian(dim=1, var=0.1, len_scale=2)
drift = SRF(drift_model, seed=101)
cond_pos = np.linspace(0.1, 8, 10)
cond_val = drift(cond_pos) + trend(cond_pos)
# resulting grid
```

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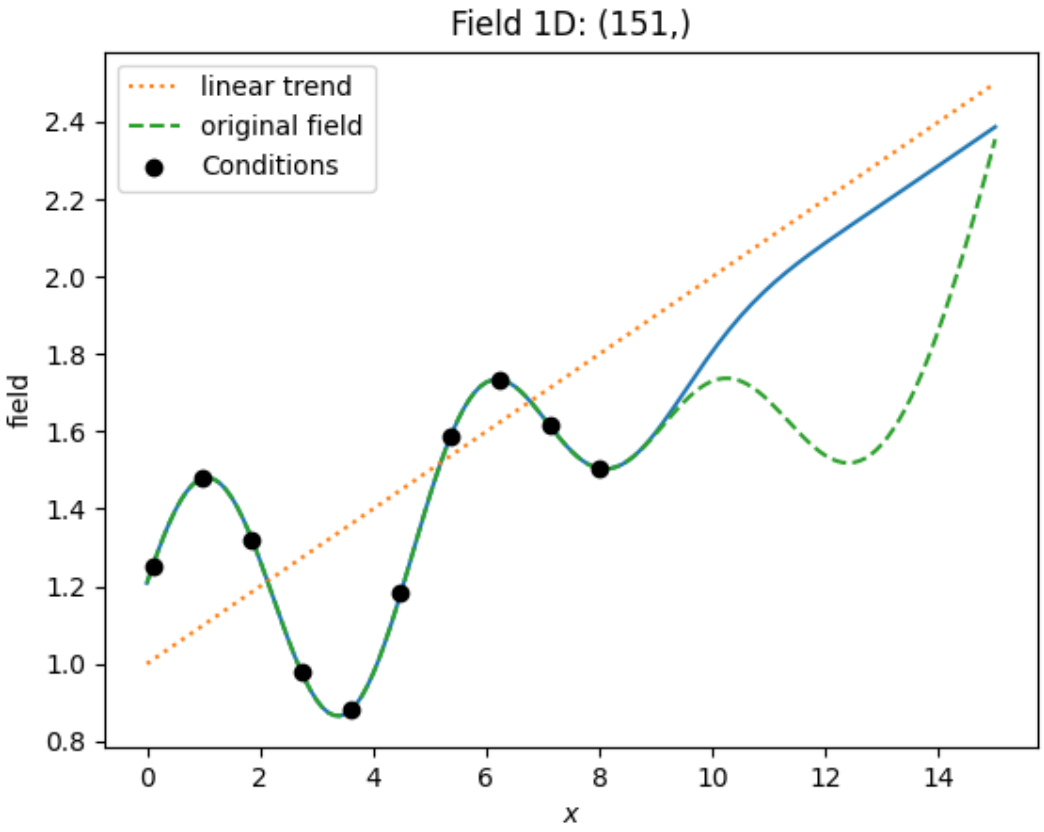
```
gridx = np.linspace(0.0, 15.0, 151)
drift_field = drift(gridx) + trend(gridx)
# kriging
model = Gaussian(dim=1, var=0.1, len_scale=2)
krig_trend = krige.Detrended(model, cond_pos, cond_val, trend)
krig_trend(gridx)
ax = krig_trend.plot()
ax.scatter(cond_pos, cond_val, color="k", zorder=10, label="Conditions")
ax.plot(gridx, trend(gridx), ":", label="linear trend")
ax.plot(gridx, drift_field, "--", label="original field")
ax.legend()
```

Total running time of the script: (0 minutes 0.132 seconds)

Detrended Ordinary Kriging




```
import numpy as np
from gstools import SRF, Gaussian, krige


def trend(x):
    """Example for a simple linear trend."""
    return x * 0.1 + 1


# synthetic condtions with trend/drift
```

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```
drift_model = Gaussian(dim=1, var=0.1, len_scale=2)
drift = SRF(drift_model, seed=101)
cond_pos = np.linspace(0.1, 8, 10)
cond_val = drift(cond_pos) + trend(cond_pos)
# resulting grid
gridx = np.linspace(0.0, 15.0, 151)
drift_field = drift(gridx) + trend(gridx)
# kriging
model = Gaussian(dim=1, var=0.1, len_scale=2)
krig_trend = krige.Ordinary(model, cond_pos, cond_val, trend=trend)
krig_trend(gridx)
ax = krig_trend.plot()
ax.scatter(cond_pos, cond_val, color="k", zorder=10, label="Conditions")
ax.plot(gridx, trend(gridx), ":", label="linear trend")
ax.plot(gridx, drift_field, "--", label="original field")
ax.legend()
```

Total running time of the script: (0 minutes 0.132 seconds)

Incorporating measurement errors

To incorporate the nugget effect and/or given measurement errors, one can set *exact* to *False* and provide either individual measurement errors for each point or set the nugget as a constant measurement error everywhere.

In the following we will show the influence of the nugget and measurement errors.

```
import numpy as np
import gstools as gs

# condtions
cond_pos = [0.3, 1.1, 1.9, 3.3, 4.7]
cond_val = [0.47, 0.74, 0.56, 1.47, 1.74]
cond_err = [0.01, 0.0, 0.1, 0.05, 0]
# resulting grid
gridx = np.linspace(0.0, 15.0, 151)
# spatial random field class
model = gs.Gaussian(dim=1, var=0.9, len_scale=1, nugget=0.1)
```

Here we will use Simple kriging (*unbiased=False*) to interpolate the given conditions.

```
krig = gs.Krige(
    model=model,
    cond_pos=cond_pos,
    cond_val=cond_val,
    mean=1,
    unbiased=False,
    exact=False,
    cond_err=cond_err,
)
krig(gridx)
```

Let's plot the data. You can see, that the estimated values differ more from the input, when the given measurement errors get bigger. In addition we plot the standard deviation.

```
ax = krig.plot()
ax.scatter(cond_pos, cond_val, color="k", zorder=10, label="Conditions")
```

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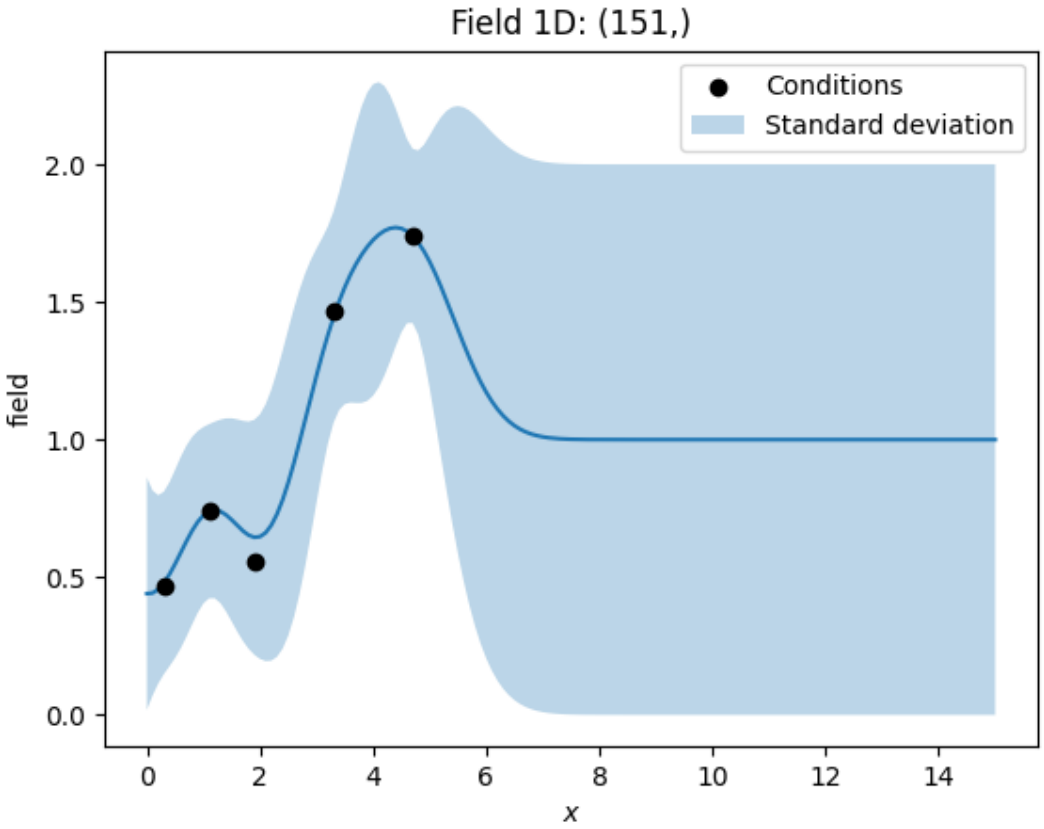
```
ax.fill_between(
    gridx,
    # plus/minus standard deviation (70 percent confidence interval)
    krig.field - np.sqrt(krig.krige_var),
    krig.field + np.sqrt(krig.krige_var),
    alpha=0.3,
    label="Standard deviation",
)
ax.legend()
```



Total running time of the script: (0 minutes 0.107 seconds)

Redundant data and pseudo-inverse

It can happen, that the kriging system gets numerically unstable. One reason could be, that the input data contains redundant conditioning points that hold different values.

To smoothly deal with such situations, you can use the pseudo inverse for the kriging matrix, which is enabled by default.

This will result in the average value for the redundant data.

Example

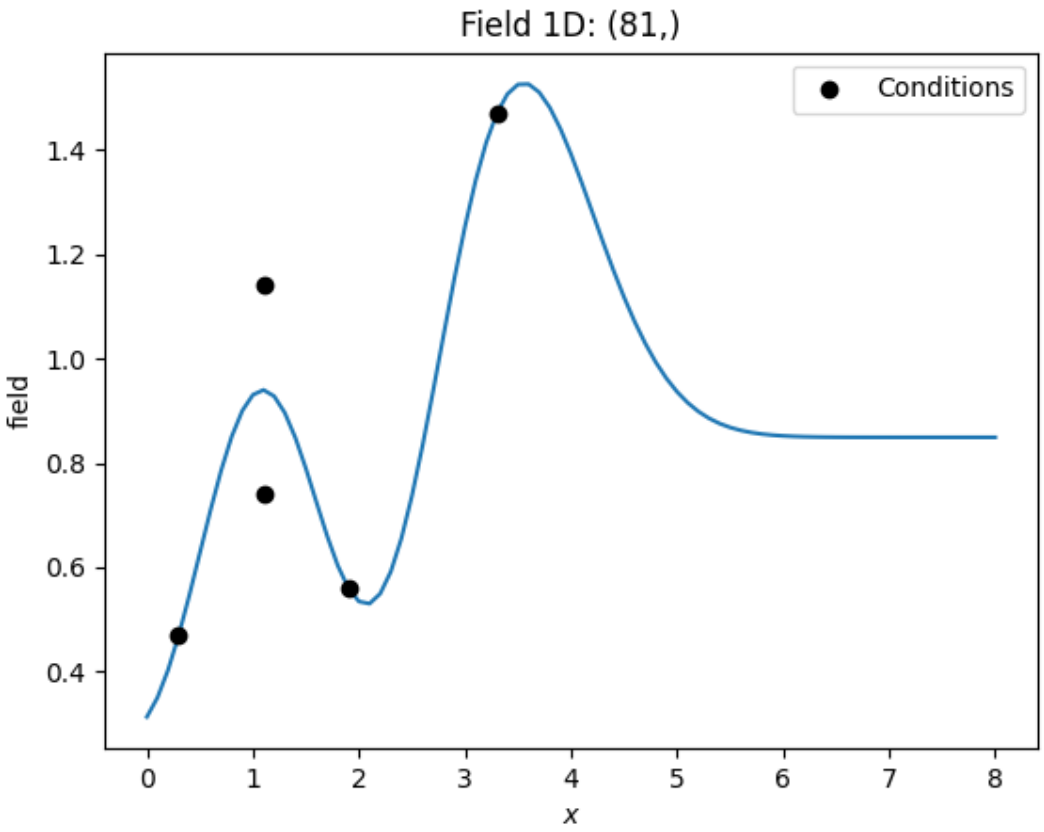
In the following we have two different values at the same location. The resulting kriging field will hold the average at this point.

```
import numpy as np
from gstools import Gaussian, krige

# condtions
cond_pos = [0.3, 1.9, 1.1, 3.3, 1.1]
cond_val = [0.47, 0.56, 0.74, 1.47, 1.14]
# resulting grid
gridx = np.linspace(0.0, 8.0, 81)
# spatial random field class
model = Gaussian(dim=1, var=0.5, len_scale=1)
```

```
krig = krige.Ordinary(model, cond_pos=cond_pos, cond_val=cond_val)
krig(gridx)
```

```
ax = krig.plot()
ax.scatter(cond_pos, cond_val, color="k", zorder=10, label="Conditions")
ax.legend()
```



Total running time of the script: (0 minutes 0.104 seconds)

2.6 Conditioned Fields

Kriged fields tend to approach the field mean outside the area of observations. To generate random fields, that coincide with given observations, but are still random according to a given covariance model away from the observations proximity, we provide the generation of conditioned random fields.

The idea behind conditioned random fields builds up on kriging. First we generate a field with a kriging method, then we generate a random field, with 0 as mean and 1 as variance that will be multiplied with the kriging standard deviation.

To do so, you can instantiate a *CondSRF* class with a configured *Krige* class.

The setup of the a conditioned random field should be as follows:

```
krige = gs.Krige(model, cond_pos, cond_val)
cond_srf = gs.CondSRF(krige)
field = cond_srf(grid)
```

Examples

Conditioning with Ordinary Kriging

Here we use ordinary kriging in 1D (for plotting reasons) with 5 given observations/conditions, to generate an ensemble of conditioned random fields.

```
import numpy as np
import matplotlib.pyplot as plt
import gstools as gs

# condtions
cond_pos = [0.3, 1.9, 1.1, 3.3, 4.7]
cond_val = [0.47, 0.56, 0.74, 1.47, 1.74]
gridx = np.linspace(0.0, 15.0, 151)
```

The conditioned spatial random field class depends on a Krige class in order to handle the conditions. This is created as described in the kriging tutorial.

Here we use a Gaussian covariance model and ordinary kriging for conditioning the spatial random field.

```
model = gs.Gaussian(dim=1, var=0.5, len_scale=1.5)
krige = gs.krige.Ordinary(model, cond_pos, cond_val)
cond_srf = gs.CondSRF(krige)
```

```
fields = []
for i in range(100):
    fields.append(cond_srf(gridx, seed=i))
    label = "Conditioned ensemble" if i == 0 else None
    plt.plot(gridx, fields[i], color="k", alpha=0.1, label=label)
plt.plot(gridx, cond_srf.krige(gridx, only_mean=True), label="estimated mean")
plt.plot(gridx, np.mean(fields, axis=0), linestyle=":", label="Ensemble mean")
plt.plot(gridx, cond_srf.krige.field, linestyle="dashed", label="kriged field")
plt.scatter(cond_pos, cond_val, color="k", zorder=10, label="Conditions")
# 99 percent confidence interval
conf = gs.tools.confidence_scaling(0.99)
plt.fill_between(
    gridx,
    cond_srf.krige.field - conf * np.sqrt(cond_srf.krige.krige_var),
```

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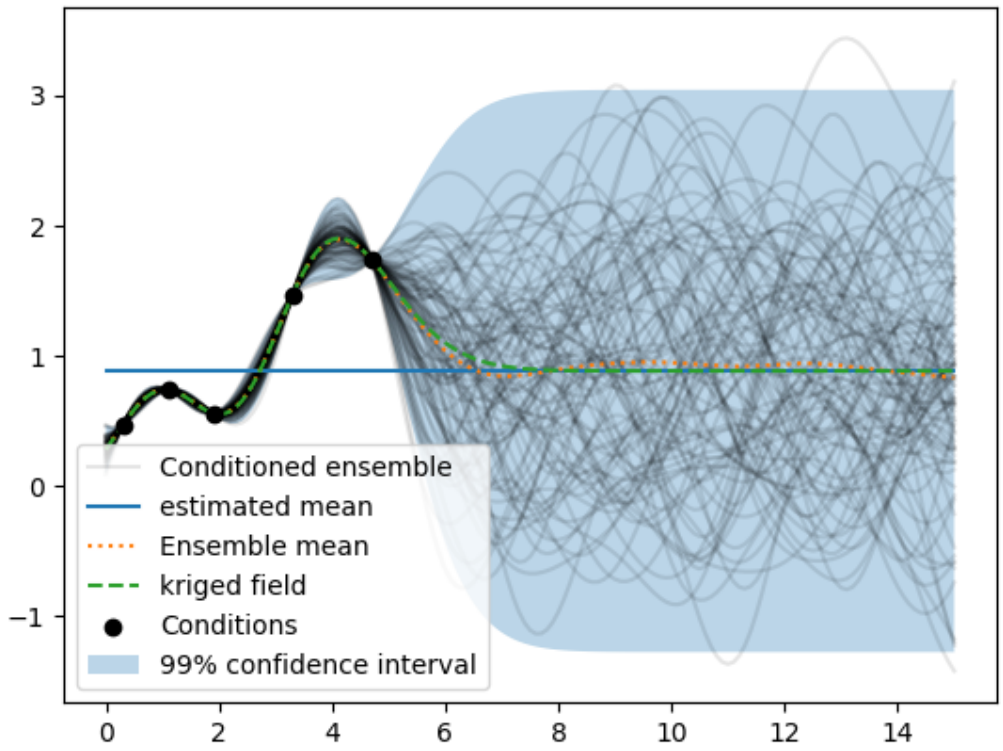
```
    cond_srf.krige.field + conf * np.sqrt(cond_srf.krige.krige_var),
    alpha=0.3,
    label="99% confidence interval",
)
plt.legend()
plt.show()
```



As you can see, the kriging field coincides with the ensemble mean of the conditioned random fields and the estimated mean is the mean of the far-field.

Total running time of the script: (0 minutes 1.308 seconds)

Creating an Ensemble of conditioned 2D Fields

Let's create an ensemble of conditioned random fields in 2D.

```
import numpy as np
import matplotlib.pyplot as plt
import gstools as gs


# conditioning data (x, y, value)
cond_pos = [[0.3, 1.9, 1.1, 3.3, 4.7], [1.2, 0.6, 3.2, 4.4, 3.8]]
cond_val = [0.47, 0.56, 0.74, 1.47, 1.74]

# grid definition for output field
x = np.arange(0, 5, 0.1)
```

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```
y = np.arange(0, 5, 0.1)

model = gs.Gaussian(dim=2, var=0.5, len_scale=5, anis=0.5, angles=-0.5)
krige = gs.Krige(model, cond_pos=cond_pos, cond_val=cond_val)
cond_srf = gs.CondSRF(krige)
```

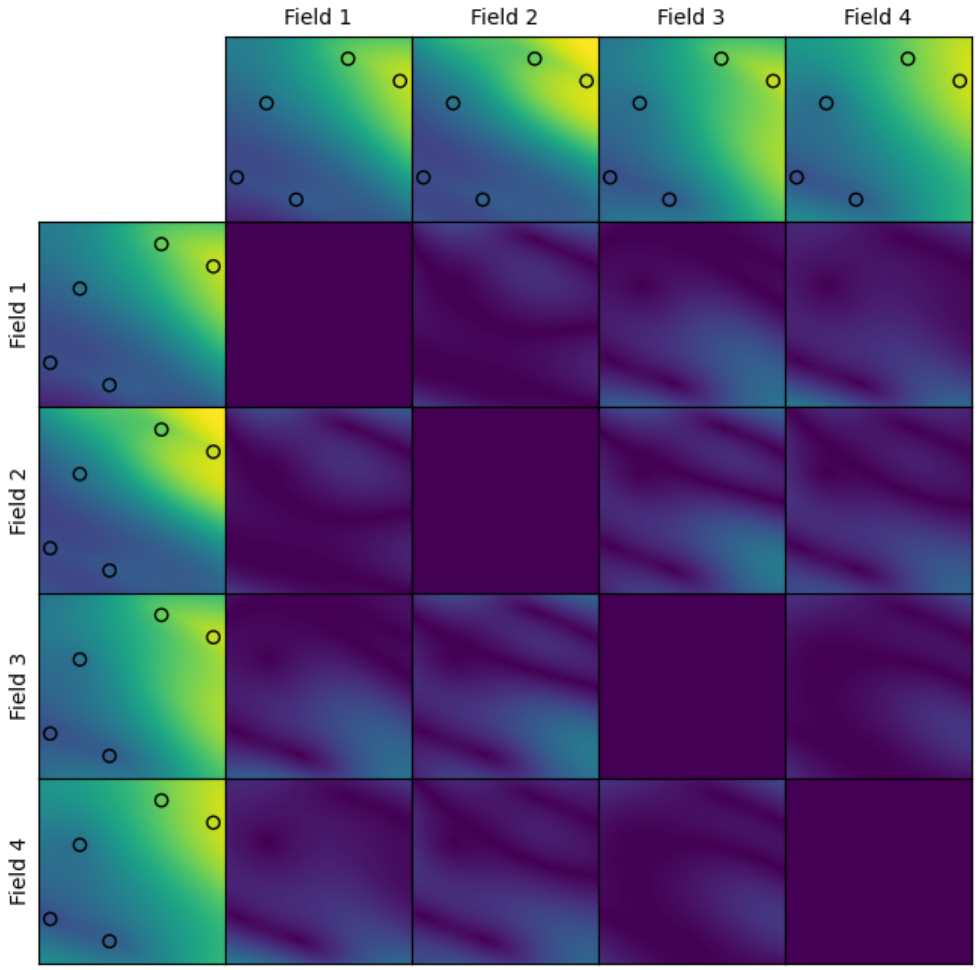
We create a list containing the generated conditioned fields.

```
ens_no = 4
field = []
for i in range(ens_no):
    field.append(cond_srf.structured([x, y], seed=i))
```

Now let's have a look at the pairwise differences between the generated fields. We will see, that they coincide at the given conditions.

```
fig, ax = plt.subplots(ens_no + 1, ens_no + 1, figsize=(8, 8))
# plotting kwargs for scatter and image
sc_kwargs = dict(c=cond_val, edgecolors="k", vmin=0, vmax=np.max(field))
im_kwargs = dict(extent=2 * [0, 5], origin="lower", vmin=0, vmax=np.max(field))
for i in range(ens_no):
    # conditioned fields and conditions
    ax[i + 1, 0].imshow(field[i].T, **im_kwargs)
    ax[i + 1, 0].scatter(*cond_pos, **sc_kwargs)
    ax[i + 1, 0].set_ylabel(f"Field {i+1}", fontsize=10)
    ax[0, i + 1].imshow(field[i].T, **im_kwargs)
    ax[0, i + 1].scatter(*cond_pos, **sc_kwargs)
    ax[0, i + 1].set_title(f"Field {i+1}", fontsize=10)
    # absolute differences
    for j in range(ens_no):
        ax[i + 1, j + 1].imshow(np.abs(field[i] - field[j]).T, **im_kwargs)

# beautify plots
ax[0, 0].axis("off")
for a in ax.flatten():
    a.set_xticklabels([]), a.set_yticklabels([])
    a.set_xticks([]), a.set_yticks([])
fig.subplots_adjust(wspace=0, hspace=0)
fig.show()
```



Total running time of the script: (0 minutes 1.248 seconds)

2.7 Field transformations

The generated fields of gstools are ordinary Gaussian random fields. In application there are several transformations to describe real world problems in an appropriate manner.

GStools provides a submodule *gstools.transform* with a range of common transformations:

binary(fld[, divide, upper, lower])	Binary transformation.
discrete(fld, values[, thresholds])	Discrete transformation.
boxcox(fld[, lmbda, shift])	(Inverse) Box-Cox transformation to denormalize data.
zinnharvey(fld[, conn])	Zinn and Harvey transformation to connect low or high values.
normal_force_moments(fld)	Force moments of a normal distributed field.

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Table 6 – continued from previous page

normal_to_lognormal(fld)	Transform normal distribution to log-normal distribution.
normal_to_uniform(fld)	Transform normal distribution to uniform distribution on [0, 1].
normal_to_arcsin(fld[, a, b])	Transform normal distribution to the bimodal arcsin distribution.
normal_to_uquad(fld[, a, b])	Transform normal distribution to U-quadratic distribution.

All the transformations take a field class, that holds a generated field, as input and will manipulate this field inplace.

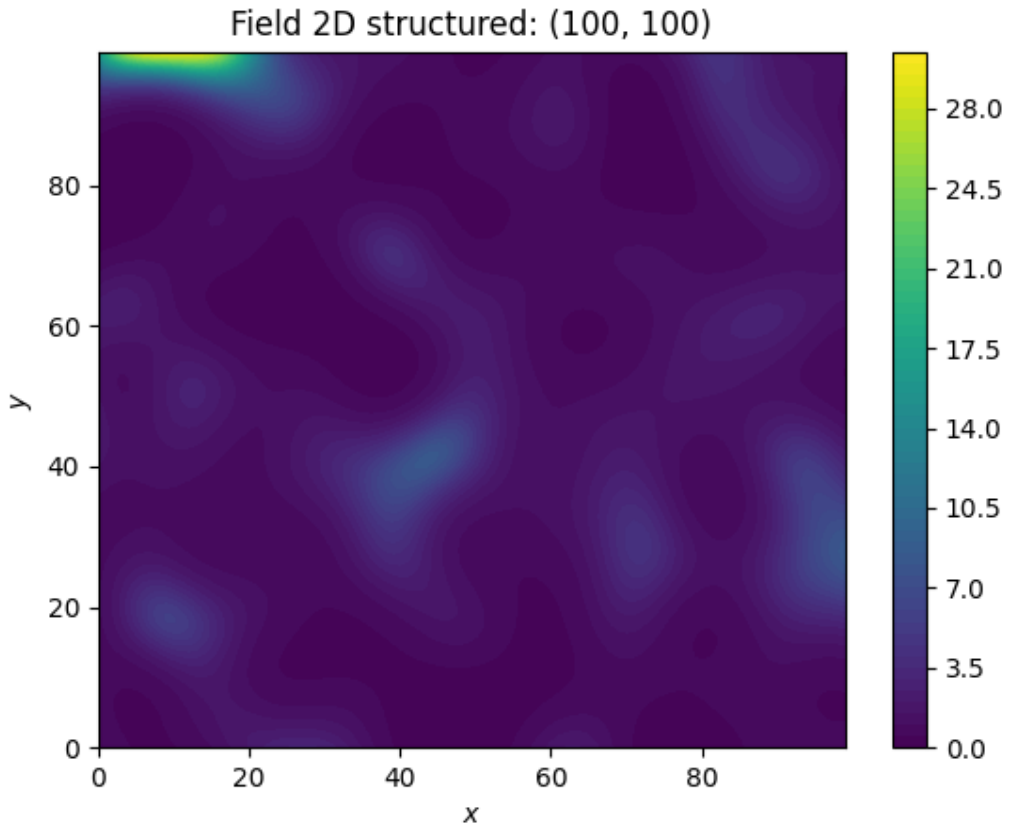
Simply import the transform submodule and apply a transformation to the srf class:

```
from gstools import transform as tf
...
tf.normal_to_lognormal(srf)
```

Examples

log-normal fields

Here we transform a field to a log-normal distribution:




```
import gstools as gs

# structured field with a size of 100x100 and a grid-size of 1x1
```

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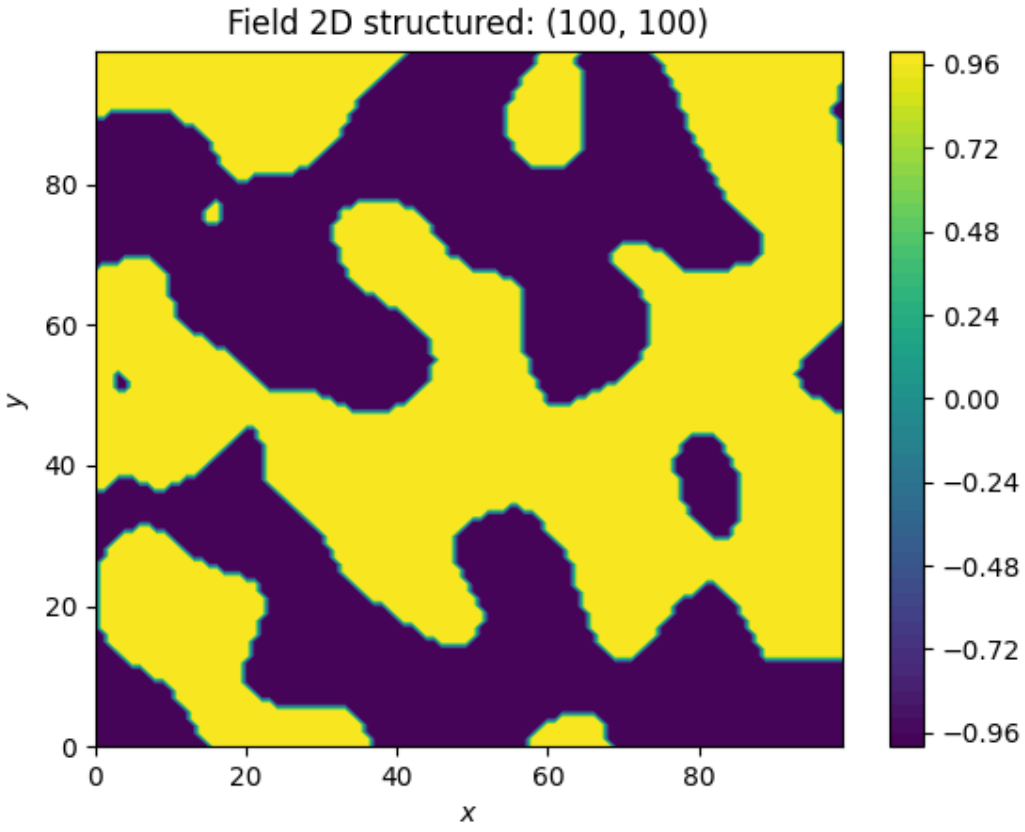
```
x = y = range(100)
model = gs.Gaussian(dim=2, var=1, len_scale=10)
srf = gs.SRF(model, seed=20170519)
srf.structured([x, y])
gs.transform.normal_to_lognormal(srf)
srf.plot()
```

Total running time of the script: (0 minutes 0.708 seconds)

binary fields

Here we transform a field to a binary field with only two values. The dividing value is the mean by default and the upper and lower values are derived to preserve the variance.

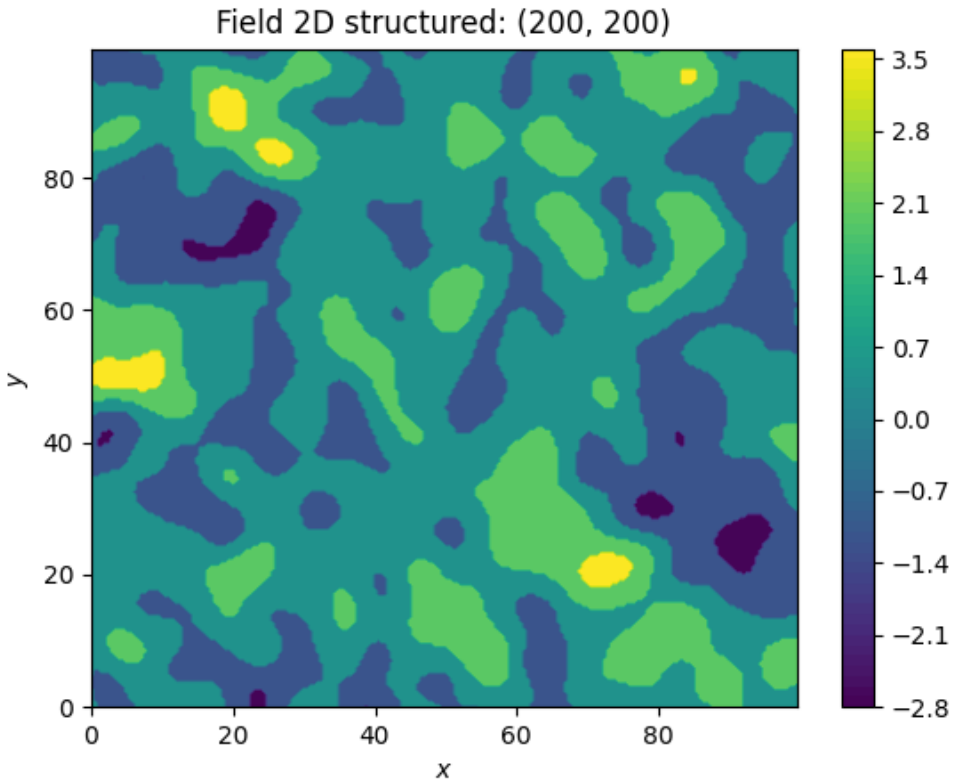


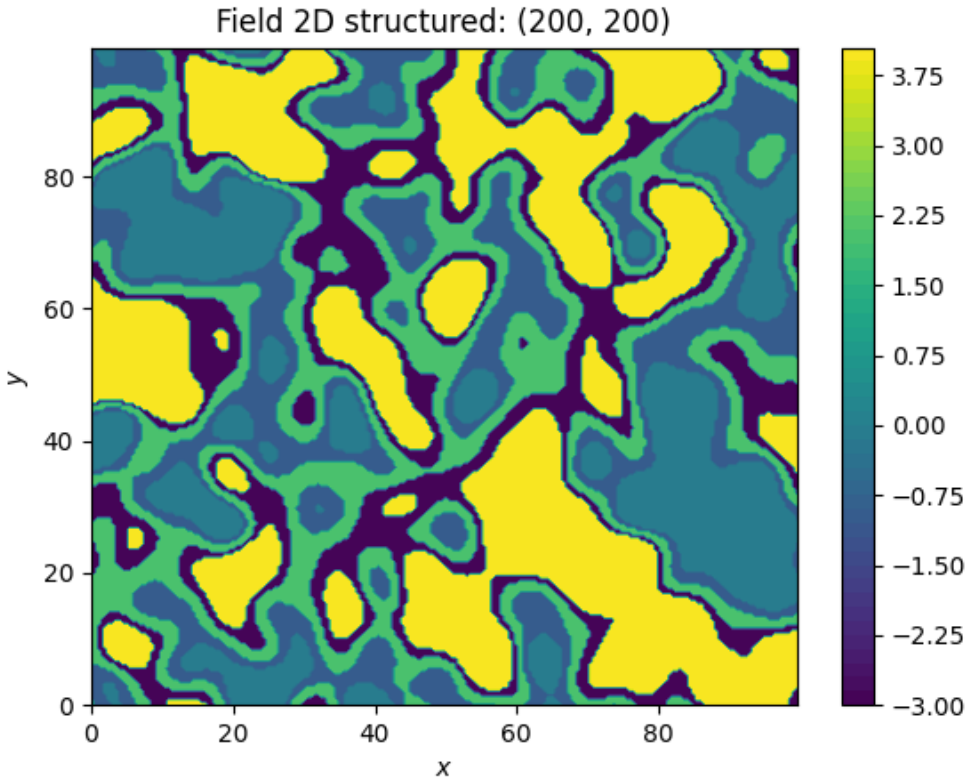

```
import gstools as gs

# structured field with a size of 100x100 and a grid-size of 1x1
x = y = range(100)
model = gs.Gaussian(dim=2, var=1, len_scale=10)
srf = gs.SRF(model, seed=20170519)
srf.structured([x, y])
gs.transform.binary(srf)
srf.plot()
```

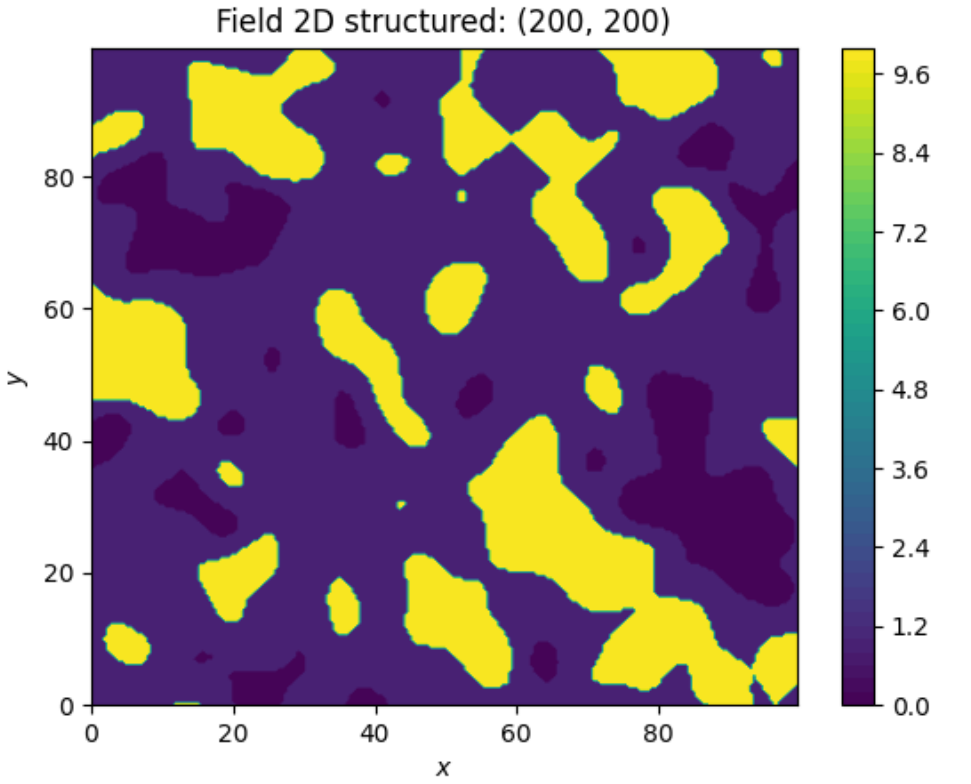
Total running time of the script: (0 minutes 0.759 seconds)

Discrete fields

Here we transform a field to a discrete field with values. If we do not give thresholds, the pairwise means of the given values are taken as thresholds. If thresholds are given, arbitrary values can be applied to the field.








```
import numpy as np
import gstools as gs
```

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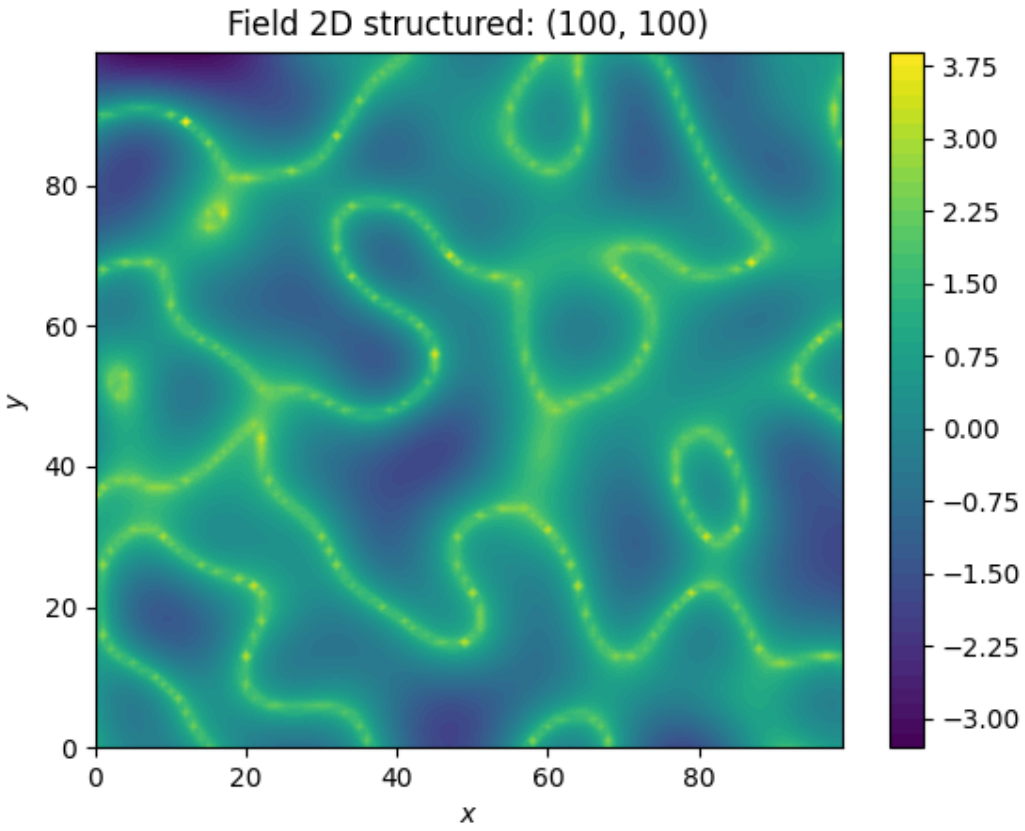
```
# structured field with a size of 100x100 and a grid-size of 0.5x0.5
x = y = np.arange(200) * 0.5
model = gs.Gaussian(dim=2, var=1, len_scale=5)
srf = gs.SRF(model, seed=20170519)

# create 5 equidistanly spaced values, thresholds are the arithmetic means
srf.structured([x, y])
discrete_values = np.linspace(np.min(srf.field), np.max(srf.field), 5)
gs.transform.discrete(srf, discrete_values)
srf.plot()

# calculate thresholds for equal shares
# but apply different values to the separated classes
discrete_values2 = [0, -1, 2, -3, 4]
srf.structured([x, y])
gs.transform.discrete(srf, discrete_values2, thresholds="equal")
srf.plot()

# user defined thresholds
thresholds = [-1, 1]
# apply different values to the separated classes
discrete_values3 = [0, 1, 10]
srf.structured([x, y])
gs.transform.discrete(srf, discrete_values3, thresholds=thresholds)
srf.plot()
```

Total running time of the script: (0 minutes 6.919 seconds)

Zinn & Harvey transformation

Here, we transform a field with the so called "Zinn & Harvey" transformation presented in Zinn & Harvey (2003). With this transformation, one could overcome the restriction that in ordinary Gaussian random fields the mean values are the ones being the most connected.




```
import gstools as gs

# structured field with a size of 100x100 and a grid-size of 1x1
x = y = range(100)
model = gs.Gaussian(dim=2, var=1, len_scale=10)
srf = gs.SRF(model, seed=20170519)
srf.structured([x, y])
gs.transform.zinnharvey(srf, conn="high")
srf.plot()
```

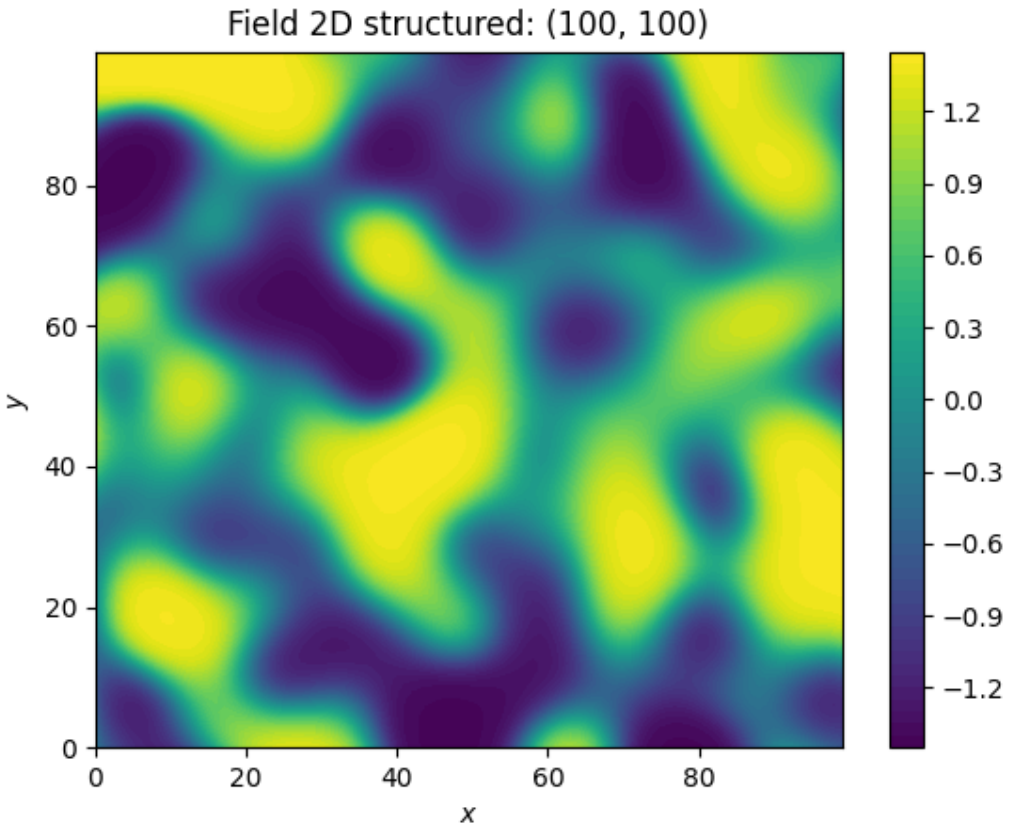
Total running time of the script: (0 minutes 0.771 seconds)

bimodal fields

We provide two transformations to obtain bimodal distributions:

- arcsin.
- uquad.

Both transformations will preserve the mean and variance of the given field by default.




```
import gstools as gs

# structured field with a size of 100x100 and a grid-size of 1x1
x = y = range(100)
model = gs.Gaussian(dim=2, var=1, len_scale=10)
srf = gs.SRF(model, seed=20170519)
field = srf.structured([x, y])
gs.transform.normal_to_arcsin(srf)
srf.plot()
```

Total running time of the script: (0 minutes 0.789 seconds)

Combinations

You can combine different transformations simply by successively applying them.

Here, we first force the single field realization to hold the given moments, namely mean and variance. Then we apply the Zinn & Harvey transformation to connect the low values. Afterwards the field is transformed to a binary field and last but not least, we transform it to log-values.

```
import gstools as gs

# structured field with a size of 100x100 and a grid-size of 1x1
x = y = range(100)
model = gs.Gaussian(dim=2, var=1, len_scale=10)
srf = gs.SRF(model, mean=-9, seed=20170519)
srf.structured([x, y])
gs.transform.normal_force_moments(srf)
```

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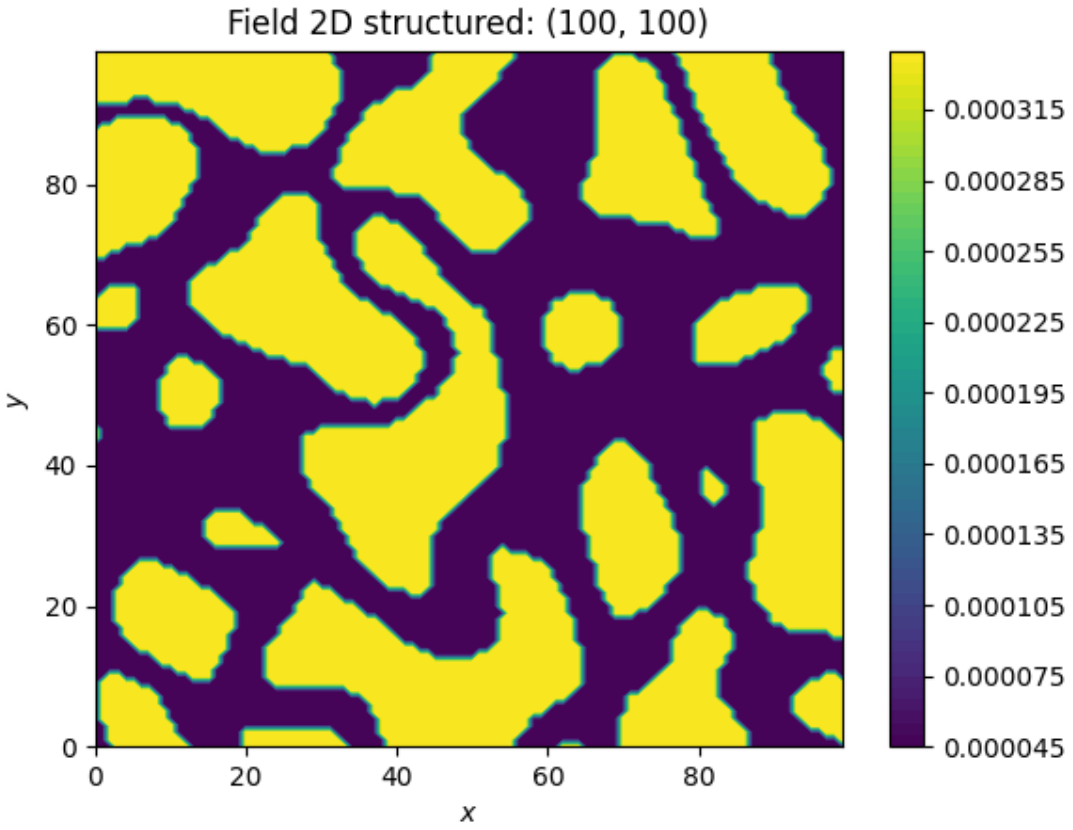
```
gs.transform.zinnharvey(srf, conn="low")
gs.transform.binary(srf)
gs.transform.normal_to_lognormal(srf)
srf.plot()
```



The resulting field could be interpreted as a transmissivity field, where the values of low permeability are the ones being the most connected and only two kinds of soil exist.

Total running time of the script: (0 minutes 0.840 seconds)

2.8 Geographic Coordinates

GSTools provides support for geographic coordinates given by:

- latitude `lat`: specifies the north–south position of a point on the Earth's surface
- longitude `lon`: specifies the east–west position of a point on the Earth's surface

If you want to use this feature for field generation or Kriging, you have to set up a geographical covariance Model by setting `latlon=True` in your desired model (see *CovModel*):

```
import numpy as np
import gstools as gs

model = gs.Gaussian(latlon=True, var=2, len_scale=np.pi / 16)
```

By doing so, the model will use the associated *Yadrenko* model on a sphere (see here). The *len_scale* is given in radians to scale the arc-length. In order to have a more meaningful length scale, one can use the `rescale` argument:

```
import gstools as gs

model = gs.Gaussian(latlon=True, var=2, len_scale=500, rescale=gs.EARTH_RADIUS)
```

Then `len_scale` can be interpreted as given in km.

A *Yadrenko* model C is derived from a valid isotropic covariance model in 3D C_{3D} by the following relation:

$$C(\zeta) = C_{3D}\left(2 \cdot \sin\left(\frac{\zeta}{2}\right)\right)$$

Where ζ is the great-circle distance.

Note: `lat` and `lon` are given in degree, whereas the great-circle distance $zeta$ is given in radians.

Note, that $2 \cdot \sin(\frac{\zeta}{2})$ is the chordal distance of two points on a sphere, which means we simply think of the earth surface as a sphere, that is cut out of the surrounding three dimensional space, when using the *Yadrenko* model.

Note: Anisotropy is not available with the geographical models, since their geometry is not euclidean. When passing values for *CovModel.anis* or *CovModel.angles*, they will be ignored.

Since the Yadrenko model comes from a 3D model, the model dimension will be 3 (see *CovModel.dim*) but the *field_dim* will be 2 in this case (see *CovModel.field_dim*).

Examples

Working with lat-lon random fields

In this example, we demonstrate how to generate a random field on geographical coordinates.

First we setup a model, with `latlon=True`, to get the associated Yadrenko model.

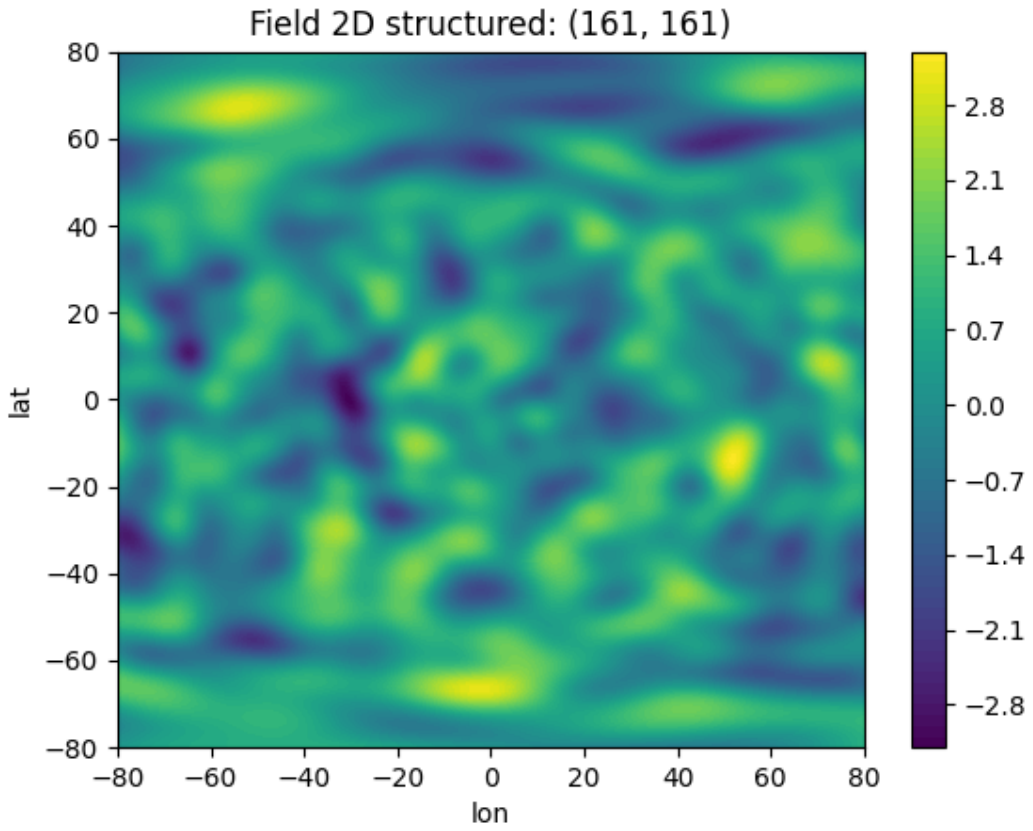
In addition, we will use the earth radius provided by *EARTH_RADIUS*, to have a meaningful length scale in km.

To generate the field, we simply pass `(lat, lon)` as the position tuple to the *SRF* class.

```
import gstools as gs

model = gs.Gaussian(latlon=True, var=1, len_scale=777, rescale=gs.EARTH_RADIUS)

lat = lon = range(-80, 81)
srf = gs.SRF(model, seed=1234)
field = srf.structured((lat, lon))
srf.plot()
```

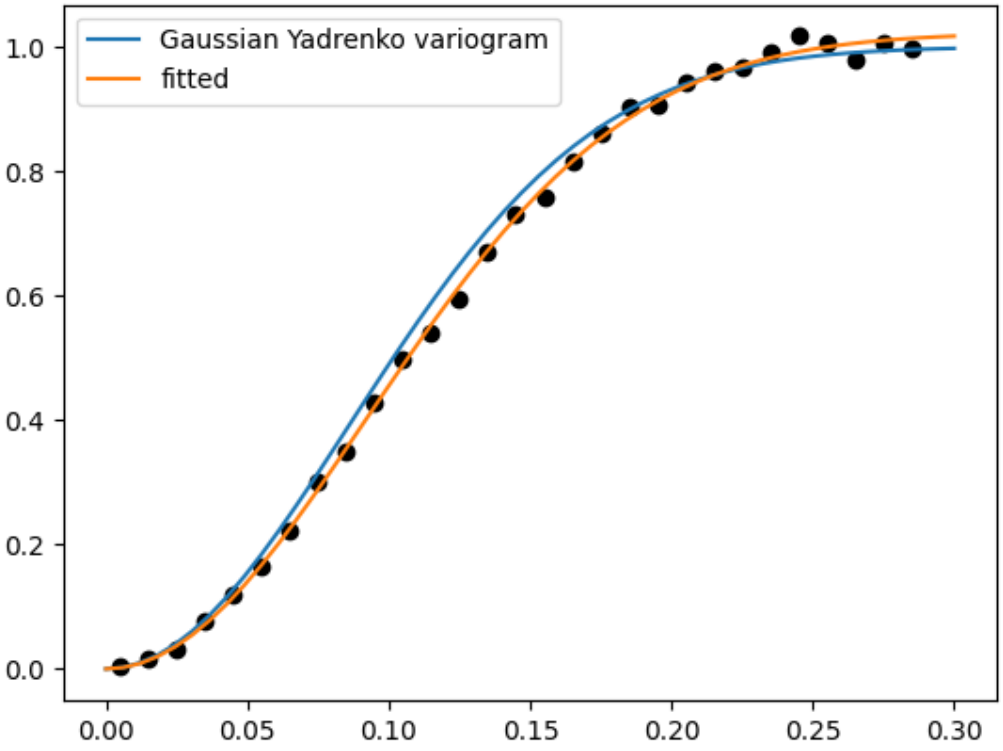


This was easy as always! Now we can use this field to estimate the empirical variogram in order to prove, that the generated field has the correct geo-statistical properties. The *vario_estimate* routine also provides a `latlon` switch to indicate, that the given field is defined on geographical variables.

As we will see, everthing went well… phew!

```
bin_edges = [0.01 * i for i in range(30)]
bin_center, emp_vario = gs.vario_estimate(
    (lat, lon),
    field,
    bin_edges,
    latlon=True,
    mesh_type="structured",
    sampling_size=2000,
    sampling_seed=12345,
)

ax = model.plot("vario_yadrenko", x_max=0.3)
model.fit_variogram(bin_center, emp_vario, nugget=False)
model.plot("vario_yadrenko", ax=ax, label="fitted", x_max=0.3)
ax.scatter(bin_center, emp_vario, color="k")
print(model)
```



Out:

```
Gaussian(latlon=True, var=1.02, len_scale=8.3e+02, nugget=0.0, rescale=6.37e+03)
```

Note: Note, that the estimated variogram coincides with the yadrenko variogram, which means it depends on the great-circle distance given in radians.

Keep that in mind when defining bins: The range is at most $\pi \approx 3.14$, which corresponds to the half globe.

Total running time of the script: (0 minutes 9.960 seconds)

Kriging geographical data

In this example we are going to interpolate actual temperature data from the German weather service DWD.

Data is retrieved utilizing the beautiful package wetterdienst, which serves as an API for the DWD data.

For better visualization, we also download a simple shapefile of the German borderline with cartopy.

In order to keep the number of dependecies low, the calls of both functions shown beneath are commented out.

```
import numpy as np
import matplotlib.pyplot as plt
import gstools as gs


def get_borders_germany():
    """Download simple german shape file with cartopy."""
    from cartopy.io import shapereader as shp_read  # version 0.18.0
```

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```
    import geopandas as gp  # 0.8.1

    shpfile = shp_read.natural_earth("50m", "cultural", "admin_0_countries")
    df = gp.read_file(shpfile)  # only use the simplest polygon
    poly = df.loc[df["ADMIN"] == "Germany"]["geometry"].values[0][0]
    np.savetxt("de_borders.txt", list(poly.exterior.coords))


def get_dwd_temperature(date="2020-06-09 12:00:00"):
    """Get air temperature from german weather stations from 9.6.20 12:00."""
    from wetterdienst.dwd import observations as obs  # version 0.13.0

    settings = dict(
        resolution=obs.DWDObservationResolution.HOURLY,
        start_date=date,
        end_date=date,
    )
    sites = obs.DWDObservationStations(
        parameter_set=obs.DWDObservationParameterSet.TEMPERATURE_AIR,
        period=obs.DWDObservationPeriod.RECENT,
        **settings,
    )
    ids, lat, lon = sites.all().loc[:, ["STATION_ID", "LAT", "LON"]].values.T
    observations = obs.DWDObservationData(
        station_ids=ids,
        parameters=obs.DWDObservationParameter.HOURLY.TEMPERATURE_AIR_200,
        periods=obs.DWDObservationPeriod.RECENT,
        **settings,
    )
    temp = observations.all().VALUE.values
    sel = np.isfinite(temp)
    # select only valid temperature data
    ids, lat, lon, temp = ids.astype(float)[sel], lat[sel], lon[sel], temp[sel]
    head = "id, lat, lon, temp"  # add a header to the file
    np.savetxt("temp_obs.txt", np.array([ids, lat, lon, temp]).T, header=head)
```

If you want to download the data again, uncomment the two following lines. We will simply load the resulting files to gain the border polygon and the observed temperature along with the station locations given by lat-lon values.

```
# get_borders_germany()
# get_dwd_temperature(date="2020-06-09 12:00:00")

border = np.loadtxt("de_borders.txt")
ids, lat, lon, temp = np.loadtxt("temp_obs.txt").T
```

First we will estimate the variogram of our temperature data. As the maximal bin distance we choose 8 degrees, which corresponds to a chordal length of about 900 km.

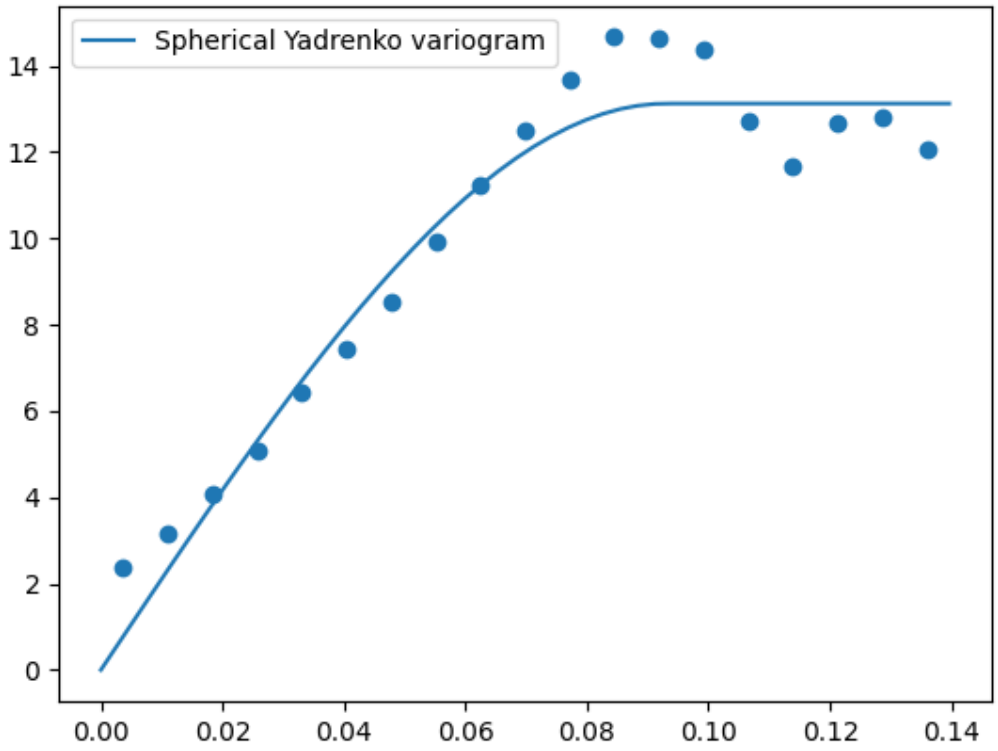
```
bins = gs.standard_bins((lat, lon), max_dist=np.deg2rad(8), latlon=True)
bin_c, vario = gs.vario_estimate((lat, lon), temp, bins, latlon=True)
```

Now we can use this estimated variogram to fit a model to it. Here we will use a *Spherical* model. We select the `latlon` option to use the *Yadrenko* variant of the model to gain a valid model for lat-lon coordinates and we rescale it to the earth-radius. Otherwise the length scale would be given in radians representing the great-circle distance.

We deselect the nugget from fitting and plot the result afterwards.

Note: You need to plot the Yadrenko variogram, since the standard variogram still holds the ordinary routine that is not respecting the great-circle distance.

```
model = gs.Spherical(latlon=True, rescale=gs.EARTH_RADIUS)
model.fit_variogram(bin_c, vario, nugget=False)
ax = model.plot("vario_yadrenko", x_max=bins[-1])
ax.scatter(bin_c, vario)
print(model)
```



Out:

```
Spherical(latlon=True, var=13.1, len_scale=5.93e+02, nugget=0.0, rescale=6.37e+03)
```

As we see, we have a rather large correlation length of 600 km.

Now we want to interpolate the data using *Universal* kriging. In order to tinker around with the data, we will use a north-south drift by assuming a linear correlation with the latitude. This can be done as follows:

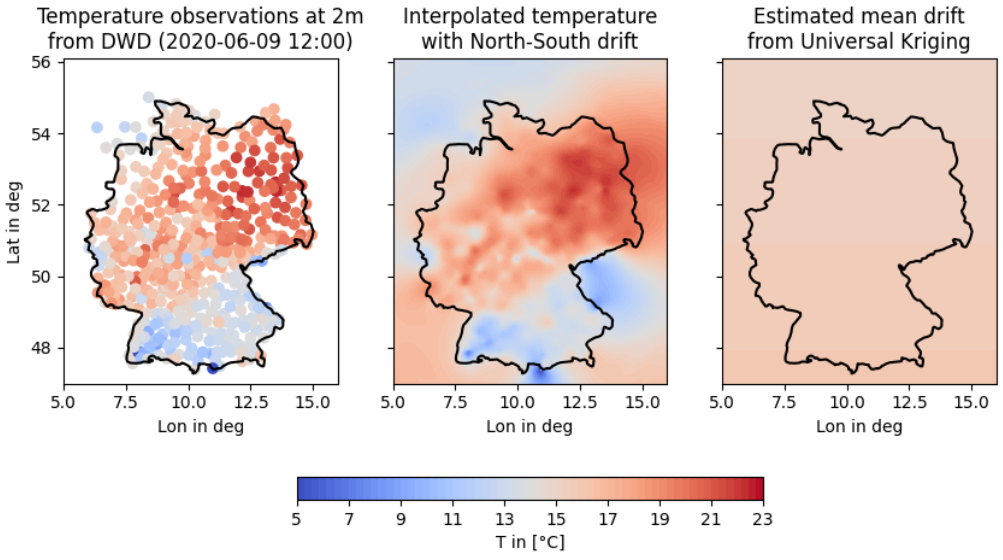
```
def north_south_drift(lat, lon):
    return lat


uk = gs.krige.Universal(
    model=model,
    cond_pos=(lat, lon),
    cond_val=temp,
    drift_functions=north_south_drift,
)
```

Now we generate the kriging field, by defining a lat-lon grid that covers the whole of Germany. The *Krige* class provides the option to only krige the mean field, so one can have a glimpse at the estimated drift.

```
g_lat = np.arange(47, 56.1, 0.1)
g_lon = np.arange(5, 16.1, 0.1)

field, k_var = uk((g_lat, g_lon), mesh_type="structured")
mean = uk((g_lat, g_lon), mesh_type="structured", only_mean=True)
```

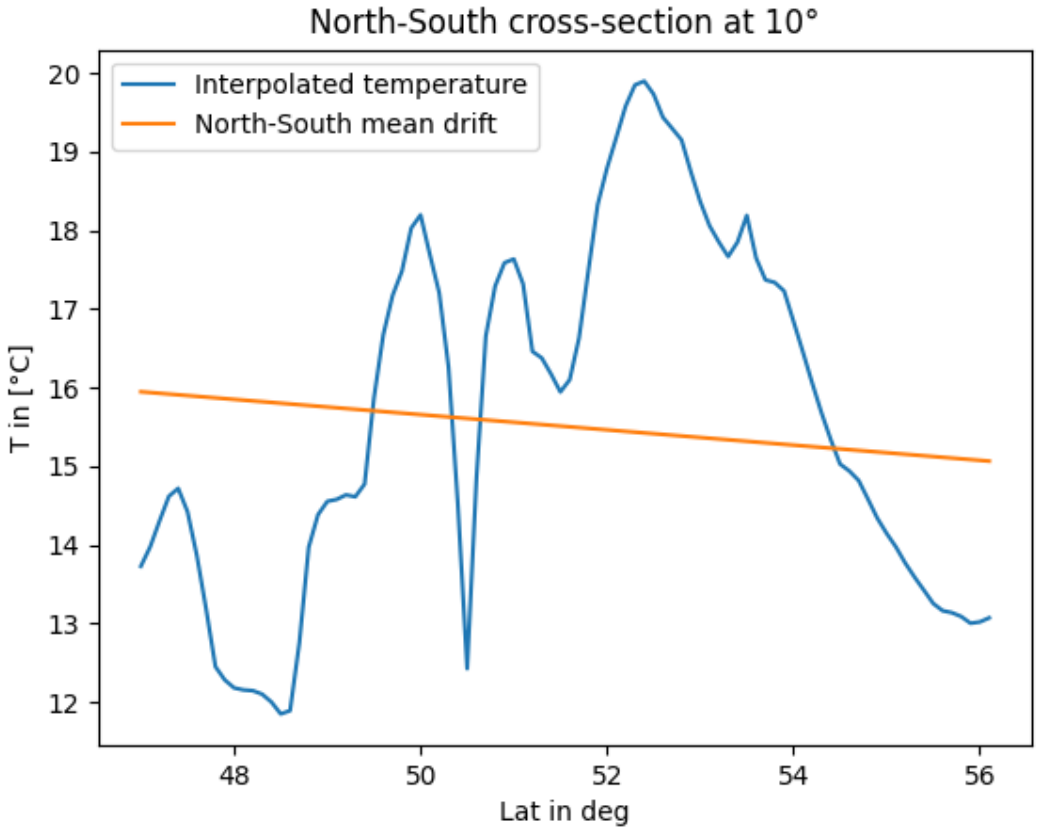
And that's it. Now let's have a look at the generated field and the input data along with the estimated mean:

```
levels = np.linspace(5, 23, 64)
fig, ax = plt.subplots(1, 3, figsize=[10, 5], sharey=True)
sca = ax[0].scatter(lon, lat, c=temp, vmin=5, vmax=23, cmap="coolwarm")
co1 = ax[1].contourf(g_lon, g_lat, field, levels, cmap="coolwarm")
co2 = ax[2].contourf(g_lon, g_lat, mean, levels, cmap="coolwarm")

[ax[i].plot(border[:, 0], border[:, 1], color="k") for i in range(3)]
[ax[i].set_xlim([5, 16]) for i in range(3)]
[ax[i].set_xlabel("Lon in deg") for i in range(3)]
ax[0].set_ylabel("Lat in deg")

ax[0].set_title("Temperature observations at 2m\nfrom DWD (2020-06-09 12:00)")
ax[1].set_title("Interpolated temperature\nwith North-South drift")
ax[2].set_title("Estimated mean drift\nfrom Universal Kriging")

fmt = dict(orientation="horizontal", shrink=0.5, fraction=0.1, pad=0.2)
fig.colorbar(co2, ax=ax, **fmt).set_label("T in [°C]")
```



To get a better impression of the estimated north-south drift, we'll take a look at a cross-section at a longitude of 10 degree:

```
fig, ax = plt.subplots()
ax.plot(g_lat, field[:, 50], label="Interpolated temperature")
ax.plot(g_lat, mean[:, 50], label="North-South mean drift")
ax.set_xlabel("Lat in deg")
ax.set_ylabel("T in [°C]")
ax.set_title("North-South cross-section at 10°")
ax.legend()
```



Interpretion of the results is now up to you! ;-)

Total running time of the script: (0 minutes 8.276 seconds)

2.9 Spatio-Temporal Modeling

Spatio-Temporal modelling can provide insights into time dependent processes like rainfall, air temperature or crop yield.

GSTools provides the metric spatio-temporal model for all covariance models by enhancing the spatial model dimension with a time dimension to result in the spatio-temporal dimension `st_dim` and setting a spatio-temporal anisotropy ratio with `st_anis`:

```
import gstools as gs
dim = 3  # spatial dimension
st_dim = dim + 1
st_anis = 0.4
st_model = gs.Exponential(dim=st_dim, anis=st_anis)
```

Since it is given in the name "spatio-temporal", we will always treat the time as last dimension. This enables us to have spatial anisotropy and rotation defined as in non-temporal models, without altering the behavior in the time dimension:

```
anis = [0.4, 0.2]  # spatial anisotropy in 3D
angles = [0.5, 0.4, 0.3]  # spatial rotation in 3D
st_model = gs.Exponential(dim=st_dim, anis=anis+[st_anis], angles=angles)
```

In order to generate spatio-temporal position tuples, GSTools provides a convenient function *generate_st_grid*. The output can be used for spatio-temporal random field generation (or kriging resp. conditioned fields):

```
pos = dim * [1, 2, 3]  # 3 points in space (1,1,1), (2,2,2) and (3,3,3)
time = range(10)  # 10 time steps
st_grid = gs.generate_st_grid(pos, time)
st_rf = gs.SRF(st_model)
st_field = st_rf(st_grid).reshape(-1, len(time))
```

Then we can access the different time-steps by the last array index.

Examples

Creating a 1D Synthetic Precipitation Field

In this example we will create a time series of a 1D synthetic precipitation field.

We'll start off by creating a Gaussian random field with an exponential variogram, which seems to reproduce the spatial correlations of precipitation fields quite well. We'll create a daily timeseries over a one dimensional cross section of 50km. This workflow is suited for sub daily precipitation time series.

```
import copy
import numpy as np
import matplotlib.pyplot as plt
import gstools as gs

# fix the seed for reproducibility
seed = 20170521
# spatial axis of 50km with a resolution of 1km
x = np.arange(0, 50, 1.0)
# half daily timesteps over three months
t = np.arange(0.0, 90.0, 0.5)

# total spatio-temporal dimension
st_dim = 1 + 1
# space-time anisotropy ratio given in units d / km
st_anis = 0.4

# an exponential variogram with a corr. lengths of 2d and 5km
model = gs.Exponential(dim=st_dim, var=1.0, len_scale=5.0, anis=st_anis)
# create a spatial random field instance
srf = gs.SRF(model, seed=seed)

pos, time = [x], [t]

# a Gaussian random field which is also saved internally for the transformations
srf.structured(pos + time)
P_gau = copy.deepcopy(srf.field)
```

Next, we could take care of the dry periods. Therefore we would simply introduce a lower threshold value. But we will combine this step with the next one. Anyway, for demonstration purposes, we will also do it with the threshold value now.

```
threshold = 0.85
P_cut = copy.deepcopy(srf.field)
P_cut[P_cut <= threshold] = 0.0
```

With the above lines of code we have created a cut off Gaussian spatial random field with an exponential variogram. But precipitation fields are not distributed Gaussian. Thus, we will now transform the field with an inverse box-cox transformation (create a non-Gaussian field) , which is often used to account for the skewness of precipitation

fields. Different values have been suggested for the transformation parameter lambda, but we will stick to 1/2. As already mentioned, we will perform the cutoff for the dry periods with this transformation implicitly with the shift. The warning will tell you that values have indeed been cut off and it can be ignored. We call the resulting field Gaussian anamorphosis.

```
# the lower this value, the more will be cut off, a value of 0.2 cuts off
# nearly everything in this example.
cutoff = 0.55
gs.transform.boxcox(srf, lmbda=0.5, shift=-1.0 / cutoff)
```

Out:

```
/home/docs/checkouts/readthedocs.org/user_builds/gstools/envs/v1.3.1/lib/python3.7/
↪site-packages/gstools/transform/field.py:164: UserWarning: Box-Cox: Some values␣
↪will be cut off!
  warn("Box-Cox: Some values will be cut off!")
```

As a last step, the amount of precipitation is set. This should of course be calibrated towards observations (the same goes for the threshold, the variance, correlation length, and so on).

```
amount = 2.0
srf.field *= amount
P_ana = srf.field
```

Finally we can have a look at the fields resulting from each step. Note, that the cutoff of the cut Gaussian only approximates the cutoff values from the box-cox transformation. For a closer look, we will examine a cross section at an arbitrary location. And afterwards we will create a contour plot for visual candy.

```
fig, axs = plt.subplots(2, 2, sharex=True, sharey=True)

axs[0, 0].set_title("Gaussian")
axs[0, 0].plot(t, P_gau[20, :])
axs[0, 0].set_ylabel(r"$P$ / mm")

axs[0, 1].set_title("Cut Gaussian")
axs[0, 1].plot(t, P_cut[20, :])

axs[1, 0].set_title("Cut Gaussian Anamorphosis")
axs[1, 0].plot(t, P_ana[20, :])
axs[1, 0].set_xlabel(r"$t$ / d")
axs[1, 0].set_ylabel(r"$P$ / mm")

axs[1, 1].set_title("Different Cross Section")
axs[1, 1].plot(t, P_ana[10, :])
axs[1, 1].set_xlabel(r"$t$ / d")

plt.tight_layout()

fig, axs = plt.subplots(2, 2, sharex=True, sharey=True)

axs[0, 0].set_title("Gaussian")
cont = axs[0, 0].contourf(t, x, P_gau, cmap="PuBu", levels=10)
cbar = fig.colorbar(cont, ax=axs[0, 0])
cbar.ax.set_ylabel(r"$P$ / mm")
axs[0, 0].set_ylabel(r"$x$ / km")

axs[0, 1].set_title("Cut Gaussian")
cont = axs[0, 1].contourf(t, x, P_cut, cmap="PuBu", levels=10)
```

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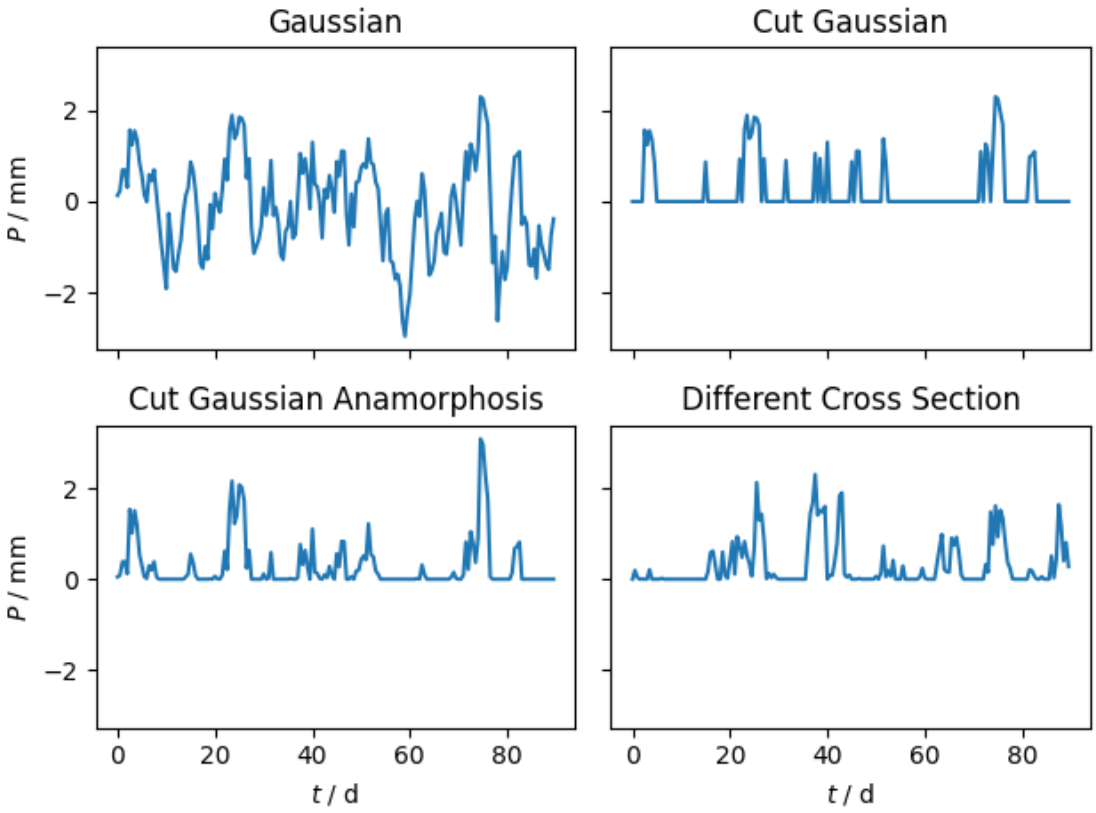
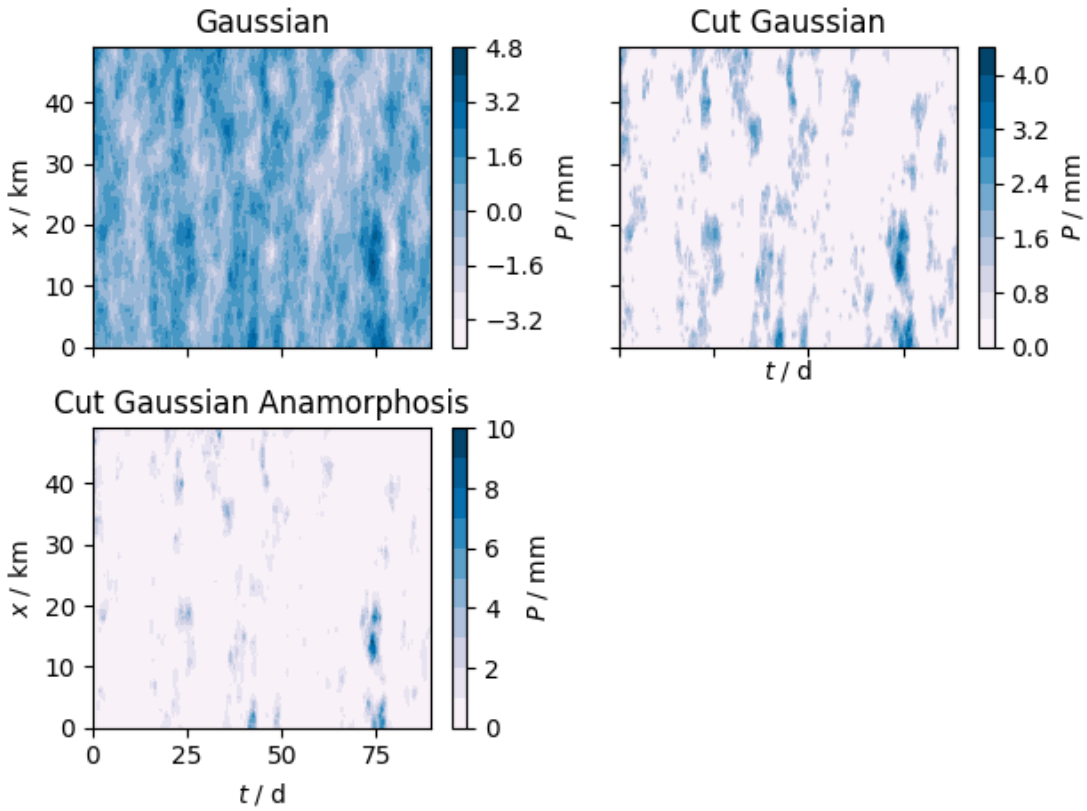
```
cbar = fig.colorbar(cont, ax=axs[0, 1])
cbar.ax.set_ylabel(r"$P$ / mm")
axs[0, 1].set_xlabel(r"$t$ / d")

axs[1, 0].set_title("Cut Gaussian Anamorphosis")
cont = axs[1, 0].contourf(t, x, P_ana, cmap="PuBu", levels=10)
cbar = fig.colorbar(cont, ax=axs[1, 0])
cbar.ax.set_ylabel(r"$P$ / mm")
axs[1, 0].set_xlabel(r"$t$ / d")
axs[1, 0].set_ylabel(r"$x$ / km")

fig.delaxes(axs[1, 1])
plt.tight_layout()
```





-

Total running time of the script: (0 minutes 1.188 seconds)

Creating a 2D Synthetic Precipitation Field

In this example we'll create a time series of a 2D synthetic precipitation field.

Very similar to the previous tutorial, we'll start off by creating a Gaussian random field with an exponential variogram, which seems to reproduce the spatial correlations of precipitation fields quite well. We'll create a daily timeseries over a two dimensional domain of 50km x 40km. This workflow is suited for sub daily precipitation time series.

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.animation as animation
import gstools as gs

# fix the seed for reproducibility
seed = 20170521
# 1st spatial axis of 50km with a resolution of 1km
x = np.arange(0, 50, 1.0)
# 2nd spatial axis of 40km with a resolution of 1km
y = np.arange(0, 40, 1.0)
# half daily timesteps over three months
t = np.arange(0.0, 90.0, 0.5)

# total spatio-temporal dimension
st_dim = 2 + 1
# space-time anisotropy ratio given in units d / km
st_anis = 0.4
```

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```
# an exponential variogram with a corr. lengths of 5km, 5km, and 2d
model = gs.Exponential(dim=st_dim, var=1.0, len_scale=5.0, anis=st_anis)
# create a spatial random field instance
srf = gs.SRF(model, seed=seed)

pos, time = [x, y], [t]

# the Gaussian random field
srf.structured(pos + time)

# account for the skewness and the dry periods
cutoff = 0.55
gs.transform.boxcox(srf, lmbda=0.5, shift=-1.0 / cutoff)

# adjust the amount of precipitation
amount = 4.0
srf.field *= amount
```

Out:

```
/home/docs/checkouts/readthedocs.org/user_builds/gstools/envs/v1.3.1/lib/python3.7/
↪site-packages/gstools/transform/field.py:164: UserWarning: Box-Cox: Some values␣
↪will be cut off!
  warn("Box-Cox: Some values will be cut off!")
```

plot the 2d precipitation field over time as an animation.

```
def _update_ani(time_step):
    im.set_array(srf.field[:, :, time_step].T)
    return (im,)


fig, ax = plt.subplots()
im = ax.imshow(
    srf.field[:, :, 0].T,
    cmap="Blues",
    interpolation="bicubic",
    origin="lower",
)
cbar = fig.colorbar(im)
cbar.ax.set_ylabel(r"Precipitation $P$ / mm")
ax.set_xlabel(r"$x$ / km")
ax.set_ylabel(r"$y$ / km")

ani = animation.FuncAnimation(
    fig, _update_ani, len(t), interval=100, blit=True
)
```

Total running time of the script: (1 minutes 30.536 seconds)

2.10 Normalizing Data

When dealing with real-world data, one can't assume it to be normal distributed. In fact, many properties are modeled by applying different transformations, for example conductivity is often assumed to be log-normal or precipitation is transformed using the famous box-cox power transformation.

These "normalizers" are often represented as parameteric power transforms and one is interested in finding the best parameter to gain normality in the input data.

This is of special interest when kriging should be applied, since the target variable of the kriging interpolation is assumed to be normal distributed.

GSTools provides a set of Normalizers and routines to automatically fit these to input data by minimizing the likelihood function.

Mean, Trend and Normalizers

All Field classes (*SRF*, *Krige* or *CondSRF*) provide the input of *mean*, *normalizer* and *trend*:

- A *trend* can be a callable function, that represents a trend in input data. For example a linear decrease of temperature with height.
- The *normalizer* will be applied after the data was detrended, i.e. the trend was substracted from the data, in order to gain normality.
- The *mean* is now interpreted as the mean of the normalized data. The user could also provide a callable mean, but it is mostly meant to be constant.

When no normalizer is given, *trend* and *mean* basically behave the same. We just decided that a trend is associated with raw data and a mean is used in the context of normally distributed data.

Provided Normalizers

The following normalizers can be passed to all Field-classes and variogram estimation routines or can be used as standalone tools to analyse data.

LogNormal([data])	Log-normal fields.
BoxCox([data])	Box-Cox (1964) transformed fields.
BoxCoxShift([data])	Box-Cox (1964) transformed fields including shifting.
YeoJohnson([data])	Yeo-Johnson (2000) transformed fields.
Modulus([data])	Modulus or John-Draper (1980) transformed fields.
Manly([data])	Manly (1971) transformed fields.

Examples

Log-Normal Kriging

Log Normal kriging is a term to describe a special workflow for kriging to deal with log-normal data, like conductivity or transmissivity in hydrogeology.

It simply means to first convert the input data to a normal distribution, i.e. applying a logarithic function, then interpolating these values with kriging and transforming the result back with the exponential function.

The resulting kriging variance describes the error variance of the log-values of the target variable.

In this example we will use ordinary kriging.

```
import numpy as np
import gstools as gs

# condtions
cond_pos = [0.3, 1.9, 1.1, 3.3, 4.7]
cond_val = [0.47, 0.56, 0.74, 1.47, 1.74]
# resulting grid
gridx = np.linspace(0.0, 15.0, 151)
# stable covariance model
model = gs.Stable(dim=1, var=0.5, len_scale=2.56, alpha=1.9)
```

In order to result in log-normal kriging, we will use the *LogNormal* Normalizer. This is a parameter-less normalizer, so we don't have to fit it.

```
normalizer = gs.normalizer.LogNormal
```

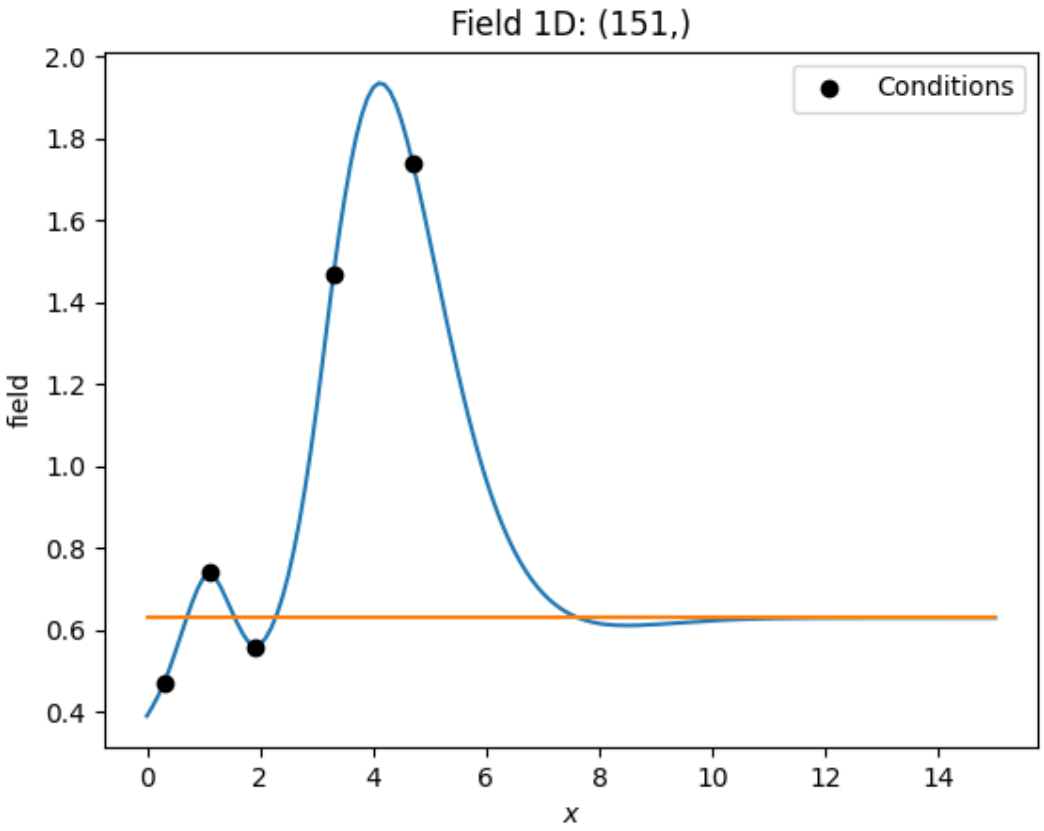
Now we generate the interpolated field as well as the mean field. This can be done by setting *only_mean=True* in *Krige.__call__*. The result is then stored as *mean_field*.

In terms of log-normal kriging, this mean represents the geometric mean of the field.

```
krige = gs.krige.Ordinary(model, cond_pos, cond_val, normalizer=normalizer)
# interpolate the field
krige(gridx)
# also generate the mean field
krige(gridx, only_mean=True)
```

And that's it. Let's have a look at the results.

```
ax = krige.plot()
# plotting the geometric mean
krige.plot("mean_field", ax=ax)
# plotting the conditioning data
ax.scatter(cond_pos, cond_val, color="k", zorder=10, label="Conditions")
ax.legend()
```



Total running time of the script: (0 minutes 0.115 seconds)

Automatic fitting

In order to demonstrate how to automatically fit normalizer and variograms, we generate synthetic log-normal data, that should be interpolated with ordinary kriging.

Normalizers are fitted by minimizing the likelihood function and variograms are fitted by estimating the empirical variogram with automatic binning and fitting the theoretical model to it. Thereby the sill is constrained to match the field variance.

Artificial data

Here we generate log-normal data following a Gaussian covariance model. We will generate the "original" field on a 60x60 mesh, from which we will take samples in order to pretend a situation of data-scarcity.

```
import numpy as np
import gstools as gs
import matplotlib.pyplot as plt

# structured field with edge length of 50
x = y = range(51)
pos = gs.generate_grid([x, y])
model = gs.Gaussian(dim=2, var=1, len_scale=10)
srf = gs.SRF(model, seed=20170519, normalizer=gs.normalizer.LogNormal())
# generate the original field
srf(pos)
```

Here, we sample 60 points and set the conditioning points and values.

```
ids = np.arange(srf.field.size)
samples = np.random.RandomState(20210201).choice(ids, size=60, replace=False)

# sample conditioning points from generated field
cond_pos = pos[:, samples]
cond_val = srf.field[samples]
```

Fitting and Interpolation

Now we want to interpolate the "measured" samples and we want to normalize the given data with the BoxCox transformation.

Here we set up the kriging routine and use a *Stable* model, that should be fitted automatically to the given data and we pass the *BoxCox* normalizer in order to gain normality.

The normalizer will be fitted automatically to the data, by setting `fit_normalizer=True`.

The covariance/variogram model will be fitted by an automatic workflow by setting `fit_variogram=True`.

```
krige = gs.krige.Ordinary(
    model=gs.Stable(dim=2),
    cond_pos=cond_pos,
    cond_val=cond_val,
    normalizer=gs.normalizer.BoxCox(),
    fit_normalizer=True,
    fit_variogram=True,
)
```

First, let's have a look at the fitting results:

```
print(krige.model)
print(krige.normalizer)
```

Out:

```
Stable(dim=2, var=0.576, len_scale=8.85, nugget=0.00682, alpha=2.0)
BoxCox(lmbda=-0.0754)
```

As we see, it went quite well. Variance is a bit underestimated, but length scale and nugget are good. The shape parameter of the stable model is correctly estimated to be close to *2*, so we result in a Gaussian like model.

The BoxCox parameter *lmbda* was estimated to be almost 0, which means, the log-normal distribution was correctly fitted.

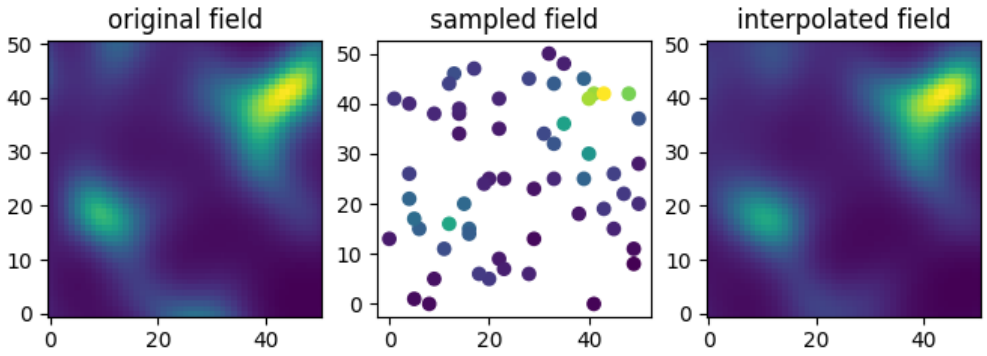
Now let's run the kriging interpolation.

```
krige(pos)
```

Plotting

Finally let's compare the original, sampled and interpolated fields. As we'll see, there is a lot of information in the covariance structure of the measurement samples and the field is reconstructed quite accurately.

```
fig, ax = plt.subplots(1, 3, figsize=[8, 3])
ax[0].imshow(srf.field.reshape(len(x), len(y)).T, origin="lower")
ax[1].scatter(*cond_pos, c=cond_val)
ax[2].imshow(krige.field.reshape(len(x), len(y)).T, origin="lower")
# titles
ax[0].set_title("original field")
ax[1].set_title("sampled field")
ax[2].set_title("interpolated field")
# set aspect ratio to equal in all plots
[ax[i].set_aspect("equal") for i in range(3)]
```



Total running time of the script: (0 minutes 0.402 seconds)

Normalizer Comparison

Let's compare the transformation behavior of the provided normalizers.

But first, we define a convenience routine and make some imports as always.

```
import numpy as np
import gstools as gs
import matplotlib.pyplot as plt


def dashes(i=1, max_n=12, width=1):
    """Return line dashes."""
    return i * [width, width] + [max_n * 2 * width - 2 * i * width, width]
```

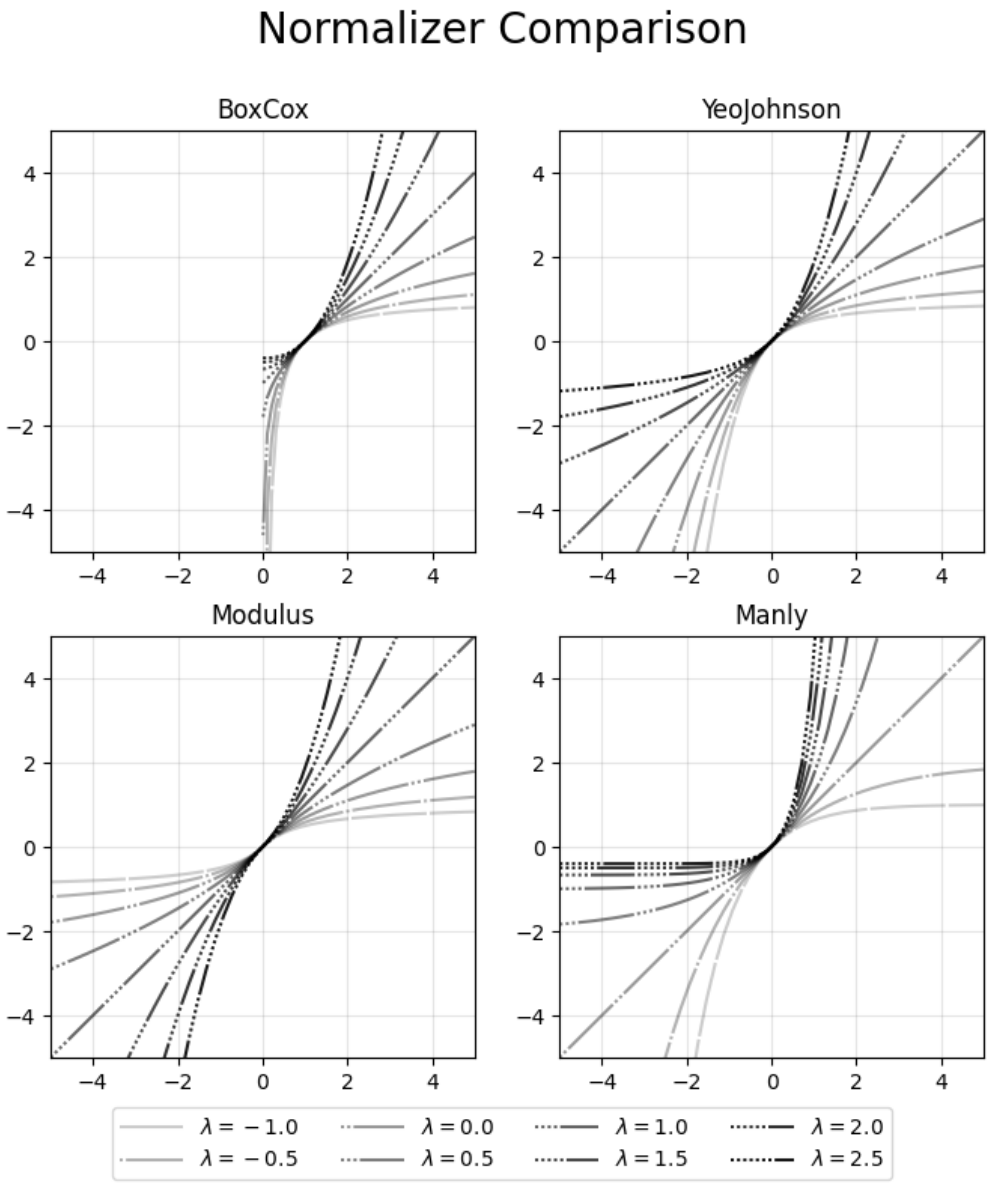
We select 4 normalizers depending on a single parameter lambda and plot their transformation behavior within the interval [-5, 5].

For the shape parameter lambda, we create a list of 8 values ranging from -1 to 2.5.

```
lmbdas = [i * 0.5 for i in range(-2, 6)]
normalizers = [
    gs.normalizer.BoxCox,
    gs.normalizer.YeoJohnson,
    gs.normalizer.Modulus,
    gs.normalizer.Manly,
]
```

Let's plot them!

```
fig, ax = plt.subplots(2, 2, figsize=[8, 8])
for i, norm in enumerate(normalizers):
    # correctly setting the data range
    x_rng = norm().normalize_range
    x = np.linspace(max(-5, x_rng[0] + 0.01), min(5, x_rng[1] - 0.01))
    for j, lmbda in enumerate(lmbdas):
        ax.flat[i].plot(
            x,
            norm(lmbda=lmbda).normalize(x),
            label=r"$\lambda=" + str(lmbda) + "$",
            color="k",
            alpha=0.2 + j * 0.1,
            dashes=dashes(j),
        )
    # axis formatting
    ax.flat[i].grid(which="both", color="grey", linestyle="-", alpha=0.2)
    ax.flat[i].set_ylim((-5, 5))
    ax.flat[i].set_xlim((-5, 5))
    ax.flat[i].set_title(norm().name)
# figure formatting
handles, labels = ax.flat[-1].get_legend_handles_labels()
fig.legend(handles, labels, loc="lower center", ncol=4, handlelength=3.0)
fig.suptitle("Normalizer Comparison", fontsize=20)
fig.show()
```



The missing *LogNormal* transformation is covered by the *BoxCox* transformation for lambda=0. The *BoxCoxShift* transformation is simply the *BoxCox* transformation shifted on the X-axis.

Total running time of the script: (0 minutes 0.344 seconds)

2.11 Miscellaneous Tutorials

More examples which do not really fit into other categories. Some are not more than a code snippet, while others are more complex and more than one part of GSTools is involved.

Examples

Truncated Power Law Variograms

GSTools also implements truncated power law variograms, which can be represented as a superposition of scale dependant modes in form of standard variograms, which are truncated by a lower- ℓ_{low} and an upper length-scale ℓ_{up}.

This example shows the truncated power law (*TPLStable*) based on the *Stable* covariance model and is given by

$$\gamma_{\ell_{\mathrm{low}},\ell_{\mathrm{up}}}(r) = \int_{\ell_{\mathrm{low}}}^{\ell_{\mathrm{up}}} \gamma(r,\lambda)\frac{\mathrm{d}\lambda}{\lambda}$$

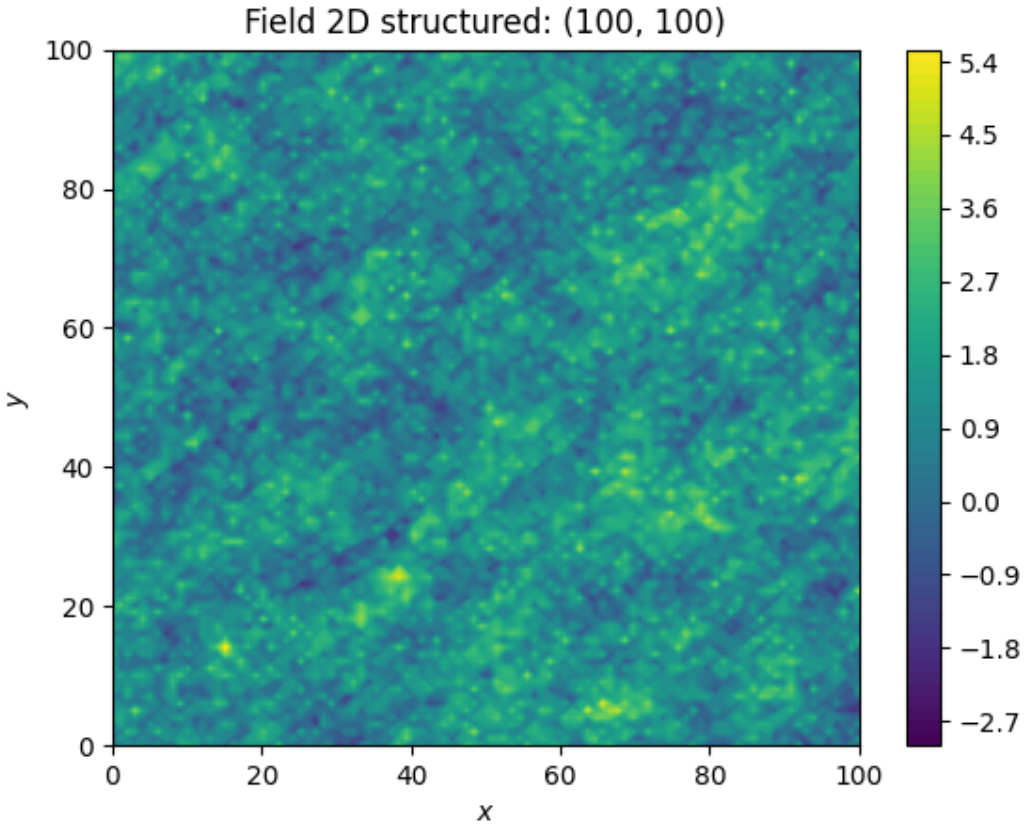
with *Stable* modes on each scale:

$$\gamma(r,\lambda) = \sigma^2(\lambda)\cdot\left(1-\exp\left[-\left(\frac{r}{\lambda}\right)^{\alpha}\right]\right)$$
$$\sigma^2(\lambda) = C\cdot\lambda^{2H}$$

which gives Gaussian modes for `alpha=2` or Exponential modes for `alpha=1`.

For $\ell_{\mathrm{low}} = 0$ this results in:

$$\gamma_{\ell_{\mathrm{up}}}(r) = \sigma^2_{\ell_{\mathrm{up}}}\cdot\left(1-\frac{2H}{\alpha}\cdot E_{1+\frac{2H}{\alpha}}\left[\left(\frac{r}{\ell_{\mathrm{up}}}\right)^{\alpha}\right]\right)$$
$$\sigma^2_{\ell_{\mathrm{up}}} = C\cdot\frac{\ell_{\mathrm{up}}^{2H}}{2H}$$



```
import numpy as np
import gstools as gs

x = y = np.linspace(0, 100, 100)
model = gs.TPLStable(
    dim=2,  # spatial dimension
    var=1,  # variance (C is calculated internally, so variance is actually 1)
    len_low=0,  # lower truncation of the power law
    len_scale=10,  # length scale (a.k.a. range), len_up = len_low + len_scale
    nugget=0.1,  # nugget
    anis=0.5,  # anisotropy between main direction and transversal ones
    angles=np.pi / 4,  # rotation angles
    alpha=1.5,  # shape parameter from the stable model
    hurst=0.7,  # hurst coefficient from the power law
)
srf = gs.SRF(model, mean=1.0, seed=19970221)
srf.structured([x, y])
srf.plot()
```

Total running time of the script: (0 minutes 15.190 seconds)

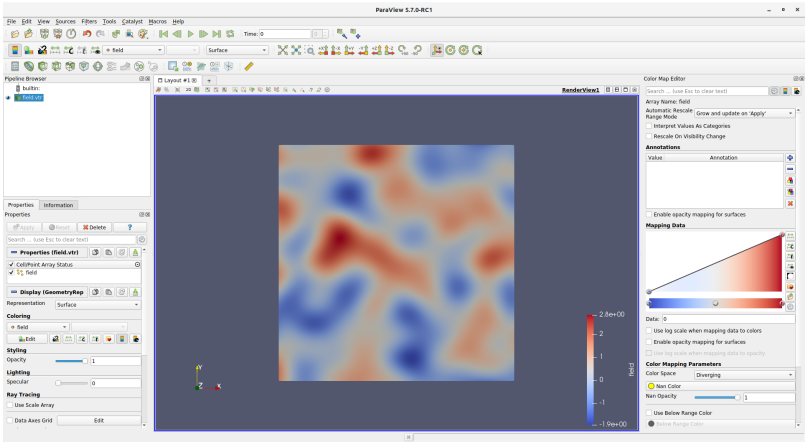
Exporting Fields

GSTools provides simple exporting routines to convert generated fields to VTK files.

These can be viewed for example with Paraview.

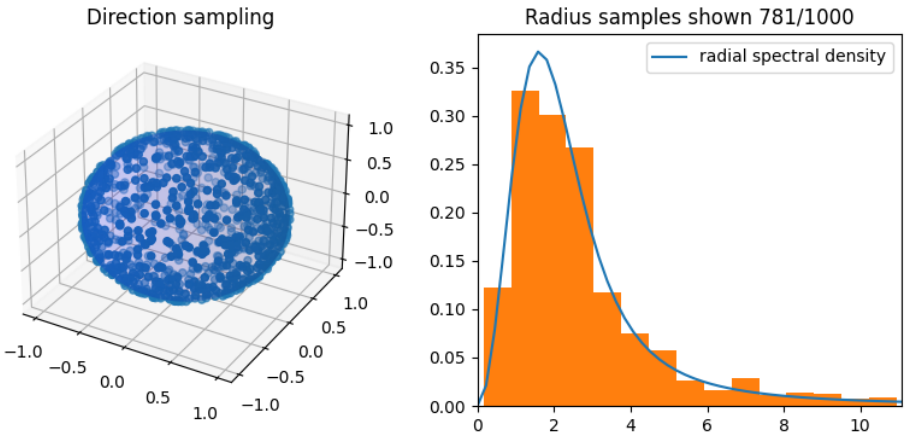
```
import gstools as gs

x = y = range(100)
model = gs.Gaussian(dim=2, var=1, len_scale=10)
srf = gs.SRF(model)
field = srf((x, y), mesh_type="structured")
srf.vtk_export(filename="field")
```

The result displayed with Paraview:



Total running time of the script: (0 minutes 0.482 seconds)

Check Random Sampling




```
import numpy as np
from matplotlib import pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import gstools as gs


def norm_rad(vec):
    """Direction on the unit sphere."""
    vec = np.array(vec, ndmin=2)
    norm = np.zeros(vec.shape[1])
    for i in range(vec.shape[0]):
        norm += vec[i] ** 2
    norm = np.sqrt(norm)
    return np.einsum("j,ij->ij", 1 / norm, vec), norm


def plot_rand_meth_samples(generator):
    """Plot the samples of the rand meth class."""
    norm, rad = norm_rad(generator._cov_sample)

    fig = plt.figure(figsize=(10, 4))

    if generator.model.dim == 3:
        ax = fig.add_subplot(121, projection=Axes3D.name)
        u = np.linspace(0, 2 * np.pi, 100)
        v = np.linspace(0, np.pi, 100)
        x = np.outer(np.cos(u), np.sin(v))
        y = np.outer(np.sin(u), np.sin(v))
        z = np.outer(np.ones(np.size(u)), np.cos(v))
        ax.plot_surface(x, y, z, rstride=4, cstride=4, color="b", alpha=0.1)
        ax.scatter(norm[0], norm[1], norm[2])
    elif generator.model.dim == 2:
        ax = fig.add_subplot(121)
        u = np.linspace(0, 2 * np.pi, 100)
        x = np.cos(u)
        y = np.sin(u)
        ax.plot(x, y, color="b", alpha=0.1)
        ax.scatter(norm[0], norm[1])
        ax.set_aspect("equal")
```

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```
    else:
        ax = fig.add_subplot(121)
        ax.bar(-1, np.sum(np.isclose(norm, -1)), color="C0")
        ax.bar(1, np.sum(np.isclose(norm, 1)), color="C0")
        ax.set_xticks([-1, 1])
        ax.set_xticklabels(("-1", "1"))
    ax.set_title("Direction sampling")

    ax = fig.add_subplot(122)
    x = np.linspace(0, 10 / generator.model.integral_scale)
    y = generator.model.spectral_rad_pdf(x)
    ax.plot(x, y, label="radial spectral density")
    sample_in = np.sum(rad <= np.max(x))
    ax.hist(rad[rad <= np.max(x)], bins=sample_in // 50, density=True)
    ax.set_xlim([0, np.max(x)])
    ax.set_title("Radius samples shown {}/{}".format(sample_in, len(rad)))
    ax.legend()
    plt.show()


model = gs.Stable(dim=3, alpha=1.5)
srf = gs.SRF(model, seed=2020)
plot_rand_meth_samples(srf.generator)
```

Total running time of the script: (0 minutes 3.656 seconds)

Analyzing the Herten Aquifer with GSTools

This example is going to be a bit more extensive and we are going to do some basic data preprocessing for the actual variogram estimation. But this example will be self-contained and all data gathering and processing will be done in this example script.

The Data

We are going to analyse the Herten aquifer, which is situated in Southern Germany. Multiple outcrop faces where surveyed and interpolated to a 3D dataset. In these publications, you can find more information about the data:

Bayer, Peter; Comunian, Alessandro; Höyng, Dominik; Mariethoz, Gregoire (2015): Physicochemical properties and 3D geostatistical simulations of the Herten and the Descalvado aquifer analogs. PANGAEA, https://doi.org/10.1594/PANGAEA.844167,

Supplement to: Bayer, P et al. (2015): Three-dimensional multi-facies realizations of sedimentary reservoir and aquifer analogs. Scientific Data, 2, 150033, https://doi.org/10.1038/sdata.2015.33

Retrieving the Data

To begin with, we need to download and extract the data. Therefore, we are going to use some built-in Python libraries. For simplicity, many values and strings will be hardcoded.

You don't have to execute the `download_herten` and `generate_transmissivity` functions, since the only produce the `herten_transmissivity.gz` and `grid_dim_origin_spacing.txt`, which are already present.

```
import os
import numpy as np
import matplotlib.pyplot as plt
import gstools as gs

VTK_PATH = os.path.join("Herten-analog", "sim-big_1000x1000x140", "sim.vtk")
```

```
def download_herten():
    """Download the data, warning: its about 250MB."""
    import zipfile
    import urllib.request

    print("Downloading Herten data")
    data_filename = "data.zip"
    data_url = (
        "http://store.pangaea.de/Publications/"
        "Bayer_et_al_2015/Herten-analog.zip"
    )
    urllib.request.urlretrieve(data_url, "data.zip")
    # extract the "big" simulation
    with zipfile.ZipFile(data_filename, "r") as zf:
        zf.extract(VTK_PATH)
```

```
def generate_transmissivity():
    """Generate a file with a transmissivity field from the HERTEN data."""
    import pyvista as pv
    import shutil

    print("Loading Herten data with pyvista")
    mesh = pv.read(VTK_PATH)
    herten = mesh.point_arrays["facies"].reshape(mesh.dimensions, order="F")
    # conductivity values per fazies from the supplementary data
    cond = 1e-4 * np.array(
        [2.5, 2.3, 0.61, 260, 1300, 950, 0.43, 0.006, 23, 1.4]
    )
    # asign the conductivities to the facies
    herten_cond = cond[herten]
    # Next, we are going to calculate the transmissivity,
    # by integrating over the vertical axis
    herten_trans = np.sum(herten_cond, axis=2) * mesh.spacing[2]
    # saving some grid informations
    grid = [mesh.dimensions[:2], mesh.origin[:2], mesh.spacing[:2]]
    print("Saving the transmissivity field and grid information")
    np.savetxt("herten_transmissivity.gz", herten_trans)
    np.savetxt("grid_dim_origin_spacing.txt", grid)
    # Some cleanup. You can comment out these lines to keep the downloaded data
    os.remove("data.zip")
    shutil.rmtree("Herten-analog")
```

Downloading and Preprocessing

You can uncomment the following two calls, so the data is downloaded and processed again.

```
# download_herten()
# generate_transmissivity()
```

Analyzing the data

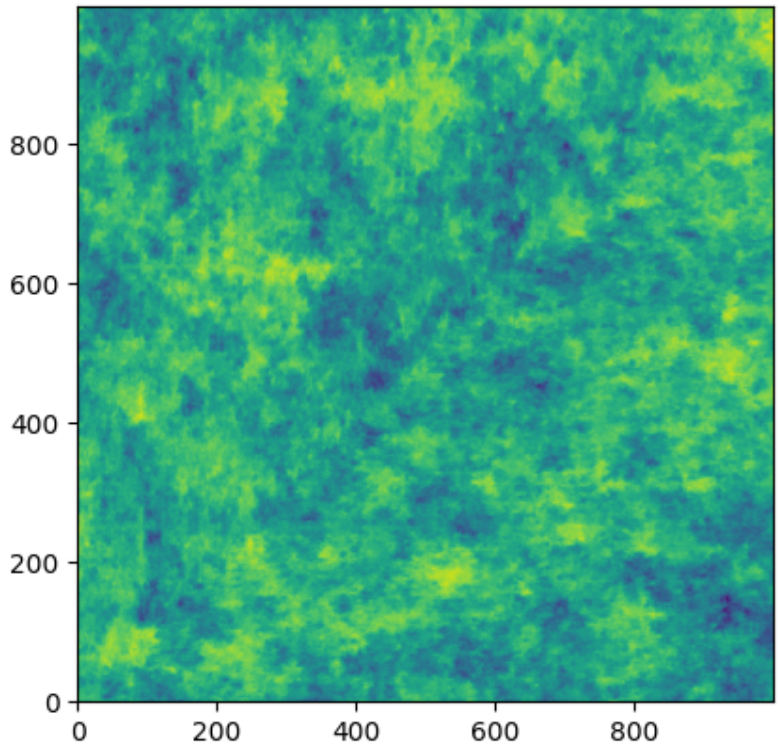
The Herten data provides information about the grid, which was already used in the previous code block. From this information, we can create our own grid on which we can estimate the variogram. As a first step, we are going to estimate an isotropic variogram, meaning that we will take point pairs from all directions into account. An unstructured grid is a natural choice for this. Therefore, we are going to create an unstructured grid from the given, structured one. For this, we are going to write another small function

```
herten_log_trans = np.log(np.loadtxt("herten_transmissivity.gz"))
dim, origin, spacing = np.loadtxt("grid_dim_origin_spacing.txt")

# create a structured grid on which the data is defined
x_s = np.arange(origin[0], origin[0] + dim[0] * spacing[0], spacing[0])
y_s = np.arange(origin[1], origin[1] + dim[1] * spacing[1], spacing[1])
# create the corresponding unstructured grid for the variogram estimation
x_u, y_u = np.meshgrid(x_s, y_s)
```

Let's have a look at the transmissivity field of the Herten aquifer

```
plt.imshow(herten_log_trans.T, origin="lower", aspect="equal")
plt.show()
```



Estimating the Variogram

Finally, everything is ready for the variogram estimation. For the unstructured method, we have to define the bins on which the variogram will be estimated. Through expert knowledge (i.e. fiddling around), we assume that the main features of the variogram will be below 10 metres distance. And because the data has a high spatial resolution, the resolution of the bins can also be high. The transmissivity data is still defined on a structured grid, but we can simply flatten it with `numpy.ndarray.flatten`, in order to bring it into the right shape. It might be more memory efficient to use `herten_log_trans.reshape(-1)`, but for better readability, we will stick to `numpy.ndarray.flatten`. Taking all data points into account would take a very long time (expert knowledge *wink*), thus we will only take 2000 datapoints into account, which are sampled randomly. In order to make the exact results reproducible, we can also set a seed.

```
bins = gs.standard_bins(pos=(x_u, y_u), max_dist=10)
bin_center, gamma = gs.vario_estimate(
    (x_u, y_u),
    herten_log_trans.reshape(-1),
    bins,
    sampling_size=2000,
    sampling_seed=19920516,
)
```

The estimated variogram is calculated on the centre of the given bins, therefore, the `bin_center` array is also returned.

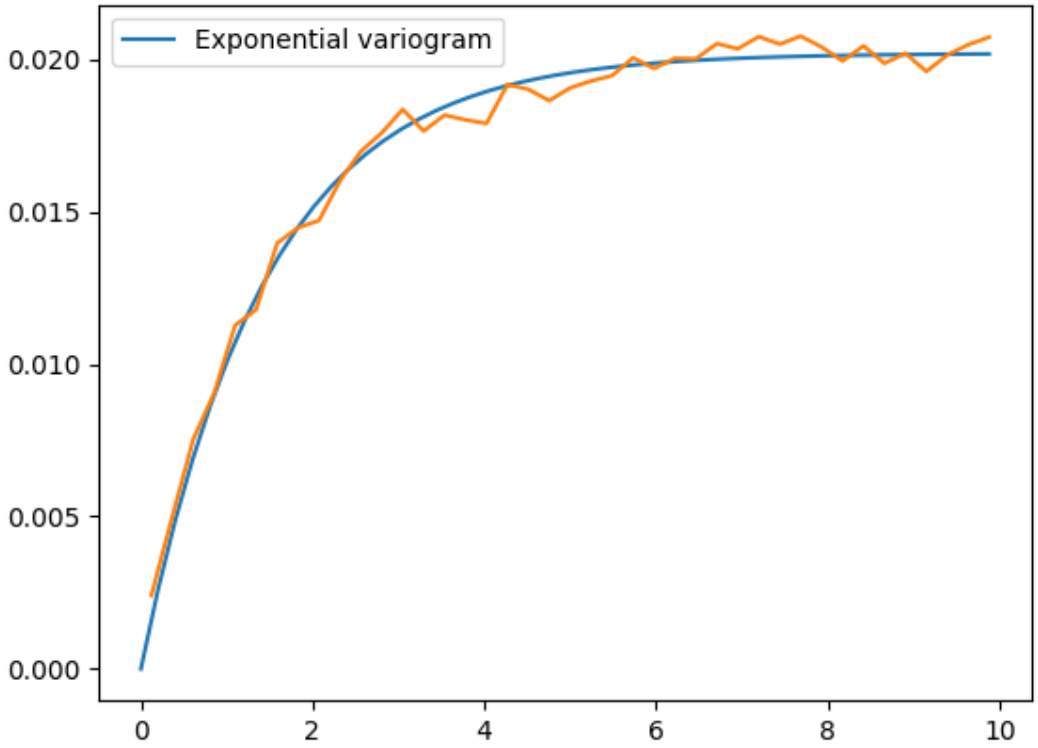
Fitting the Variogram

Now, we can see, if the estimated variogram can be modelled by a common variogram model. Let's try the *Exponential* model.

```
# fit an exponential model
fit_model = gs.Exponential(dim=2)
fit_model.fit_variogram(bin_center, gamma, nugget=False)
```

Finally, we can visualise some results. For quickly plotting a covariance model, GSTools provides some helper functions.

```
ax = fit_model.plot(x_max=max(bin_center))
ax.plot(bin_center, gamma)
```



That looks like a pretty good fit! By printing the model, we can directly see the fitted parameters

```
print(fit_model)
```

Out:

```
Exponential(dim=2, var=0.0202, len_scale=1.45, nugget=0.0)
```

With this data, we could start generating new ensembles of the Herten aquifer with the *SRF* class.

Estimating the Variogram in Specific Directions

Estimating a variogram on a structured grid gives us the possibility to only consider values in a specific direction. This could be a first test, to see if the data is anisotropic. In order to speed up the calculations, we are going to only use every 10th datapoint and for a comparison with the isotropic variogram calculated earlier, we only need the first 21 array items.

```
# estimate the variogram on a structured grid
# use only every 10th value, otherwise calculations would take very long
x_s_skip = np.ravel(x_s)[::10]
y_s_skip = np.ravel(y_s)[::10]
herten_trans_skip = herten_log_trans[::10, ::10]
```

With this much smaller data set, we can immediately estimate the variogram in the x- and y-axis

```
gamma_x = gs.vario_estimate_axis(herten_trans_skip, direction="x")
gamma_y = gs.vario_estimate_axis(herten_trans_skip, direction="y")
```

With these two estimated variograms, we can start fitting *Exponential* covariance models

```
x_plot = x_s_skip[:21]
y_plot = y_s_skip[:21]
# fit an exponential model
fit_model_x = gs.Exponential(dim=2)
fit_model_x.fit_variogram(x_plot, gamma_x[:21], nugget=False)
fit_model_y = gs.Exponential(dim=2)
fit_model_y.fit_variogram(y_plot, gamma_y[:21], nugget=False)
```

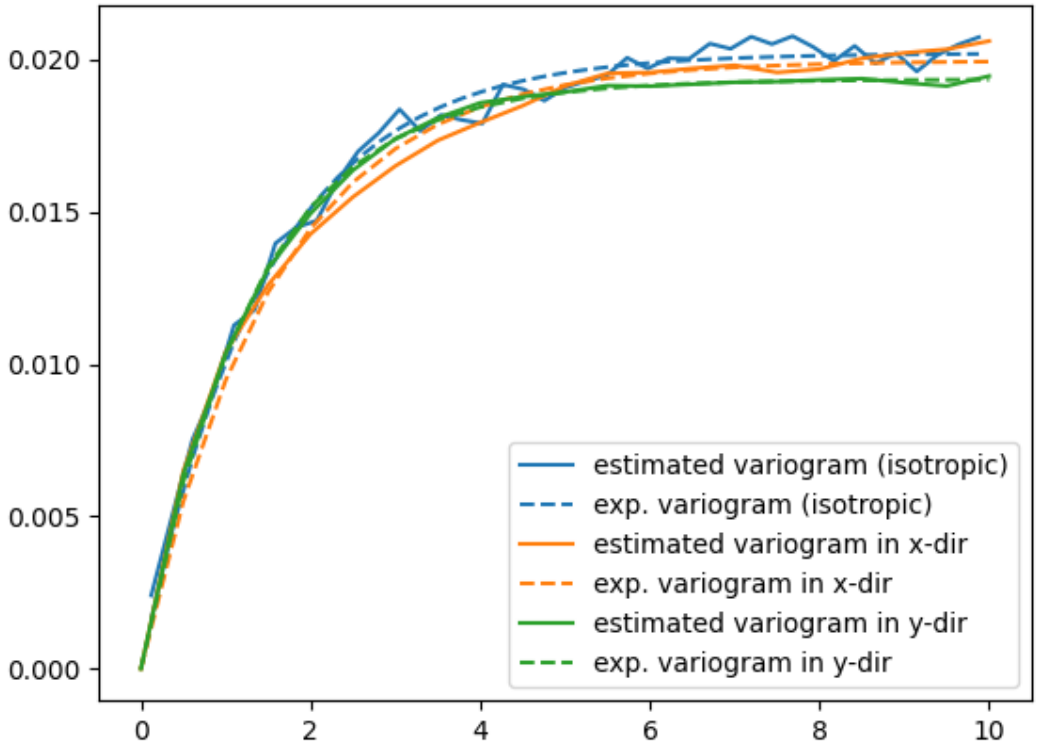
Now, the isotropic variogram and the two variograms in x- and y-direction can be plotted together with their respective models, which will be plotted with dashed lines.

```
plt.figure()  # new figure
(line,) = plt.plot(bin_center, gamma, label="estimated variogram (isotropic)")
plt.plot(
    bin_center,
    fit_model.variogram(bin_center),
    color=line.get_color(),
    linestyle="--",
    label="exp. variogram (isotropic)",
)

(line,) = plt.plot(x_plot, gamma_x[:21], label="estimated variogram in x-dir")
plt.plot(
    x_plot,
    fit_model_x.variogram(x_plot),
    color=line.get_color(),
    linestyle="--",
    label="exp. variogram in x-dir",
)

(line,) = plt.plot(y_plot, gamma_y[:21], label="estimated variogram in y-dir")
plt.plot(
    y_plot,
    fit_model_y.variogram(y_plot),
    color=line.get_color(),
    linestyle="--",
    label="exp. variogram in y-dir",
)

plt.legend()
plt.show()
```



The plot might be a bit cluttered, but at least it is pretty obvious that the Herten aquifer has no apparent anisotropies in its spatial structure.

```
print("semivariogram model (isotropic):\n", fit_model)
print("semivariogram model (in x-dir.):\n", fit_model_x)
print("semivariogram model (in y-dir.):\n", fit_model_y)
```

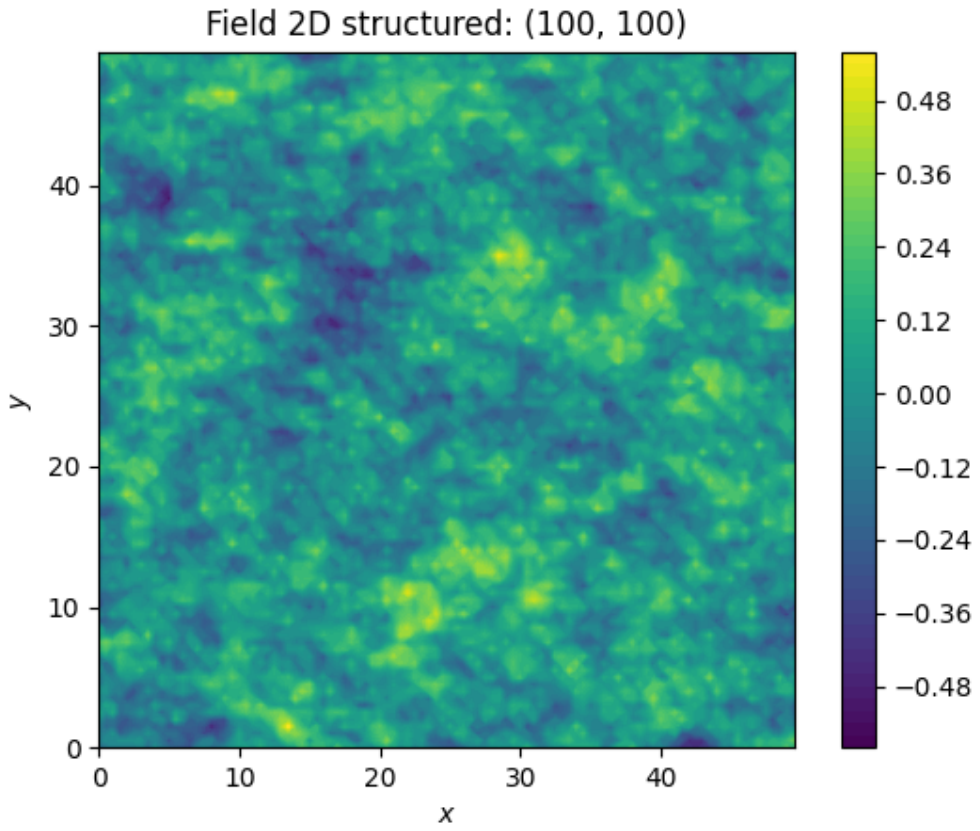
Out:

```
semivariogram model (isotropic):
 Exponential(dim=2, var=0.0202, len_scale=1.45, nugget=0.0)
semivariogram model (in x-dir.):
 Exponential(dim=2, var=0.0199, len_scale=1.55, nugget=0.0)
semivariogram model (in y-dir.):
 Exponential(dim=2, var=0.0193, len_scale=1.31, nugget=0.0)
```

Creating a Spatial Random Field from the Herten Parameters

With all the hard work done, it's straight forward now, to generate new *Herten-like realisations*

```
# create a spatial random field on the low-resolution grid
srf = gs.SRF(fit_model, seed=19770928)
srf.structured([x_s_skip, y_s_skip])
ax = srf.plot()
ax.set_aspect("equal")
```



That's pretty neat!

Total running time of the script: (0 minutes 3.576 seconds)

Standalone Field class

The *Field* class of GSTools can be used to plot arbitrary data in nD.

In the following example we will produce 10000 random points in 4D with random values and plot them.

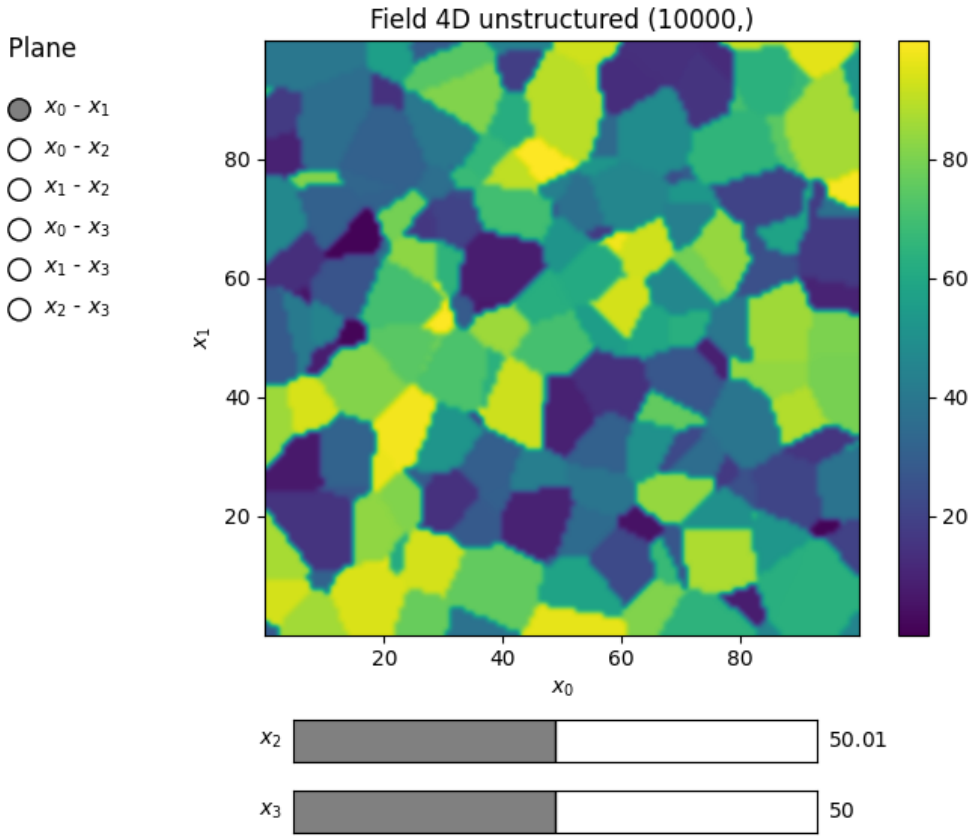
```
import numpy as np
import gstools as gs

rng = np.random.RandomState(19970221)
x0 = rng.rand(10000) * 100.0
x1 = rng.rand(10000) * 100.0
x2 = rng.rand(10000) * 100.0
x3 = rng.rand(10000) * 100.0
values = rng.rand(10000) * 100.0
```

Only thing needed to instantiate the Field is the dimension.

Afterwards we can call the instance like all other Fields (*SRF*, *Krige* or *CondSRF*), but with an additional field.

```
plotter = gs.field.Field(dim=4)
plotter(pos=(x0, x1, x2, x3), field=values)
plotter.plot()
```



Total running time of the script: (0 minutes 0.664 seconds)

CHAPTER 3

GSTOOLS API

3.1 Purpose

GeoStatTools is a library providing geostatistical tools for random field generation, conditioned field generation, kriging and variogram estimation based on a list of provided or even user-defined covariance models.

The following functionalities are directly provided on module-level.

3.2 Subpackages

covmodel	GStools subpackage providing a set of handy covariance models.
field	GStools subpackage providing tools for spatial random fields.
variogram	GStools subpackage providing tools for estimating and fitting variograms.
krige	GStools subpackage providing kriging.
random	GStools subpackage for random number generation.
tools	GStools subpackage providing miscellaneous tools.
transform	GStools subpackage providing transformations to post-process normal fields.
normalizer	GStools subpackage providing normalization routines.

3.3 Classes

Kriging

Swiss-Army-Knife for Kriging. For short cut classes see: *gstools.krige*

Krige(model, cond_pos, cond_val[, ...])	A Swiss Army knife for kriging.

Spatial Random Field

Classes for (conditioned) random field generation

SRF(model[, mean, normalizer, trend, ...])	A class to generate spatial random fields (SRF).
CondSRF(krige[, generator])	A class to generate conditioned spatial random fields (SRF).

Covariance Base-Class

Class to construct user defined covariance models

CovModel([dim, var, len_scale, nugget, ...])	Base class for the GSTools covariance models.

Covariance Models

Standard Covariance Models

Gaussian([dim, var, len_scale, nugget, ...])	The Gaussian covariance model.
Exponential([dim, var, len_scale, nugget, ...])	The Exponential covariance model.
Matern([dim, var, len_scale, nugget, anis, ...])	The Matérn covariance model.
Stable([dim, var, len_scale, nugget, anis, ...])	The stable covariance model.
Rational([dim, var, len_scale, nugget, ...])	The rational quadratic covariance model.
Cubic([dim, var, len_scale, nugget, anis, ...])	The Cubic covariance model.
Linear([dim, var, len_scale, nugget, anis, ...])	The bounded linear covariance model.
Circular([dim, var, len_scale, nugget, ...])	The circular covariance model.
Spherical([dim, var, len_scale, nugget, ...])	The Spherical covariance model.
HyperSpherical([dim, var, len_scale, ...])	The Hyper-Spherical covariance model.
SuperSpherical([dim, var, len_scale, ...])	The Super-Spherical covariance model.
JBessel([dim, var, len_scale, nugget, anis, ...])	The J-Bessel hole model.

Truncated Power Law Covariance Models

TPLGaussian([dim, var, len_scale, nugget, ...])	Truncated-Power-Law with Gaussian modes.
TPLExponential([dim, var, len_scale, ...])	Truncated-Power-Law with Exponential modes.
TPLStable([dim, var, len_scale, nugget, ...])	Truncated-Power-Law with Stable modes.
TPLSimple([dim, var, len_scale, nugget, ...])	The simply truncated power law model.

3.4 Functions

VTK-Export

Routines to export fields to the vtk format

vtk_export(filename, pos, fields[, mesh_type])	Export a field to vtk.
to_vtk(pos, fields[, mesh_type])	Create a VTK/PyVista grid.

Geometric

Some convenient functions for geometric operations

rotated_main_axes(dim, angles)	Create list of the main axis defined by the given system rotations.
generate_grid(pos)	Generate grid from a structured position tuple.
generate_st_grid(pos, time[, mesh_type])	Generate spatio-temporal grid from a position tuple and time array.

Variogram Estimation

Estimate the variogram of a given field with these routines

vario_estimate(pos, field[, bin_edges, ...])	Estimates the empirical variogram.
vario_estimate_axis(field[, direction, ...])	Estimates the variogram along array axis.
standard_bins([pos, dim, latlon, mesh_type, ...])	Get standard binning.

3.5 Misc

EARTH_RADIUS	earth radius for WGS84 ellipsoid in km

3.6 gstools.covmodel

GStools subpackage providing a set of handy covariance models.

Subpackages

plot	GStools subpackage providing plotting routines for the covariance models.

Covariance Base-Class

Class to construct user defined covariance models

CovModel([dim, var, len_scale, nugget, ...])	Base class for the GSTools covariance models.

gstools.covmodel.CovModel

class gstools.covmodel.**CovModel**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: `object`

Base class for the GSTools covariance models.

Parameters

- **dim** (`int`, optional) – dimension of the model. Default: `3`
- **var** (`float`, optional) – variance of the model (the nugget is not included in "this" variance) Default: `1.0`
- **len_scale** (`float` or `list`, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: `1.0`
- **nugget** (`float`, optional) – nugget of the model. Default: `0.0`
- **anis** (`float` or `list`, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: `1.0`
- **angles** (`float` or `list`, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: `0.0`
- **integral_scale** (`float` or `list` or `None`, optional) – If given, `len_scale` will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: `None`

- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None
- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False
- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, var will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to {"a": -1, "b": 1, "N": 1000, "h": 0.001}. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds list: Bounds for the lenght scale.

len_scale_vec numpy.ndarray: The length scales in each direction.

name str: The name of the CovModel class.

nugget `float`: The nugget of the model.

nugget_bounds `list`: Bounds for the nugget.

opt_arg `list` of `str`: Names of the optional arguments.

opt_arg_bounds `dict`: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Check the given dimension.
check_opt_arg()	Run checks for the optional arguments.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Provide default optional arguments by the user.
default_opt_arg_bounds()	Provide default boundaries for optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.
fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.

continues on next page

Table 13 – continued from previous page

ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for the variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Check the given dimension.

check_opt_arg()
Run checks for the optional arguments.

This is in addition to the bound-checks

Notes

- You can use this to raise a ValueError/warning
- Any return value will be ignored
- This method will only be run once, when the class is initialized

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Provide default optional arguments by the user.

Should be given as a dictionary when overridden.

default_opt_arg_bounds()
Provide default boundaries for optional arguments.

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

  If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None
- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

  Default: "default"
- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None

- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'
- **loss** (`str` or `callable`, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'
- **max_eval** (`int` or `None`, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (`bool`, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (`dict`, optional) – Other keyword arguments passed to scipys curve_fit. Default: None
- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (`dict`) – Dictonary with the fitted parameter values
- **pcov** (`numpy.ndarray`) – The estimated covariance of *popt* from `scipy.optimize.curve_fit`. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.
- **r2_score** (`float`, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

---

fix_dim()

Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"
  - "vario_axis"
  - "cov_axis"
  - "cor_axis"
  - "spectrum"
  - "spectral_density"
  - "spectral_rad_pdf"
- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in *gstools.covmodel.plot*.

See also:

gstools.covmodel.plot

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: [a, b] or [a, b, <type>] where <type> is one of "oo", "cc", "oc" or "co" to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (float) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (float) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for the variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

property angles
Rotation angles (in rad) of the model.

Type numpy.ndarray

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: [a, b] or [a, b, <type>] where <type> is one of "oo", "cc", "oc" or "co" to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- `integral_scale_vec[0] = integral_scale`
- `integral_scale_vec[1] = integral_scale*anis[0]`
- `integral_scale_vec[2] = integral_scale*anis[1]`

Type `numpy.ndarray`

property is_isotropic
State if a model is isotropic.

Type `bool`

property iso_arg
Names of isotropic arguments.

Type `list` of `str`

property iso_arg_list
Values of isotropic arguments.

Type `list` of `float`

property latlon
Whether the model depends on geographical coords.

Type `bool`

property len_rescaled
The rescaled main length scale of the model.

Type `float`

property len_scale
The main length scale of the model.

Type `float`

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `list`

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type `numpy.ndarray`

property name
The name of the CovModel class.

Type str

property nugget
The nugget of the model.

Type float

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property opt_arg
Names of the optional arguments.

Type list of str

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type `float`

property var
The variance of the model.

Type `float`

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `list`

property var_raw
The raw variance of the model without factor.

(See. CovModel.var_factor)

Type `float`

Covariance Models

Standard Covariance Models

Gaussian([dim, var, len_scale, nugget, ...])	The Gaussian covariance model.
Exponential([dim, var, len_scale, nugget, ...])	The Exponential covariance model.
Matern([dim, var, len_scale, nugget, anis, ...])	The Matérn covariance model.
Stable([dim, var, len_scale, nugget, anis, ...])	The stable covariance model.
Rational([dim, var, len_scale, nugget, ...])	The rational quadratic covariance model.
Cubic([dim, var, len_scale, nugget, anis, ...])	The Cubic covariance model.
Linear([dim, var, len_scale, nugget, anis, ...])	The bounded linear covariance model.
Circular([dim, var, len_scale, nugget, ...])	The circular covariance model.
Spherical([dim, var, len_scale, nugget, ...])	The Spherical covariance model.
HyperSpherical([dim, var, len_scale, ...])	The Hyper-Spherical covariance model.
SuperSpherical([dim, var, len_scale, ...])	The Super-Spherical covariance model.
JBessel([dim, var, len_scale, nugget, anis, ...])	The J-Bessel hole model.

gstools.covmodel.Gaussian

class gstools.covmodel.**Gaussian**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: *gstools.covmodel.base.CovModel*

The Gaussian covariance model.

Notes

This model is given by the following variogram [Webster2007]:

$$\gamma(r) = \sigma^2 \left(1 - \exp\left(-\left(s \cdot \frac{r}{\ell}\right)^2\right)\right) + n$$

Where the standard rescale factor is $s = \frac{\sqrt{\pi}}{2}$.

References

Parameters

- **dim** (int, optional) – dimension of the model. Default: 3
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: 1.0
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: 1.0
- **nugget** (float, optional) – nugget of the model. Default: 0.0
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: `1.0`

- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: `0.0`
- **integral_scale** (float or list or None, optional) – If given, `len_scale` will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: None
- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None
- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False
- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, `var` will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to `{"a": -1, "b": 1, "N": 1000, "h": 0.001}`. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg `list` of `str`: Names of isotropic arguments.

iso_arg_list `list` of `float`: Values of isotropic arguments.

latlon `bool`: Whether the model depends on geographical coords.

len_rescaled `float`: The rescaled main length scale of the model.

len_scale `float`: The main length scale of the model.

len_scale_bounds `list`: Bounds for the lenght scale.

len_scale_vec `numpy.ndarray`: The length scales in each direction.

name `str`: The name of the CovModel class.

nugget `float`: The nugget of the model.

nugget_bounds `list`: Bounds for the nugget.

opt_arg `list` of `str`: Names of the optional arguments.

opt_arg_bounds `dict`: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Check the given dimension.
check_opt_arg()	Run checks for the optional arguments.
cor(h)	Gaussian normalized correlation function.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	Correlation function of the model.
cov_axis(r[, axis])	Covariance along axis of anisotropy.

continues on next page

Table 15 – continued from previous page

cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Provide default optional arguments by the user.
default_opt_arg_bounds()	Provide default boundaries for optional arguments.
default_rescale()	Gaussian rescaling factor to result in integral scale.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.
fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.
ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_cdf(r)	Gaussian radial spectral cdf.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectral_rad_ppf(u)	Gaussian radial spectral ppf.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for the variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Check the given dimension.

check_opt_arg()
Run checks for the optional arguments.

This is in addition to the bound-checks

Notes

- You can use this to raise a ValueError/warning
- Any return value will be ignored

- This method will only be run once, when the class is initialized

cor(*h*)
Gaussian normalized correlation function.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
Correlation function of the model.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Provide default optional arguments by the user.

Should be given as a dictionary when overridden.

default_opt_arg_bounds()
Provide default boundaries for optional arguments.

default_rescale()
Gaussian rescaling factor to result in integral scale.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True

- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

  If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None
- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

  Default: "default"
- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None
- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'
- **loss** (str or callable, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'

- **max_eval** (int or None, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (bool, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (dict, optional) – Other keyword arguments passed to scipys curve_fit. Default: None
- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (dict) – Dictonary with the fitted parameter values
- **pcov** (numpy.ndarray) – The estimated covariance of *popt* from scipy.optimize.curve_fit. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.
- **r2_score** (float, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

---

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"

- "cor_yadrenko"
- "vario_axis"
- "cov_axis"
- "cor_axis"
- "spectrum"
- "spectral_density"
- "spectral_rad_pdf"

- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in *gstools.covmodel.plot*.

See also:

gstools.covmodel.plot

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

spectral_density(k)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (*float*) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_cdf(r)
Gaussian radial spectral cdf.

spectral_rad_pdf(r)
Radial spectral density of the model.

spectral_rad_ppf(u)
Gaussian radial spectral ppf.

Notes

Not defined for 3D.

---

spectrum(k)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (float) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for the variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type numpy.ndarray

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: [a, b] or [a, b, <type>] where <type> is one of "oo", "cc", "oc" or "co" to define if the bounds are open (“o”) or closed (“c”).

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: [a, b] or [a, b, <type>] where <type> is one of "oo", "cc", "oc" or "co" to define if the bounds are open (“o”) or closed (“c”).

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- integral_scale_vec[0] = integral_scale
- integral_scale_vec[1] = integral_scale*anis[0]
- integral_scale_vec[2] = integral_scale*anis[1]

Type numpy.ndarray

property is_isotropic
State if a model is isotropic.

Type bool

property iso_arg
Names of isotropic arguments.

Type list of str

property iso_arg_list
Values of isotropic arguments.

Type list of float

property latlon
Whether the model depends on geographical coords.

Type bool

property len_rescaled
The rescaled main length scale of the model.

Type float

property len_scale
The main length scale of the model.

Type float

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type numpy.ndarray

property name
The name of the CovModel class.

Type str

property nugget
The nugget of the model.

Type float

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property opt_arg
Names of the optional arguments.

Type list of str

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `list`

property var_raw
The raw variance of the model without factor.

(See. CovModel.var_factor)

Type `float`

gstools.covmodel.Exponential

class gstools.covmodel.**Exponential**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: *gstools.covmodel.base.CovModel*

The Exponential covariance model.

Notes

This model is given by the following variogram [Webster2007]:

$$\gamma(r) = \sigma^2 \left(1 - \exp\left(-s \cdot \frac{r}{\ell}\right)\right) + n$$

Where the standard rescale factor is $s = 1$.

References

Parameters

- **dim** (int, optional) – dimension of the model. Default: 3
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: 1.0
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: 1.0
- **nugget** (float, optional) – nugget of the model. Default: 0.0
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: 1.0
- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: 0.0
- **integral_scale** (float or list or None, optional) – If given, len_scale will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: None
- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None
- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal

to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False

- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, `var` will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to `{"a": -1, "b": 1, "N": 1000, "h": 0.001}`. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds list: Bounds for the lenght scale.

len_scale_vec numpy.ndarray: The length scales in each direction.

name str: The name of the CovModel class.

nugget float: The nugget of the model.

nugget_bounds list: Bounds for the nugget.

opt_arg list of str: Names of the optional arguments.

opt_arg_bounds dict: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Check the given dimension.
check_opt_arg()	Run checks for the optional arguments.
cor(h)	Exponential normalized correlation function.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	Correlation function of the model.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Provide default optional arguments by the user.
default_opt_arg_bounds()	Provide default boundaries for optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.
fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.
ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.

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Table 16 – continued from previous page

pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_cdf(r)	Exponential radial spectral cdf.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectral_rad_ppf(u)	Exponential radial spectral ppf.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for the variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Check the given dimension.

check_opt_arg()
Run checks for the optional arguments.

This is in addition to the bound-checks

Notes

- You can use this to raise a ValueError/warning
- Any return value will be ignored
- This method will only be run once, when the class is initialized

cor(*h*)
Exponential normalized correlation function.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
Correlation function of the model.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Provide default optional arguments by the user.

Should be given as a dictionary when overridden.

default_opt_arg_bounds()
Provide default boundaries for optional arguments.

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

  If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None
- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

Default: "default"

- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None
- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'
- **loss** (str or callable, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'
- **max_eval** (int or None, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (bool, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (dict, optional) – Other keyword arguments passed to scipys curve_fit. Default: None
- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (dict) – Dictonary with the fitted parameter values
- **pcov** (numpy.ndarray) – The estimated covariance of *popt* from scipy.optimize.curve_fit. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.
- **r2_score** (float, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"
  - "vario_axis"
  - "cov_axis"
  - "cor_axis"
  - "spectrum"
  - "spectral_density"
  - "spectral_rad_pdf"
- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in *gstools.covmodel.plot*.

See also:

gstools.covmodel.plot

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_cdf(*r*)
Exponential radial spectral cdf.

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectral_rad_ppf(*u*)
Exponential radial spectral ppf.

Notes

Not defined for 3D.

---

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for the variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type numpy.ndarray

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- integral_scale_vec[0] = integral_scale
- integral_scale_vec[1] = integral_scale*anis[0]
- integral_scale_vec[2] = integral_scale*anis[1]

Type numpy.ndarray

property is_isotropic
State if a model is isotropic.

Type bool

property iso_arg
Names of isotropic arguments.

Type list of str

property iso_arg_list
Values of isotropic arguments.

Type list of float

property latlon
Whether the model depends on geographical coords.

Type bool

property len_rescaled
The rescaled main length scale of the model.

Type float

property len_scale
The main length scale of the model.

Type float

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `list`

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type `numpy.ndarray`

property name
The name of the CovModel class.

Type `str`

property nugget
The nugget of the model.

Type `float`

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `list`

property opt_arg
Names of the optional arguments.

Type `list` of `str`

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `dict`

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property var_raw
The raw variance of the model without factor.

(See. CovModel.var_factor)

Type float

gstools.covmodel.Matern

class gstools.covmodel.**Matern**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: *gstools.covmodel.base.CovModel*

The Matérn covariance model.

Notes

This model is given by the following correlation function [Rasmussen2003]:

$$\rho(r) = \frac{2^{1-\nu}}{\Gamma(\nu)} \cdot \left(\sqrt{\nu} \cdot s \cdot \frac{r}{\ell}\right)^{\nu} \cdot \mathrm{K}_{\nu}\left(\sqrt{\nu} \cdot s \cdot \frac{r}{\ell}\right)$$

Where the standard rescale factor is $s = 1$. Γ is the gamma function and K_{ν} is the modified Bessel function of the second kind.

ν is a shape parameter and should be >= 0.2.

If $\nu > 20$, a gaussian model is used, since it represents the limiting case:

$$\rho(r) = \exp\left(-\left(s \cdot \frac{r}{2\ell}\right)^2\right)$$

References

Parameters

- **nu** (float, optional) – Shape parameter. Standard range: `[0.2, 30]` Default: `1.0`
- **dim** (int, optional) – dimension of the model. Default: `3`
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: `1.0`
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: `1.0`
- **nugget** (float, optional) – nugget of the model. Default: `0.0`
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: `1.0`
- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: `0.0`
- **integral_scale** (float or list or None, optional) – If given, `len_scale` will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: None

- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None
- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False
- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, var will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to {"a": -1, "b": 1, "N": 1000, "h": 0.001}. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds list: Bounds for the lenght scale.

len_scale_vec numpy.ndarray: The length scales in each direction.

name str: The name of the CovModel class.

nugget `float`: The nugget of the model.

nugget_bounds `list`: Bounds for the nugget.

opt_arg `list` of `str`: Names of the optional arguments.

opt_arg_bounds `dict`: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Check the given dimension.
check_opt_arg()	Run checks for the optional arguments.
cor(h)	Matérn normalized correlation function.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	Correlation function of the model.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Defaults for the optional arguments.
default_opt_arg_bounds()	Defaults for boundaries of the optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.

continues on next page

Table 17 – continued from previous page

fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.
ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for the variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Check the given dimension.

check_opt_arg()
Run checks for the optional arguments.

This is in addition to the bound-checks

Notes

- You can use this to raise a ValueError/warning
- Any return value will be ignored
- This method will only be run once, when the class is initialized

cor(*h*)
Matérn normalized correlation function.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
Correlation function of the model.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Defaults for the optional arguments.

- `{"nu": 1.0}`

Returns Defaults for optional arguments

Return type dict

default_opt_arg_bounds()
Defaults for boundaries of the optional arguments.

- `{"nu": [0.2, 30.0, "cc"]}`

Returns Boundaries for optional arguments

Return type dict

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None

- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

  Default: "default"
- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None
- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'
- **loss** (str or callable, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'
- **max_eval** (int or None, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (bool, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (dict, optional) – Other keyword arguments passed to scipys curve_fit. Default: None

- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (dict) – Dictonary with the fitted parameter values
- **pcov** (numpy.ndarray) – The estimated covariance of *popt* from scipy.optimize.curve_fit. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.
- **r2_score** (float, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

---

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"
  - "vario_axis"
  - "cov_axis"
  - "cor_axis"
  - "spectrum"

- "spectral_density"
- "spectral_rad_pdf"

- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in `gstools.covmodel.plot`.

See also:

`gstools.covmodel.plot`

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for the variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type numpy.ndarray

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- integral_scale_vec[0] = integral_scale
- integral_scale_vec[1] = integral_scale*anis[0]
- integral_scale_vec[2] = integral_scale*anis[1]

Type numpy.ndarray

property is_isotropic
State if a model is isotropic.

Type bool

property iso_arg
Names of isotropic arguments.

Type list of str

property iso_arg_list
Values of isotropic arguments.

Type list of float

property latlon
Whether the model depends on geographical coords.

Type bool

property len_rescaled
The rescaled main length scale of the model.

Type float

property len_scale
The main length scale of the model.

Type float

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type numpy.ndarray

property name
The name of the CovModel class.

Type str

property nugget
The nugget of the model.

Type float

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property opt_arg
Names of the optional arguments.

Type list of str

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property var_raw
The raw variance of the model without factor.

(See. CovModel.var_factor)

Type float

gstools.covmodel.Stable

class gstools.covmodel.**Stable**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: *gstools.covmodel.base.CovModel*

The stable covariance model.

Notes

This model is given by the following correlation function [Wackernagel2003]:

$$\rho(r) = \exp\left(-\left(s \cdot \frac{r}{\ell}\right)^{\alpha}\right)$$

Where the standard rescale factor is $s = 1$. α is a shape parameter with $\alpha \in (0, 2]$

References

Parameters

- **alpha** (float, optional) – Shape parameter. Standard range: `(0, 2]` Default: `1.5`
- **dim** (int, optional) – dimension of the model. Default: `3`
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: `1.0`
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: `1.0`
- **nugget** (float, optional) – nugget of the model. Default: `0.0`
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: `1.0`
- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: `0.0`
- **integral_scale** (float or list or None, optional) – If given, `len_scale` will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: None
- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None
- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance

r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False

- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, `var` will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to `{"a": -1, "b": 1, "N": 1000, "h": 0.001}`. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds list: Bounds for the lenght scale.

len_scale_vec numpy.ndarray: The length scales in each direction.

name str: The name of the CovModel class.

nugget float: The nugget of the model.

nugget_bounds list: Bounds for the nugget.

opt_arg list of str: Names of the optional arguments.

opt_arg_bounds dict: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Check the given dimension.
check_opt_arg()	Check the optional arguments.
cor(h)	Stable normalized correlation function.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	Correlation function of the model.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Defaults for the optional arguments.
default_opt_arg_bounds()	Defaults for boundaries of the optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.
fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.
ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.

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Table 18 – continued from previous page

plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for the variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Check the given dimension.

check_opt_arg()
Check the optional arguments.

Warns **alpha** – If alpha is < 0.3, the model tends to a nugget model and gets numerically unstable.

cor(*h*)
Stable normalized correlation function.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
Correlation function of the model.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Defaults for the optional arguments.

- `{"alpha": 1.5}`

Returns Defaults for optional arguments

Return type dict

default_opt_arg_bounds()
Defaults for boundaries of the optional arguments.

- `{"alpha": [0, 2, "oc"]}`

Returns Boundaries for optional arguments

Return type dict

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

  If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None
- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as

given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

Default: "default"

- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None
- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'
- **loss** (str or callable, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'
- **max_eval** (int or None, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (bool, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (dict, optional) – Other keyword arguments passed to scipys curve_fit. Default: None
- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (dict) – Dictonary with the fitted parameter values
- **pcov** (numpy.ndarray) – The estimated covariance of *popt* from scipy.optimize.curve_fit. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.

- **r2_score** (float, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"
  - "vario_axis"
  - "cov_axis"
  - "cor_axis"
  - "spectrum"
  - "spectral_density"
  - "spectral_rad_pdf"
- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in *gstools.covmodel.plot*.

See also:

gstools.covmodel.plot

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (*float*) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (*float*) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for the variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type numpy.ndarray

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- integral_scale_vec[0] = integral_scale
- integral_scale_vec[1] = integral_scale*anis[0]
- integral_scale_vec[2] = integral_scale*anis[1]

Type numpy.ndarray

property is_isotropic
State if a model is isotropic.

Type bool

property iso_arg
Names of isotropic arguments.

Type list of str

property iso_arg_list
Values of isotropic arguments.

Type list of float

property latlon
Whether the model depends on geographical coords.

Type bool

property len_rescaled
The rescaled main length scale of the model.

Type float

property len_scale
The main length scale of the model.

Type float

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: [a, b] or [a, b, <type>] where <type> is one of "oo", "cc", "oc" or "co" to define if the bounds are open ("o") or closed ("c").

Type list

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type numpy.ndarray

property name
The name of the CovModel class.

Type str

property nugget
The nugget of the model.

Type float

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property opt_arg
Names of the optional arguments.

Type list of str

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property var_raw
The raw variance of the model without factor.

(See. CovModel.var_factor)

Type float

gstools.covmodel.Rational

class gstools.covmodel.**Rational**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: *gstools.covmodel.base.CovModel*

The rational quadratic covariance model.

Notes

This model is given by the following correlation function [Rasmussen2003]:

$$\rho(r) = \left(1 + \frac{1}{\alpha} \cdot \left(s \cdot \frac{r}{\ell}\right)^2\right)^{-\alpha}$$

Where the standard rescale factor is $s = 1$. α is a shape parameter and should be > 0.5.

For $\alpha \to \infty$ this model converges to the Gaussian model:

$$\rho(r) = \exp\left(-\left(s \cdot \frac{r}{\ell}\right)^2\right)$$

References

Parameters

- **alpha** (float, optional) – Shape parameter. Standard range: `[0.5, 50]` Default: `1.0`
- **dim** (int, optional) – dimension of the model. Default: `3`
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: `1.0`
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: `1.0`
- **nugget** (float, optional) – nugget of the model. Default: `0.0`
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: `1.0`
- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: `0.0`
- **integral_scale** (float or list or None, optional) – If given, `len_scale` will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: None

- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None
- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False
- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, var will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to {"a": -1, "b": 1, "N": 1000, "h": 0.001}. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds list: Bounds for the lenght scale.

len_scale_vec numpy.ndarray: The length scales in each direction.

name str: The name of the CovModel class.

nugget `float`: The nugget of the model.

nugget_bounds `list`: Bounds for the nugget.

opt_arg `list` of `str`: Names of the optional arguments.

opt_arg_bounds `dict`: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Check the given dimension.
check_opt_arg()	Run checks for the optional arguments.
cor(h)	Rational normalized correlation function.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	Correlation function of the model.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Defaults for the optional arguments.
default_opt_arg_bounds()	Defaults for boundaries of the optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.

continues on next page

Table 19 – continued from previous page

fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.
ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for the variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Check the given dimension.

check_opt_arg()
Run checks for the optional arguments.

This is in addition to the bound-checks

Notes

- You can use this to raise a ValueError/warning
- Any return value will be ignored
- This method will only be run once, when the class is initialized

cor(*h*)
Rational normalized correlation function.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
Correlation function of the model.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Defaults for the optional arguments.

- `{"alpha": 1.0}`

Returns Defaults for optional arguments

Return type `dict`

default_opt_arg_bounds()
Defaults for boundaries of the optional arguments.

- `{"alpha": [0.5, 50.0]}`

Returns Boundaries for optional arguments

Return type `dict`

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (`numpy.ndarray`) – The bin-centers of the empirical variogram.
- **y_data** (`numpy.ndarray`) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (`bool`, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (`float` or `bool`, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None

- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

  Default: "default"
- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None
- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'
- **loss** (str or callable, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'
- **max_eval** (int or None, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (bool, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (dict, optional) – Other keyword arguments passed to scipys curve_fit. Default: None

- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (dict) – Dictonary with the fitted parameter values
- **pcov** (numpy.ndarray) – The estimated covariance of *popt* from scipy.optimize.curve_fit. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.
- **r2_score** (float, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

---

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"
  - "vario_axis"
  - "cov_axis"
  - "cor_axis"
  - "spectrum"

- "spectral_density"
- "spectral_rad_pdf"

- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in `gstools.covmodel.plot`.

See also:

`gstools.covmodel.plot`

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for the variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type numpy.ndarray

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- integral_scale_vec[0] = integral_scale
- integral_scale_vec[1] = integral_scale*anis[0]
- integral_scale_vec[2] = integral_scale*anis[1]

Type numpy.ndarray

property is_isotropic
State if a model is isotropic.

Type bool

property iso_arg
Names of isotropic arguments.

Type list of str

property iso_arg_list
Values of isotropic arguments.

Type list of float

property latlon
Whether the model depends on geographical coords.

Type bool

property len_rescaled
The rescaled main length scale of the model.

Type float

property len_scale
The main length scale of the model.

Type float

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `list`

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type `numpy.ndarray`

property name
The name of the CovModel class.

Type `str`

property nugget
The nugget of the model.

Type `float`

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `list`

property opt_arg
Names of the optional arguments.

Type `list` of `str`

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `dict`

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property var_raw
The raw variance of the model without factor.

(See. CovModel.var_factor)

Type float

gstools.covmodel.Cubic

class gstools.covmodel.**Cubic**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: *gstools.covmodel.base.CovModel*

The Cubic covariance model.

A model with reverse curvature near the origin and a finite range of correlation.

Notes

This model is given by the following correlation function [Chiles2009]:

$$\rho(r) = \begin{cases} 1 - 7\left(s \cdot \frac{r}{\ell}\right)^2 + \frac{35}{4}\left(s \cdot \frac{r}{\ell}\right)^3 - \frac{7}{2}\left(s \cdot \frac{r}{\ell}\right)^5 + \frac{3}{4}\left(s \cdot \frac{r}{\ell}\right)^7 & r < \frac{\ell}{s} \\ 0 & r \geq \frac{\ell}{s} \end{cases}$$

Where the standard rescale factor is $s = 1$.

References

Parameters

- **dim** (int, optional) – dimension of the model. Default: 3
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: 1.0
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: 1.0
- **nugget** (float, optional) – nugget of the model. Default: 0.0
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: 1.0
- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: 0.0
- **integral_scale** (float or list or None, optional) – If given, len_scale will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: None
- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None

- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False
- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, `var` will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to `{"a": -1, "b": 1, "N": 1000, "h": 0.001}`. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds list: Bounds for the lenght scale.

len_scale_vec numpy.ndarray: The length scales in each direction.

name str: The name of the CovModel class.

nugget float: The nugget of the model.

nugget_bounds list: Bounds for the nugget.

opt_arg `list` of `str`: Names of the optional arguments.

opt_arg_bounds `dict`: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Check the given dimension.
check_opt_arg()	Run checks for the optional arguments.
cor(h)	Spherical normalized correlation function.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	Correlation function of the model.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Provide default optional arguments by the user.
default_opt_arg_bounds()	Provide default boundaries for optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.
fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.

continues on next page

Table 20 – continued from previous page

ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for the variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Check the given dimension.

check_opt_arg()
Run checks for the optional arguments.

This is in addition to the bound-checks

Notes

- You can use this to raise a ValueError/warning
- Any return value will be ignored
- This method will only be run once, when the class is initialized

cor(*h*)
Spherical normalized correlation function.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
Correlation function of the model.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Provide default optional arguments by the user.

Should be given as a dictionary when overridden.

default_opt_arg_bounds()
Provide default boundaries for optional arguments.

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

  If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None
- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as

given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

Default: "default"

- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None
- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'
- **loss** (str or callable, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'
- **max_eval** (int or None, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (bool, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (dict, optional) – Other keyword arguments passed to scipys curve_fit. Default: None
- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (dict) – Dictonary with the fitted parameter values
- **pcov** (numpy.ndarray) – The estimated covariance of *popt* from scipy.optimize.curve_fit. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.

- **r2_score** (float, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"
  - "vario_axis"
  - "cov_axis"
  - "cor_axis"
  - "spectrum"
  - "spectral_density"
  - "spectral_rad_pdf"
- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in *gstools.covmodel.plot*.

See also:

gstools.covmodel.plot

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (*float*) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (*float*) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for the variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type numpy.ndarray

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- integral_scale_vec[0] = integral_scale
- integral_scale_vec[1] = integral_scale*anis[0]
- integral_scale_vec[2] = integral_scale*anis[1]

Type numpy.ndarray

property is_isotropic
State if a model is isotropic.

Type bool

property iso_arg
Names of isotropic arguments.

Type list of str

property iso_arg_list
Values of isotropic arguments.

Type list of float

property latlon
Whether the model depends on geographical coords.

Type bool

property len_rescaled
The rescaled main length scale of the model.

Type float

property len_scale
The main length scale of the model.

Type float

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: [a, b] or [a, b, <type>] where <type> is one of "oo", "cc", "oc" or "co" to define if the bounds are open ("o") or closed ("c").

Type list

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type numpy.ndarray

property name
The name of the CovModel class.

Type str

property nugget
The nugget of the model.

Type float

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property opt_arg
Names of the optional arguments.

Type list of str

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property var_raw
The raw variance of the model without factor.

(See. CovModel.var_factor)

Type float

gstools.covmodel.Linear

class gstools.covmodel.**Linear**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: *gstools.covmodel.base.CovModel*

The bounded linear covariance model.

This model is derived from the relative intersection area of two lines in 1D, where the middle points have a distance of r and the line lengths are ℓ.

Notes

This model is given by the following correlation function [Webster2007]:

$$\rho(r) = \begin{cases} 1 - s \cdot \frac{r}{\ell} & r < \frac{\ell}{s} \\ 0 & r \geq \frac{\ell}{s} \end{cases}$$

Where the standard rescale factor is $s = 1$.

References

Parameters

- **dim** (int, optional) – dimension of the model. Default: 3
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: 1.0
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: 1.0
- **nugget** (float, optional) – nugget of the model. Default: 0.0
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: 1.0
- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: 0.0
- **integral_scale** (float or list or None, optional) – If given, len_scale will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: None
- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None

- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False
- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, `var` will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to `{"a": -1, "b": 1, "N": 1000, "h": 0.001}`. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds list: Bounds for the lenght scale.

len_scale_vec numpy.ndarray: The length scales in each direction.

name str: The name of the CovModel class.

nugget float: The nugget of the model.

nugget_bounds list: Bounds for the nugget.

opt_arg `list` of `str`: Names of the optional arguments.

opt_arg_bounds `dict`: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Linear model is only valid in 1D.
check_opt_arg()	Run checks for the optional arguments.
cor(h)	Linear normalized correlation function.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	Correlation function of the model.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Provide default optional arguments by the user.
default_opt_arg_bounds()	Provide default boundaries for optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.
fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.

continues on next page

Table 21 – continued from previous page

ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for the variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Linear model is only valid in 1D.

check_opt_arg()
Run checks for the optional arguments.

This is in addition to the bound-checks

Notes

- You can use this to raise a ValueError/warning
- Any return value will be ignored
- This method will only be run once, when the class is initialized

cor(*h*)
Linear normalized correlation function.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
Correlation function of the model.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Provide default optional arguments by the user.

Should be given as a dictionary when overridden.

default_opt_arg_bounds()
Provide default boundaries for optional arguments.

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

  If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None
- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as

given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

Default: "default"

- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None
- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'
- **loss** (str or callable, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'
- **max_eval** (int or None, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (bool, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (dict, optional) – Other keyword arguments passed to scipys curve_fit. Default: None
- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (dict) – Dictonary with the fitted parameter values
- **pcov** (numpy.ndarray) – The estimated covariance of *popt* from scipy.optimize.curve_fit. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.

- **r2_score** (float, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"
  - "vario_axis"
  - "cov_axis"
  - "cor_axis"
  - "spectrum"
  - "spectral_density"
  - "spectral_rad_pdf"
- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in *gstools.covmodel.plot*.

See also:

gstools.covmodel.plot

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (*float*) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (*float*) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for the variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type numpy.ndarray

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- integral_scale_vec[0] = integral_scale
- integral_scale_vec[1] = integral_scale*anis[0]
- integral_scale_vec[2] = integral_scale*anis[1]

Type numpy.ndarray

property is_isotropic
State if a model is isotropic.

Type bool

property iso_arg
Names of isotropic arguments.

Type list of str

property iso_arg_list
Values of isotropic arguments.

Type list of float

property latlon
Whether the model depends on geographical coords.

Type bool

property len_rescaled
The rescaled main length scale of the model.

Type float

property len_scale
The main length scale of the model.

Type float

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: [a, b] or [a, b, <type>] where <type> is one of "oo", "cc", "oc" or "co" to define if the bounds are open ("o") or closed ("c").

Type list

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type numpy.ndarray

property name
The name of the CovModel class.

Type str

property nugget
The nugget of the model.

Type float

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property opt_arg
Names of the optional arguments.

Type list of str

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property var_raw
The raw variance of the model without factor.

(See. CovModel.var_factor)

Type float

gstools.covmodel.Circular

class gstools.covmodel.**Circular**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: *gstools.covmodel.base.CovModel*

The circular covariance model.

This model is derived as the relative intersection area of two discs in 2D, where the middle points have a distance of r and the diameters are given by ℓ.

Notes

This model is given by the following correlation function [Webster2007]:

$$\rho(r) = \begin{cases} \frac{2}{\pi} \cdot \left(\cos^{-1}\left(s \cdot \frac{r}{\ell}\right) - s \cdot \frac{r}{\ell} \cdot \sqrt{1 - \left(s \cdot \frac{r}{\ell}\right)^2} \right) & r < \frac{\ell}{s} \\ 0 & r \geq \frac{\ell}{s} \end{cases}$$

Where the standard rescale factor is $s = 1$.

References

Parameters

- **dim** (int, optional) – dimension of the model. Default: 3
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: 1.0
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: 1.0
- **nugget** (float, optional) – nugget of the model. Default: 0.0
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: 1.0
- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: 0.0
- **integral_scale** (float or list or None, optional) – If given, len_scale will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: None
- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None

- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False
- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, `var` will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to `{"a": -1, "b": 1, "N": 1000, "h": 0.001}`. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds list: Bounds for the lenght scale.

len_scale_vec numpy.ndarray: The length scales in each direction.

name str: The name of the CovModel class.

nugget float: The nugget of the model.

nugget_bounds list: Bounds for the nugget.

opt_arg `list` of `str`: Names of the optional arguments.

opt_arg_bounds `dict`: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Circular model is only valid in 1D and 2D.
check_opt_arg()	Run checks for the optional arguments.
cor(h)	Circular normalized correlation function.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	Correlation function of the model.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Provide default optional arguments by the user.
default_opt_arg_bounds()	Provide default boundaries for optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.
fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.

continues on next page

Table 22 – continued from previous page

ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for the variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Circular model is only valid in 1D and 2D.

check_opt_arg()
Run checks for the optional arguments.

This is in addition to the bound-checks

Notes

- You can use this to raise a ValueError/warning
- Any return value will be ignored
- This method will only be run once, when the class is initialized

cor(*h*)
Circular normalized correlation function.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
Correlation function of the model.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Provide default optional arguments by the user.

Should be given as a dictionary when overridden.

default_opt_arg_bounds()
Provide default boundaries for optional arguments.

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

  If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None
- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as

given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

Default: "default"

- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None
- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'
- **loss** (str or callable, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'
- **max_eval** (int or None, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (bool, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (dict, optional) – Other keyword arguments passed to scipys curve_fit. Default: None
- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (dict) – Dictonary with the fitted parameter values
- **pcov** (numpy.ndarray) – The estimated covariance of *popt* from scipy.optimize.curve_fit. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.

- **r2_score** (float, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"
  - "vario_axis"
  - "cov_axis"
  - "cor_axis"
  - "spectrum"
  - "spectral_density"
  - "spectral_rad_pdf"
- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in *gstools.covmodel.plot*.

See also:

gstools.covmodel.plot

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (*float*) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (*float*) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for the variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type numpy.ndarray

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- integral_scale_vec[0] = integral_scale
- integral_scale_vec[1] = integral_scale*anis[0]
- integral_scale_vec[2] = integral_scale*anis[1]

Type numpy.ndarray

property is_isotropic
State if a model is isotropic.

Type bool

property iso_arg
Names of isotropic arguments.

Type list of str

property iso_arg_list
Values of isotropic arguments.

Type list of float

property latlon
Whether the model depends on geographical coords.

Type bool

property len_rescaled
The rescaled main length scale of the model.

Type float

property len_scale
The main length scale of the model.

Type float

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: [a, b] or [a, b, <type>] where <type> is one of "oo", "cc", "oc" or "co" to define if the bounds are open ("o") or closed ("c").

Type list

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type numpy.ndarray

property name
The name of the CovModel class.

Type str

property nugget
The nugget of the model.

Type float

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property opt_arg
Names of the optional arguments.

Type list of str

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property var_raw
The raw variance of the model without factor.

(See. CovModel.var_factor)

Type float

gstools.covmodel.Spherical

class gstools.covmodel.**Spherical**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: *gstools.covmodel.base.CovModel*

The Spherical covariance model.

This model is derived from the relative intersection area of two spheres in 3D, where the middle points have a distance of r and the diameters are given by ℓ.

Notes

This model is given by the following correlation function [Webster2007]:

$$\rho(r) = \begin{cases} 1 - \frac{3}{2} \cdot s \cdot \frac{r}{\ell} + \frac{1}{2} \cdot \left(s \cdot \frac{r}{\ell}\right)^3 & r < \frac{\ell}{s} \\ 0 & r \geq \frac{\ell}{s} \end{cases}$$

Where the standard rescale factor is $s = 1$.

References

Parameters

- **dim** (int, optional) – dimension of the model. Default: 3
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: 1.0
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: 1.0
- **nugget** (float, optional) – nugget of the model. Default: 0.0
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: 1.0
- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: 0.0
- **integral_scale** (float or list or None, optional) – If given, len_scale will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: None
- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None

- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False
- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, `var` will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to `{"a": -1, "b": 1, "N": 1000, "h": 0.001}`. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds list: Bounds for the lenght scale.

len_scale_vec numpy.ndarray: The length scales in each direction.

name str: The name of the CovModel class.

nugget float: The nugget of the model.

nugget_bounds list: Bounds for the nugget.

opt_arg `list` of `str`: Names of the optional arguments.

opt_arg_bounds `dict`: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Spherical model is only valid in 1D, 2D and 3D.
check_opt_arg()	Run checks for the optional arguments.
cor(h)	Spherical normalized correlation function.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	Correlation function of the model.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Provide default optional arguments by the user.
default_opt_arg_bounds()	Provide default boundaries for optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.
fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.

continues on next page

Table 23 – continued from previous page

ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for the variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Spherical model is only valid in 1D, 2D and 3D.

check_opt_arg()
Run checks for the optional arguments.

This is in addition to the bound-checks

Notes

- You can use this to raise a ValueError/warning
- Any return value will be ignored
- This method will only be run once, when the class is initialized

cor(*h*)
Spherical normalized correlation function.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
Correlation function of the model.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Provide default optional arguments by the user.

Should be given as a dictionary when overridden.

default_opt_arg_bounds()
Provide default boundaries for optional arguments.

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

  If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None
- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as

given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

Default: "default"

- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None
- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'
- **loss** (str or callable, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'
- **max_eval** (int or None, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (bool, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (dict, optional) – Other keyword arguments passed to scipys curve_fit. Default: None
- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (dict) – Dictonary with the fitted parameter values
- **pcov** (numpy.ndarray) – The estimated covariance of *popt* from scipy.optimize.curve_fit. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.

- **r2_score** (float, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"
  - "vario_axis"
  - "cov_axis"
  - "cor_axis"
  - "spectrum"
  - "spectral_density"
  - "spectral_rad_pdf"
- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in *gstools.covmodel.plot*.

See also:

gstools.covmodel.plot

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (*float*) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (*float*) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for the variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type numpy.ndarray

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- integral_scale_vec[0] = integral_scale
- integral_scale_vec[1] = integral_scale*anis[0]
- integral_scale_vec[2] = integral_scale*anis[1]

Type numpy.ndarray

property is_isotropic
State if a model is isotropic.

Type bool

property iso_arg
Names of isotropic arguments.

Type list of str

property iso_arg_list
Values of isotropic arguments.

Type list of float

property latlon
Whether the model depends on geographical coords.

Type bool

property len_rescaled
The rescaled main length scale of the model.

Type float

property len_scale
The main length scale of the model.

Type float

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: [a, b] or [a, b, <type>] where <type> is one of "oo", "cc", "oc" or "co" to define if the bounds are open ("o") or closed ("c").

Type list

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type numpy.ndarray

property name
The name of the CovModel class.

Type str

property nugget
The nugget of the model.

Type float

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property opt_arg
Names of the optional arguments.

Type list of str

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property var_raw
The raw variance of the model without factor.

(See. CovModel.var_factor)

Type float

gstools.covmodel.HyperSpherical

class gstools.covmodel.**HyperSpherical**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: *gstools.covmodel.base.CovModel*

The Hyper-Spherical covariance model.

This model is derived from the relative intersection area of two d-dimensional hyperspheres, where the middle points have a distance of r and the diameters are given by ℓ.

In 1D this is the Linear model, in 2D the Circular model and in 3D the Spherical model.

Notes

This model is given by the following correlation function [Matern1960]:

$$\rho(r) = \begin{cases} 1 - s \cdot \frac{r}{\ell} \cdot \frac{{}_2F_1\left(\frac{1}{2}, -\frac{d-1}{2}, \frac{3}{2}, \left(s \cdot \frac{r}{\ell}\right)^2\right)}{{}_2F_1\left(\frac{1}{2}, -\frac{d-1}{2}, \frac{3}{2}, 1\right)} & r < \frac{\ell}{s} \\ 0 & r \geq \frac{\ell}{s} \end{cases}$$

Where the standard rescale factor is $s = 1$. d is the dimension.

References

Parameters

- **dim** (int, optional) – dimension of the model. Default: 3
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: 1.0
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: 1.0
- **nugget** (float, optional) – nugget of the model. Default: 0.0
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: 1.0
- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: 0.0
- **integral_scale** (float or list or None, optional) – If given, len_scale will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: None
- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None

- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False
- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, `var` will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to `{"a": -1, "b": 1, "N": 1000, "h": 0.001}`. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds list: Bounds for the lenght scale.

len_scale_vec numpy.ndarray: The length scales in each direction.

name str: The name of the CovModel class.

nugget float: The nugget of the model.

nugget_bounds list: Bounds for the nugget.

opt_arg `list` of `str`: Names of the optional arguments.

opt_arg_bounds `dict`: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Check the given dimension.
check_opt_arg()	Run checks for the optional arguments.
cor(h)	Hyper-Spherical normalized correlation function.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	Correlation function of the model.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Provide default optional arguments by the user.
default_opt_arg_bounds()	Provide default boundaries for optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.
fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.

continues on next page

Table 24 – continued from previous page

ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for the variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Check the given dimension.

check_opt_arg()
Run checks for the optional arguments.

This is in addition to the bound-checks

Notes

- You can use this to raise a ValueError/warning
- Any return value will be ignored
- This method will only be run once, when the class is initialized

cor(*h*)
Hyper-Spherical normalized correlation function.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
Correlation function of the model.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Provide default optional arguments by the user.

Should be given as a dictionary when overridden.

default_opt_arg_bounds()
Provide default boundaries for optional arguments.

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

  If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None
- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as

given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

Default: "default"

- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None
- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'
- **loss** (str or callable, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'
- **max_eval** (int or None, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (bool, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (dict, optional) – Other keyword arguments passed to scipys curve_fit. Default: None
- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (dict) – Dictonary with the fitted parameter values
- **pcov** (numpy.ndarray) – The estimated covariance of *popt* from scipy.optimize.curve_fit. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.

- **r2_score** (float, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"
  - "vario_axis"
  - "cov_axis"
  - "cor_axis"
  - "spectrum"
  - "spectral_density"
  - "spectral_rad_pdf"
- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in *gstools.covmodel.plot*.

See also:

gstools.covmodel.plot

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (*float*) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (*float*) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for the variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type numpy.ndarray

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: [a, b] or [a, b, <type>] where <type> is one of "oo", "cc", "oc" or "co" to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: [a, b] or [a, b, <type>] where <type> is one of "oo", "cc", "oc" or "co" to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- integral_scale_vec[0] = integral_scale
- integral_scale_vec[1] = integral_scale*anis[0]
- integral_scale_vec[2] = integral_scale*anis[1]

Type numpy.ndarray

property is_isotropic
State if a model is isotropic.

Type bool

property iso_arg
Names of isotropic arguments.

Type list of str

property iso_arg_list
Values of isotropic arguments.

Type list of float

property latlon
Whether the model depends on geographical coords.

Type bool

property len_rescaled
The rescaled main length scale of the model.

Type float

property len_scale
The main length scale of the model.

Type float

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: [a, b] or [a, b, <type>] where <type> is one of "oo", "cc", "oc" or "co" to define if the bounds are open ("o") or closed ("c").

Type list

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type `numpy.ndarray`

property name
The name of the CovModel class.

Type `str`

property nugget
The nugget of the model.

Type `float`

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `list`

property opt_arg
Names of the optional arguments.

Type `list` of `str`

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `dict`

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property var_raw
The raw variance of the model without factor.

(See. CovModel.var_factor)

Type float

gstools.covmodel.SuperSpherical

class gstools.covmodel.**SuperSpherical**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: *gstools.covmodel.base.CovModel*

The Super-Spherical covariance model.

This model is derived from the relative intersection area of two d-dimensional hyperspheres, where the middle points have a distance of r and the diameters are given by ℓ. It is than valid in all lower dimensions. By default it coincides with the Hyper-Spherical model.

Notes

This model is given by the following correlation function [Matern1960]:

$$\rho(r) = \begin{cases} 1 - s \cdot \frac{r}{\ell} \cdot \frac{{}_2F_1\left(\frac{1}{2}, -\nu, \frac{3}{2}, \left(s \cdot \frac{r}{\ell}\right)^2\right)}{{}_2F_1\left(\frac{1}{2}, -\nu, \frac{3}{2}, 1\right)} & r < \frac{\ell}{s} \\ 0 & r \geq \frac{\ell}{s} \end{cases}$$

Where the standard rescale factor is $s = 1$. $\nu \geq \frac{d-1}{2}$ is a shape parameter.

References

Parameters

- **nu** (float, optional) – Shape parameter. Standard range: `[(dim-1)/2, 50]` Default: `(dim-1)/2`
- **dim** (int, optional) – dimension of the model. Default: `3`
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: `1.0`
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: `1.0`
- **nugget** (float, optional) – nugget of the model. Default: `0.0`
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: `1.0`
- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: `0.0`
- **integral_scale** (float or list or None, optional) – If given, `len_scale` will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: `None`

- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None
- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False
- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, var will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to {"a": -1, "b": 1, "N": 1000, "h": 0.001}. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds list: Bounds for the lenght scale.

len_scale_vec numpy.ndarray: The length scales in each direction.

name str: The name of the CovModel class.

nugget `float`: The nugget of the model.

nugget_bounds `list`: Bounds for the nugget.

opt_arg `list` of `str`: Names of the optional arguments.

opt_arg_bounds `dict`: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Check the given dimension.
check_opt_arg()	Run checks for the optional arguments.
cor(h)	Super-Spherical normalized correlation function.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	Correlation function of the model.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Defaults for the optional arguments.
default_opt_arg_bounds()	Defaults for boundaries of the optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.

continues on next page

Table 25 – continued from previous page

fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.
ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for the variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Check the given dimension.

check_opt_arg()
Run checks for the optional arguments.

This is in addition to the bound-checks

Notes

- You can use this to raise a ValueError/warning
- Any return value will be ignored
- This method will only be run once, when the class is initialized

cor(*h*)
Super-Spherical normalized correlation function.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
Correlation function of the model.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Defaults for the optional arguments.

- `{"nu": (dim-1)/2}`

Returns Defaults for optional arguments

Return type dict

default_opt_arg_bounds()
Defaults for boundaries of the optional arguments.

- `{"nu": [(dim-1)/2, 50.0]}`

Returns Boundaries for optional arguments

Return type dict

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None

- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

  Default: "default"
- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None
- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'
- **loss** (str or callable, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'
- **max_eval** (int or None, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (bool, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (dict, optional) – Other keyword arguments passed to scipys curve_fit. Default: None

- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (dict) – Dictonary with the fitted parameter values
- **pcov** (numpy.ndarray) – The estimated covariance of *popt* from scipy.optimize.curve_fit. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.
- **r2_score** (float, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

---

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"
  - "vario_axis"
  - "cov_axis"
  - "cor_axis"
  - "spectrum"

- "spectral_density"
- "spectral_rad_pdf"

- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in `gstools.covmodel.plot`.

See also:

`gstools.covmodel.plot`

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for the variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type numpy.ndarray

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- integral_scale_vec[0] = integral_scale
- integral_scale_vec[1] = integral_scale*anis[0]
- integral_scale_vec[2] = integral_scale*anis[1]

Type numpy.ndarray

property is_isotropic
State if a model is isotropic.

Type bool

property iso_arg
Names of isotropic arguments.

Type list of str

property iso_arg_list
Values of isotropic arguments.

Type list of float

property latlon
Whether the model depends on geographical coords.

Type bool

property len_rescaled
The rescaled main length scale of the model.

Type float

property len_scale
The main length scale of the model.

Type float

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `list`

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type `numpy.ndarray`

property name
The name of the CovModel class.

Type `str`

property nugget
The nugget of the model.

Type `float`

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `list`

property opt_arg
Names of the optional arguments.

Type `list` of `str`

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `dict`

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property var_raw
The raw variance of the model without factor.

(See. CovModel.var_factor)

Type float

gstools.covmodel.JBessel

class gstools.covmodel.**JBessel**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: *gstools.covmodel.base.CovModel*

The J-Bessel hole model.

This covariance model is a valid hole model, meaning it has areas of negative correlation but a valid spectral density.

Notes

This model is given by the following correlation function [Chiles2009]:

$$\rho(r) = \Gamma(\nu+1) \cdot \frac{\mathrm{J}_\nu\left(s \cdot \frac{r}{\ell}\right)}{\left(s \cdot \frac{r}{2\ell}\right)^\nu}$$

Where the standard rescale factor is $s = 1$. Γ is the gamma function and J_ν is the Bessel functions of the first kind. $\nu \geq \frac{d}{2} - 1$ is a shape parameter, which defaults to $\nu = \frac{d}{2}$, since the spectrum of the model gets instable for $\nu \to \frac{d}{2} - 1$.

For $\nu = \frac{1}{2}$ (valid in d=1,2,3) we get the so-called 'Wave' model:

$$\rho(r) = \frac{\sin\left(s \cdot \frac{r}{\ell}\right)}{s \cdot \frac{r}{\ell}}$$

References

Parameters

- **nu** (float, optional) – Shape parameter. Standard range: `[dim/2 - 1, 50]` Default: `dim/2`
- **dim** (int, optional) – dimension of the model. Default: `3`
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: `1.0`
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: `1.0`
- **nugget** (float, optional) – nugget of the model. Default: `0.0`
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: `1.0`
- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: `0.0`

- **integral_scale** (float or list or None, optional) – If given, len_scale will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: None
- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None
- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False
- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, var will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to {"a": -1, "b": 1, "N": 1000, "h": 0.001}. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds `list`: Bounds for the lenght scale.

len_scale_vec `numpy.ndarray`: The length scales in each direction.

name `str`: The name of the CovModel class.

nugget `float`: The nugget of the model.

nugget_bounds `list`: Bounds for the nugget.

opt_arg `list` of `str`: Names of the optional arguments.

opt_arg_bounds `dict`: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Check the given dimension.
check_opt_arg()	Check the optional arguments.
cor(h)	J-Bessel correlation.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	Correlation function of the model.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Defaults for the optional arguments.

continues on next page

Table 26 – continued from previous page

default_opt_arg_bounds()	Defaults for boundaries of the optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.
fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.
ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for the variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Check the given dimension.

check_opt_arg()
Check the optional arguments.

Warns **nu** – If nu is close to dim/2 - 1, the model tends to get unstable.

cor(*h*)
J-Bessel correlation.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
Correlation function of the model.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Defaults for the optional arguments.

- `{"nu": dim/2}`

Returns Defaults for optional arguments

Return type dict

default_opt_arg_bounds()
Defaults for boundaries of the optional arguments.

- `{"nu": [dim/2 - 1, 50.0]}`

Returns Boundaries for optional arguments

Return type dict

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

  If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None

- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - ”default”: using the default values of the covariance model (“len_scale” will be mean of given bin centers; “var” and “nugget” will be mean of given variogram values (if in given bounds))
  - ”current”: using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate “default” can bet set to “default” or “current” for unspecified values to get same behavior as given above (“default” by default)) Example: `{"len_scale": 10, "default": "current"}`

  Default: “default”
- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - ’inv’: inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None
- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - ’trf’ : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - ’dogbox’ : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: ‘trf’
- **loss** (str or callable, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - ’linear’ (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - ’soft_l1’ : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - ’huber’ : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to ‘soft_l1’.
  - ’cauchy’ : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - ’arctan’ : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to ‘cauchy’.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: ‘soft_l1’
- **max_eval** (int or None, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (bool, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (dict, optional) – Other keyword arguments passed to scipys curve_fit. Default: None

- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (dict) – Dictonary with the fitted parameter values
- **pcov** (numpy.ndarray) – The estimated covariance of *popt* from scipy.optimize.curve_fit. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.
- **r2_score** (float, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

---

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"
  - "vario_axis"
  - "cov_axis"
  - "cor_axis"
  - "spectrum"

- "spectral_density"
- "spectral_rad_pdf"

- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in `gstools.covmodel.plot`.

See also:

`gstools.covmodel.plot`

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for the variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type numpy.ndarray

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- integral_scale_vec[0] = integral_scale
- integral_scale_vec[1] = integral_scale*anis[0]
- integral_scale_vec[2] = integral_scale*anis[1]

Type numpy.ndarray

property is_isotropic
State if a model is isotropic.

Type bool

property iso_arg
Names of isotropic arguments.

Type list of str

property iso_arg_list
Values of isotropic arguments.

Type list of float

property latlon
Whether the model depends on geographical coords.

Type bool

property len_rescaled
The rescaled main length scale of the model.

Type float

property len_scale
The main length scale of the model.

Type float

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type numpy.ndarray

property name
The name of the CovModel class.

Type str

property nugget
The nugget of the model.

Type float

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property opt_arg
Names of the optional arguments.

Type list of str

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property var_raw
The raw variance of the model without factor.

(See. CovModel.var_factor)

Type float

Truncated Power Law Covariance Models

TPLGaussian([dim, var, len_scale, nugget, ...])	Truncated-Power-Law with Gaussian modes.
TPLExponential([dim, var, len_scale, ...])	Truncated-Power-Law with Exponential modes.
TPLStable([dim, var, len_scale, nugget, ...])	Truncated-Power-Law with Stable modes.
TPLSimple([dim, var, len_scale, nugget, ...])	The simply truncated power law model.

gstools.covmodel.TPLGaussian

class gstools.covmodel.**TPLGaussian**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: gstools.covmodel.tpl_models.TPLCovModel

Truncated-Power-Law with Gaussian modes.

Notes

The truncated power law is given by a superposition of scale-dependent variograms [Federico1997]:

$$\gamma_{\ell_{\mathrm{low}},\ell_{\mathrm{up}}}(r) = \int_{\ell_{\mathrm{low}}}^{\ell_{\mathrm{up}}} \gamma(r,\lambda)\frac{\mathrm{d}\lambda}{\lambda}$$

with *Gaussian* modes on each scale:

$$\gamma(r,\lambda) = \sigma^2(\lambda)\cdot\left(1-\exp\left[-\left(\frac{r}{\lambda}\right)^2\right]\right)$$
$$\sigma^2(\lambda) = C\cdot\lambda^{2H}$$

This results in:

$$\gamma_{\ell_{\mathrm{low}},\ell_{\mathrm{up}}}(r) = \sigma^2_{\ell_{\mathrm{low}},\ell_{\mathrm{up}}}\cdot\left(1 - H\cdot\frac{\ell_{\mathrm{up}}^{2H}\cdot E_{1+H}\left[\left(\frac{r}{\ell_{\mathrm{up}}}\right)^2\right] - \ell_{\mathrm{low}}^{2H}\cdot E_{1+H}\left[\left(\frac{r}{\ell_{\mathrm{low}}}\right)^2\right]}{\ell_{\mathrm{up}}^{2H}-\ell_{\mathrm{low}}^{2H}}\right)$$
$$\sigma^2_{\ell_{\mathrm{low}},\ell_{\mathrm{up}}} = \frac{C\cdot\left(\ell_{\mathrm{up}}^{2H}-\ell_{\mathrm{low}}^{2H}\right)}{2H}$$

The "length scale" of this model is equivalent by the integration range:

$$\ell = \ell_{\mathrm{up}} - \ell_{\mathrm{low}}$$

If you want to define an upper scale truncation, you should set `len_low` and `len_scale` accordingly.

The following Parameters occur:

- $C > 0$: scaling factor from the Power-Law (intensity of variation) This parameter will be calculated internally by the given variance. You can access C directly by `model.var_raw`
- $0 < H < 1$: hurst coefficient (`model.hurst`)
- $\ell_{\mathrm{low}} \geq 0$: lower length scale truncation of the model (`model.len_low`)
- $\ell_{\mathrm{up}} \geq 0$: upper length scale truncation of the model (`model.len_up`)

  This will be calculated internally by:

  - `len_up = len_low + len_scale`

  That means, that the `len_scale` in this model actually represents the integration range for the truncated power law.
- $E_s(x)$ is the exponential integral.

References

Parameters

- **hurst** (float, optional) – Hurst coefficient of the power law. Standard range: `(0, 1)`. Default: `0.5`
- **len_low** (float, optional) – The lower length scale truncation of the model. Standard range: `[0, inf]`. Default: `0.0`
- **dim** (int, optional) – dimension of the model. Default: `3`
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: `1.0`
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: `1.0`
- **nugget** (float, optional) – nugget of the model. Default: `0.0`
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: `1.0`
- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: `0.0`
- **integral_scale** (float or list or None, optional) – If given, `len_scale` will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: None
- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None
- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False
- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, `var` will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to `{"a": -1, "b": 1, "N": 1000, "h": 0.001}`. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_low_rescaled float: Lower length scale truncation rescaled.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds list: Bounds for the lenght scale.

len_scale_vec numpy.ndarray: The length scales in each direction.

len_up float: Upper length scale truncation of the model.

len_up_rescaled float: Upper length scale truncation rescaled.

name str: The name of the CovModel class.

nugget float: The nugget of the model.

nugget_bounds list: Bounds for the nugget.

opt_arg list of str: Names of the optional arguments.

opt_arg_bounds dict: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Check the given dimension.
check_opt_arg()	Run checks for the optional arguments.
cor(h)	TPL with Gaussian modes - normalized correlation function.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	TPL with Gaussian modes - correlation function.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Defaults for the optional arguments.
default_opt_arg_bounds()	Defaults for boundaries of the optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.
fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.
ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for C (intensity of variation) to result in variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.

continues on next page

Table 28 – continued from previous page

vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Check the given dimension.

check_opt_arg()
Run checks for the optional arguments.

This is in addition to the bound-checks

Notes

- You can use this to raise a ValueError/warning
- Any return value will be ignored
- This method will only be run once, when the class is initialized

cor(*h*)
TPL with Gaussian modes - normalized correlation function.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
TPL with Gaussian modes - correlation function.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Defaults for the optional arguments.

- `{"hurst": 0.5, "len_low": 0.0}`

Returns Defaults for optional arguments

Return type dict

default_opt_arg_bounds()
Defaults for boundaries of the optional arguments.

- `{"hurst": [0, 1, "oo"], "len_low": [0, inf, "cc"]}`

Returns Boundaries for optional arguments

Return type dict

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

  If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None
- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as

given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

Default: "default"

- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None
- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'
- **loss** (str or callable, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'
- **max_eval** (int or None, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (bool, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (dict, optional) – Other keyword arguments passed to scipys curve_fit. Default: None
- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (dict) – Dictonary with the fitted parameter values
- **pcov** (numpy.ndarray) – The estimated covariance of *popt* from scipy.optimize.curve_fit. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.

- **r2_score** (float, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"
  - "vario_axis"
  - "cov_axis"
  - "cor_axis"
  - "spectrum"
  - "spectral_density"
  - "spectral_rad_pdf"
- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in *gstools.covmodel.plot*.

See also:

gstools.covmodel.plot

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for C (intensity of variation) to result in variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type `numpy.ndarray`

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- integral_scale_vec[0] = integral_scale
- integral_scale_vec[1] = integral_scale*anis[0]
- integral_scale_vec[2] = integral_scale*anis[1]

Type numpy.ndarray

property is_isotropic
State if a model is isotropic.

Type bool

property iso_arg
Names of isotropic arguments.

Type list of str

property iso_arg_list
Values of isotropic arguments.

Type list of float

property latlon
Whether the model depends on geographical coords.

Type bool

property len_low_rescaled
Lower length scale truncation rescaled.

- len_low_rescaled = len_low / rescale

Type float

property len_rescaled
The rescaled main length scale of the model.

Type float

property len_scale
The main length scale of the model.

Type float

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `list`

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type `numpy.ndarray`

property len_up
Upper length scale truncation of the model.

- `len_up = len_low + len_scale`

Type `float`

property len_up_rescaled
Upper length scale truncation rescaled.

- `len_up_rescaled = (len_low + len_scale) / rescale`

Type `float`

property name
The name of the CovModel class.

Type `str`

property nugget
The nugget of the model.

Type `float`

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `list`

property opt_arg
Names of the optional arguments.

Type `list` of `str`

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property var_raw

The raw variance of the model without factor.

(See. CovModel.var_factor)

Type float

gstools.covmodel.TPLExponential

class gstools.covmodel.**TPLExponential**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: gstools.covmodel.tpl_models.TPLCovModel

Truncated-Power-Law with Exponential modes.

Notes

The truncated power law is given by a superposition of scale-dependent variograms [Federico1997]:

$$\gamma_{\ell_{\mathrm{low}},\ell_{\mathrm{up}}}(r) = \int_{\ell_{\mathrm{low}}}^{\ell_{\mathrm{up}}} \gamma(r,\lambda)\frac{\mathrm{d}\lambda}{\lambda}$$

with *Exponential* modes on each scale:

$$\gamma(r,\lambda) = \sigma^2(\lambda)\cdot\left(1-\exp\left[-\frac{r}{\lambda}\right]\right)$$
$$\sigma^2(\lambda) = C\cdot\lambda^{2H}$$

This results in:

$$\gamma_{\ell_{\mathrm{low}},\ell_{\mathrm{up}}}(r) = \sigma^2_{\ell_{\mathrm{low}},\ell_{\mathrm{up}}}\cdot\left(1-2H\cdot\frac{\ell_{\mathrm{up}}^{2H}\cdot E_{1+2H}\left[\frac{r}{\ell_{\mathrm{up}}}\right]-\ell_{\mathrm{low}}^{2H}\cdot E_{1+2H}\left[\frac{r}{\ell_{\mathrm{low}}}\right]}{\ell_{\mathrm{up}}^{2H}-\ell_{\mathrm{low}}^{2H}}\right)$$
$$\sigma^2_{\ell_{\mathrm{low}},\ell_{\mathrm{up}}} = \frac{C\cdot\left(\ell_{\mathrm{up}}^{2H}-\ell_{\mathrm{low}}^{2H}\right)}{2H}$$

The "length scale" of this model is equivalent by the integration range:

$$\ell = \ell_{\mathrm{up}} - \ell_{\mathrm{low}}$$

If you want to define an upper scale truncation, you should set `len_low` and `len_scale` accordingly.

The following Parameters occur:

- $C > 0$: scaling factor from the Power-Law (intensity of variation) This parameter will be calculated internally by the given variance. You can access C directly by `model.var_raw`
- $0 < H < \frac{1}{2}$: hurst coefficient (`model.hurst`)
- $\ell_{\mathrm{low}} \geq 0$: lower length scale truncation of the model (`model.len_low`)
- $\ell_{\mathrm{up}} \geq 0$: upper length scale truncation of the model (`model.len_up`)

  This will be calculated internally by:

  - `len_up = len_low + len_scale`

  That means, that the `len_scale` in this model actually represents the integration range for the truncated power law.
- $E_s(x)$ is the exponential integral.

References

Parameters

- **hurst** (float, optional) – Hurst coefficient of the power law. Standard range: `(0, 1)`. Default: `0.5`
- **len_low** (float, optional) – The lower length scale truncation of the model. Standard range: `[0, inf]`. Default: `0.0`
- **dim** (int, optional) – dimension of the model. Default: `3`
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: `1.0`
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: `1.0`
- **nugget** (float, optional) – nugget of the model. Default: `0.0`
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: `1.0`
- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: `0.0`
- **integral_scale** (float or list or None, optional) – If given, `len_scale` will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: None
- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None
- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False
- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, `var` will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to `{"a": -1, "b": 1, "N": 1000, "h": 0.001}`. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_low_rescaled float: Lower length scale truncation rescaled.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds list: Bounds for the lenght scale.

len_scale_vec numpy.ndarray: The length scales in each direction.

len_up float: Upper length scale truncation of the model.

len_up_rescaled float: Upper length scale truncation rescaled.

name str: The name of the CovModel class.

nugget float: The nugget of the model.

nugget_bounds list: Bounds for the nugget.

opt_arg list of str: Names of the optional arguments.

opt_arg_bounds dict: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Check the given dimension.
check_opt_arg()	Run checks for the optional arguments.
cor(h)	TPL with Exponential modes - normalized correlation function.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	TPL with Exponential modes - correlation function.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Defaults for the optional arguments.
default_opt_arg_bounds()	Defaults for boundaries of the optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.
fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.
ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for C (intensity of variation) to result in variance.

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Table 29 – continued from previous page

vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Check the given dimension.

check_opt_arg()
Run checks for the optional arguments.

This is in addition to the bound-checks

Notes

- You can use this to raise a ValueError/warning
- Any return value will be ignored
- This method will only be run once, when the class is initialized

cor(*h*)
TPL with Exponential modes - normalized correlation function.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
TPL with Exponential modes - correlation function.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Defaults for the optional arguments.

- `{"hurst": 0.25, "len_low": 0.0}`

Returns Defaults for optional arguments

Return type dict

default_opt_arg_bounds()
Defaults for boundaries of the optional arguments.

- `{"hurst": [0, 1, "oo"], "len_low": [0, inf, "cc"]}`

Returns Boundaries for optional arguments

Return type dict

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

  If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None
- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as

given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

Default: "default"

- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None
- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'
- **loss** (str or callable, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'
- **max_eval** (int or None, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (bool, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (dict, optional) – Other keyword arguments passed to scipys curve_fit. Default: None
- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (dict) – Dictonary with the fitted parameter values
- **pcov** (numpy.ndarray) – The estimated covariance of *popt* from scipy.optimize.curve_fit. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.

- **r2_score** (float, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"
  - "vario_axis"
  - "cov_axis"
  - "cor_axis"
  - "spectrum"
  - "spectral_density"
  - "spectral_rad_pdf"
- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in *gstools.covmodel.plot*.

See also:

gstools.covmodel.plot

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for C (intensity of variation) to result in variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type `numpy.ndarray`

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- integral_scale_vec[0] = integral_scale
- integral_scale_vec[1] = integral_scale*anis[0]
- integral_scale_vec[2] = integral_scale*anis[1]

Type numpy.ndarray

property is_isotropic
State if a model is isotropic.

Type bool

property iso_arg
Names of isotropic arguments.

Type list of str

property iso_arg_list
Values of isotropic arguments.

Type list of float

property latlon
Whether the model depends on geographical coords.

Type bool

property len_low_rescaled
Lower length scale truncation rescaled.

- len_low_rescaled = len_low / rescale

Type float

property len_rescaled
The rescaled main length scale of the model.

Type float

property len_scale
The main length scale of the model.

Type float

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `list`

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type `numpy.ndarray`

property len_up
Upper length scale truncation of the model.

- `len_up = len_low + len_scale`

Type `float`

property len_up_rescaled
Upper length scale truncation rescaled.

- `len_up_rescaled = (len_low + len_scale) / rescale`

Type `float`

property name
The name of the CovModel class.

Type `str`

property nugget
The nugget of the model.

Type `float`

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `list`

property opt_arg
Names of the optional arguments.

Type `list` of `str`

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property var_raw

The raw variance of the model without factor.

(See. CovModel.var_factor)

Type float

gstools.covmodel.TPLStable

class gstools.covmodel.**TPLStable**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: gstools.covmodel.tpl_models.TPLCovModel

Truncated-Power-Law with Stable modes.

Notes

The truncated power law is given by a superposition of scale-dependent variograms:

$$\gamma_{\ell_{\mathrm{low}},\ell_{\mathrm{up}}}(r) = \int_{\ell_{\mathrm{low}}}^{\ell_{\mathrm{up}}} \gamma(r,\lambda)\frac{\mathrm{d}\lambda}{\lambda}$$

with *Stable* modes on each scale:

$$\gamma(r,\lambda) = \sigma^2(\lambda)\cdot\left(1-\exp\left[-\left(\frac{r}{\lambda}\right)^{\alpha}\right]\right)$$
$$\sigma^2(\lambda) = C\cdot\lambda^{2H}$$

This results in:

$$\gamma_{\ell_{\mathrm{low}},\ell_{\mathrm{up}}}(r) = \sigma^2_{\ell_{\mathrm{low}},\ell_{\mathrm{up}}}\cdot\left(1-\frac{2H}{\alpha}\cdot\frac{\ell_{\mathrm{up}}^{2H}\cdot E_{1+\frac{2H}{\alpha}}\left[\left(\frac{r}{\ell_{\mathrm{up}}}\right)^{\alpha}\right]-\ell_{\mathrm{low}}^{2H}\cdot E_{1+\frac{2H}{\alpha}}\left[\left(\frac{r}{\ell_{\mathrm{low}}}\right)^{\alpha}\right]}{\ell_{\mathrm{up}}^{2H}-\ell_{\mathrm{low}}^{2H}}\right)$$
$$\sigma^2_{\ell_{\mathrm{low}},\ell_{\mathrm{up}}} = \frac{C\cdot\left(\ell_{\mathrm{up}}^{2H}-\ell_{\mathrm{low}}^{2H}\right)}{2H}$$

The "length scale" of this model is equivalent by the integration range:

$$\ell = \ell_{\mathrm{up}} - \ell_{\mathrm{low}}$$

If you want to define an upper scale truncation, you should set `len_low` and `len_scale` accordingly.

The following Parameters occur:

- $0 < \alpha \leq 2$: The shape parameter of the Stable model.
  - $\alpha = 1$: Exponential modes
  - $\alpha = 2$: Gaussian modes
- $C > 0$: scaling factor from the Power-Law (intensity of variation) This parameter will be calculated internally by the given variance. You can access C directly by `model.var_raw`
- $0 < H < \frac{\alpha}{2}$: hurst coefficient (`model.hurst`)
- $\ell_{\mathrm{low}} \geq 0$: lower length scale truncation of the model (`model.len_low`)
- $\ell_{\mathrm{up}} \geq 0$: upper length scale truncation of the model (`model.len_up`)

  This will be calculated internally by:

  - `len_up = len_low + len_scale`

  That means, that the `len_scale` in this model actually represents the integration range for the truncated power law.
- $E_s(x)$ is the exponential integral.

Parameters

- **hurst** (float, optional) – Hurst coefficient of the power law. Standard range: `(0, 1)`. Default: `0.5`
- **alpha** (float, optional) – Shape parameter of the stable model. Standard range: `(0, 2]`. Default: `1.5`
- **len_low** (float, optional) – The lower length scale truncation of the model. Standard range: `[0, inf]`. Default: `0.0`
- **dim** (int, optional) – dimension of the model. Default: `3`
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: `1.0`
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: `1.0`
- **nugget** (float, optional) – nugget of the model. Default: `0.0`
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: `1.0`
- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: `0.0`
- **integral_scale** (float or list or None, optional) – If given, `len_scale` will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: None
- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None
- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False
- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, `var` will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to `{"a": -1, "b": 1, "N": 1000, "h": 0.001}`. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_low_rescaled float: Lower length scale truncation rescaled.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds list: Bounds for the lenght scale.

len_scale_vec numpy.ndarray: The length scales in each direction.

len_up float: Upper length scale truncation of the model.

len_up_rescaled float: Upper length scale truncation rescaled.

name str: The name of the CovModel class.

nugget float: The nugget of the model.

nugget_bounds list: Bounds for the nugget.

opt_arg list of str: Names of the optional arguments.

opt_arg_bounds dict: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Check the given dimension.
check_opt_arg()	Check the optional arguments.
cor(h)	TPL with Stable modes - normalized correlation function.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	TPL with Stable modes - correlation function.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Defaults for the optional arguments.
default_opt_arg_bounds()	Defaults for boundaries of the optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.
fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.
ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for C (intensity of variation) to result in variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.

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Table 30 – continued from previous page

vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Check the given dimension.

check_opt_arg()
Check the optional arguments.

Warns **alpha** – If alpha is < 0.3, the model tends to a nugget model and gets numerically unstable.

cor(*h*)
TPL with Stable modes - normalized correlation function.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
TPL with Stable modes - correlation function.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Defaults for the optional arguments.

- `{"hurst": 0.5, "alpha": 1.5, "len_low": 0.0}`

Returns Defaults for optional arguments

Return type `dict`

default_opt_arg_bounds()
Defaults for boundaries of the optional arguments.

- `{"hurst": [0, 1, "oo"], "alpha": [0, 2, "oc"], "len_low": [0, inf, "cc"]}`

Returns Boundaries for optional arguments

Return type dict

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:
  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

  If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None
- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

  Default: "default"
- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None

- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'
- **loss** (`str` or `callable`, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'
- **max_eval** (`int` or `None`, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.
- **return_r2** (`bool`, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (`dict`, optional) – Other keyword arguments passed to scipys curve_fit. Default: None
- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (`dict`) – Dictonary with the fitted parameter values
- **pcov** (`numpy.ndarray`) – The estimated covariance of *popt* from `scipy.optimize.curve_fit`. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.
- **r2_score** (`float`, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *`CovModel.set_arg_bounds`*.

The fitted parameters will be instantly set in the model.

---

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"
  - "vario_axis"
  - "cov_axis"
  - "cor_axis"
  - "spectrum"
  - "spectral_density"
  - "spectral_rad_pdf"
- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in *gstools.covmodel.plot*.

See also:

gstools.covmodel.plot

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: [a, b] or [a, b, <type>] where <type> is one of "oo", "cc", "oc" or "co" to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (float) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (float) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for C (intensity of variation) to result in variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type numpy.ndarray

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: [a, b] or [a, b, <type>] where <type> is one of "oo", "cc", "oc" or "co" to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- `integral_scale_vec[0] = integral_scale`
- `integral_scale_vec[1] = integral_scale*anis[0]`
- `integral_scale_vec[2] = integral_scale*anis[1]`

Type `numpy.ndarray`

property is_isotropic
State if a model is isotropic.

Type `bool`

property iso_arg
Names of isotropic arguments.

Type `list` of `str`

property iso_arg_list
Values of isotropic arguments.

Type `list` of `float`

property latlon
Whether the model depends on geographical coords.

Type `bool`

property len_low_rescaled
Lower length scale truncation rescaled.

- `len_low_rescaled = len_low / rescale`

Type `float`

property len_rescaled
The rescaled main length scale of the model.

Type `float`

property len_scale
The main length scale of the model.

Type `float`

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type `list`

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type numpy.ndarray

property len_up
Upper length scale truncation of the model.

- `len_up = len_low + len_scale`

Type float

property len_up_rescaled
Upper length scale truncation rescaled.

- `len_up_rescaled = (len_low + len_scale) / rescale`

Type float

property name
The name of the CovModel class.

Type str

property nugget
The nugget of the model.

Type float

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property opt_arg
Names of the optional arguments.

Type list of str

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property var_raw
The raw variance of the model without factor.

(See. CovModel.var_factor)

Type float

gstools.covmodel.TPLSimple

class gstools.covmodel.**TPLSimple**(*dim=3*, *var=1.0*, *len_scale=1.0*, *nugget=0.0*, *anis=1.0*, *angles=0.0*, *integral_scale=None*, *rescale=None*, *latlon=False*, *var_raw=None*, *hankel_kw=None*, ***opt_arg*)

Bases: *gstools.covmodel.base.CovModel*

The simply truncated power law model.

This model describes a simple truncated power law with a finite length scale. In contrast to other models, this one is not derived from super-positioning modes.

Notes

This model is given by the following correlation function [Wendland1995]:

$$\rho(r) = \begin{cases} \left(1 - s \cdot \frac{r}{\ell}\right)^{\nu} & r < \frac{\ell}{s} \\ 0 & r \geq \frac{\ell}{s} \end{cases}$$

Where the standard rescale factor is $s = 1$. $\nu \geq \frac{d+1}{2}$ is a shape parameter, which defaults to $\nu = \frac{d+1}{2}$,

For $\nu = 1$ (valid only in d=1) this coincides with the truncated linear model:

$$\rho(r) = \begin{cases} 1 - s \cdot \frac{r}{\ell} & r < \frac{\ell}{s} \\ 0 & r \geq \frac{\ell}{s} \end{cases}$$

References

Parameters

- **nu** (float, optional) – Shape parameter. Standard range: `[(dim+1)/2, 50]` Default: `dim/2`
- **dim** (int, optional) – dimension of the model. Default: `3`
- **var** (float, optional) – variance of the model (the nugget is not included in "this" variance) Default: `1.0`
- **len_scale** (float or list, optional) – length scale of the model. If a single value is given, the same length-scale will be used for every direction. If multiple values (for main and transversal directions) are given, *anis* will be recalculated accordingly. If only two values are given in 3D, the latter one will be used for both transversal directions. Default: `1.0`
- **nugget** (float, optional) – nugget of the model. Default: `0.0`
- **anis** (float or list, optional) – anisotropy ratios in the transversal directions [e_y, e_z].
  - e_y = l_y / l_x
  - e_z = l_z / l_x

  If only one value is given in 3D, e_y will be set to 1. This value will be ignored, if multiple len_scales are given. Default: `1.0`
- **angles** (float or list, optional) – angles of rotation (given in rad):
  - in 2D: given as rotation around z-axis
  - in 3D: given by yaw, pitch, and roll (known as Tait–Bryan angles)

  Default: `0.0`

- **integral_scale** (float or list or None, optional) – If given, len_scale will be ignored and recalculated, so that the integral scale of the model matches the given one. Default: None
- **rescale** (float or None, optional) – Optional rescaling factor to divide the length scale with. This could be used for unit convertion or rescaling the length scale to coincide with e.g. the integral scale. Will be set by each model individually. Default: None
- **latlon** (bool, optional) – Whether the model is describing 2D fields on earths surface described by latitude and longitude. When using this, the model will internally use the associated 'Yadrenko' model to represent a valid model. This means, the spatial distance r will be replaced by $2\sin(\alpha/2)$, where α is the great-circle distance, which is equal to the spatial distance of two points in 3D. As a consequence, *dim* will be set to *3* and anisotropy will be disabled. *rescale* can be set to e.g. earth's radius, to have a meaningful *len_scale* parameter. Default: False
- **var_raw** (float or None, optional) – raw variance of the model which will be multiplied with *CovModel.var_factor* to result in the actual variance. If given, var will be ignored. (This is just for models that override *CovModel.var_factor*) Default: None
- **hankel_kw** (dict or None, optional) – Modify the init-arguments of hankel.SymmetricFourierTransform used for the spectrum calculation. Use with caution (Better: Don't!). None is equivalent to {"a": -1, "b": 1, "N": 1000, "h": 0.001}. Default: None
- ****opt_arg** – Optional arguments are covered by these keyword arguments. If present, they are described in the section *Other Parameters*.

Attributes

angles numpy.ndarray: Rotation angles (in rad) of the model.

anis numpy.ndarray: The anisotropy factors of the model.

anis_bounds list: Bounds for the nugget.

arg list of str: Names of all arguments.

arg_bounds dict: Bounds for all parameters.

arg_list list of float: Values of all arguments.

dim int: The dimension of the model.

dist_func tuple of callable: pdf, cdf and ppf.

do_rotation bool: State if a rotation is performed.

field_dim int: The field dimension of the model.

hankel_kw dict: hankel.SymmetricFourierTransform kwargs.

has_cdf bool: State if a cdf is defined by the user.

has_ppf bool: State if a ppf is defined by the user.

integral_scale float: The main integral scale of the model.

integral_scale_vec numpy.ndarray: The integral scales in each direction.

is_isotropic bool: State if a model is isotropic.

iso_arg list of str: Names of isotropic arguments.

iso_arg_list list of float: Values of isotropic arguments.

latlon bool: Whether the model depends on geographical coords.

len_rescaled float: The rescaled main length scale of the model.

len_scale float: The main length scale of the model.

len_scale_bounds `list`: Bounds for the lenght scale.

len_scale_vec `numpy.ndarray`: The length scales in each direction.

name `str`: The name of the CovModel class.

nugget `float`: The nugget of the model.

nugget_bounds `list`: Bounds for the nugget.

opt_arg `list` of `str`: Names of the optional arguments.

opt_arg_bounds `dict`: Bounds for the optional arguments.

pykrige_angle 2D rotation angle for pykrige.

pykrige_angle_x 3D rotation angle around x for pykrige.

pykrige_angle_y 3D rotation angle around y for pykrige.

pykrige_angle_z 3D rotation angle around z for pykrige.

pykrige_anis 2D anisotropy ratio for pykrige.

pykrige_anis_y 3D anisotropy ratio in y direction for pykrige.

pykrige_anis_z 3D anisotropy ratio in z direction for pykrige.

pykrige_kwargs Keyword arguments for pykrige routines.

rescale `float`: Rescale factor for the length scale of the model.

sill `float`: The sill of the variogram.

var `float`: The variance of the model.

var_bounds `list`: Bounds for the variance.

var_raw `float`: The raw variance of the model without factor.

Methods

anisometrize(pos)	Bring a position tuple into the anisotropic coordinate-system.
calc_integral_scale()	Calculate the integral scale of the isotrope model.
check_arg_bounds()	Check arguments to be within their given bounds.
check_dim(dim)	Check the given dimension.
check_opt_arg()	Run checks for the optional arguments.
cor(h)	TPL Simple - normalized correlation function.
cor_axis(r[, axis])	Correlation along axis of anisotropy.
cor_spatial(pos)	Spatial correlation respecting anisotropy and rotation.
cor_yadrenko(zeta)	Yadrenko correlation for great-circle distance from latlon-pos.
correlation(r)	Correlation function of the model.
cov_axis(r[, axis])	Covariance along axis of anisotropy.
cov_nugget(r)	Isotropic covariance of the model respecting the nugget at r=0.
cov_spatial(pos)	Spatial covariance respecting anisotropy and rotation.
cov_yadrenko(zeta)	Yadrenko covariance for great-circle distance from latlon-pos.
covariance(r)	Covariance of the model.
default_arg_bounds()	Provide default boundaries for arguments.
default_opt_arg()	Defaults for the optional arguments.

continues on next page

Table 31 – continued from previous page

default_opt_arg_bounds()	Defaults for boundaries of the optional arguments.
default_rescale()	Provide default rescaling factor.
fit_variogram(x_data, y_data[, anis, sill, ...])	Fiting the variogram-model to an empirical variogram.
fix_dim()	Set a fix dimension for the model.
isometrize(pos)	Make a position tuple ready for isotropic operations.
ln_spectral_rad_pdf(r)	Log radial spectral density of the model.
main_axes()	Axes of the rotated coordinate-system.
percentile_scale([per])	Calculate the percentile scale of the isotrope model.
plot([func])	Plot a function of a the CovModel.
pykrige_vario([args, r])	Isotropic variogram of the model for pykrige.
set_arg_bounds([check_args])	Set bounds for the parameters of the model.
spectral_density(k)	Spectral density of the covariance model.
spectral_rad_pdf(r)	Radial spectral density of the model.
spectrum(k)	Spectrum of the covariance model.
var_factor()	Factor for the variance.
vario_axis(r[, axis])	Variogram along axis of anisotropy.
vario_nugget(r)	Isotropic variogram of the model respecting the nugget at r=0.
vario_spatial(pos)	Spatial variogram respecting anisotropy and rotation.
vario_yadrenko(zeta)	Yadrenko variogram for great-circle distance from latlon-pos.
variogram(r)	Isotropic variogram of the model.

anisometrize(*pos*)
Bring a position tuple into the anisotropic coordinate-system.

calc_integral_scale()
Calculate the integral scale of the isotrope model.

check_arg_bounds()
Check arguments to be within their given bounds.

check_dim(*dim*)
Check the given dimension.

check_opt_arg()
Run checks for the optional arguments.

This is in addition to the bound-checks

Notes

- You can use this to raise a ValueError/warning
- Any return value will be ignored
- This method will only be run once, when the class is initialized

---

cor(*h*)
TPL Simple - normalized correlation function.

cor_axis(*r*, *axis=0*)
Correlation along axis of anisotropy.

cor_spatial(*pos*)
Spatial correlation respecting anisotropy and rotation.

cor_yadrenko(*zeta*)
Yadrenko correlation for great-circle distance from latlon-pos.

correlation(*r*)
Correlation function of the model.

cov_axis(*r*, *axis=0*)
Covariance along axis of anisotropy.

cov_nugget(*r*)
Isotropic covariance of the model respecting the nugget at r=0.

cov_spatial(*pos*)
Spatial covariance respecting anisotropy and rotation.

cov_yadrenko(*zeta*)
Yadrenko covariance for great-circle distance from latlon-pos.

covariance(*r*)
Covariance of the model.

default_arg_bounds()
Provide default boundaries for arguments.

Given as a dictionary.

default_opt_arg()
Defaults for the optional arguments.

- `{"nu": dim/2}`

Returns Defaults for optional arguments

Return type dict

default_opt_arg_bounds()
Defaults for boundaries of the optional arguments.

- `{"nu": [dim/2 - 1, 50.0]}`

Returns Boundaries for optional arguments

Return type dict

default_rescale()
Provide default rescaling factor.

fit_variogram(*x_data*, *y_data*, *anis=True*, *sill=None*, *init_guess='default'*, *weights=None*, *method='trf'*, *loss='soft_l1'*, *max_eval=None*, *return_r2=False*, *curve_fit_kwargs=None*, ***para_select*)
Fiting the variogram-model to an empirical variogram.

Parameters

- **x_data** (numpy.ndarray) – The bin-centers of the empirical variogram.
- **y_data** (numpy.ndarray) – The messured variogram If multiple are given, they are interpreted as the directional variograms along the main axis of the associated rotated coordinate system. Anisotropy ratios will be estimated in that case.
- **anis** (bool, optional) – In case of a directional variogram, you can control anisotropy by this argument. Deselect the parameter from fitting, by setting it "False". You could also pass a fixed value to be set in the model. Then the anisotropy ratios wont be altered during fitting. Default: True
- **sill** (float or bool, optional) – Here you can provide a fixed sill for the variogram. It needs to be in a fitting range for the var and nugget bounds. If variance or nugget are not selected for estimation, the nugget will be recalculated to fulfill:

  - sill = var + nugget
  - if the variance is bigger than the sill, nugget will bet set to its lower bound and the variance will be set to the fitting partial sill.

  If variance is deselected, it needs to be less than the sill, otherwise a ValueError comes up. Same for nugget. If sill=False, it will be deslected from estimation and set to the current sill of the model. Then, the procedure above is applied. Default: None

- **init_guess** (str or dict, optional) – Initial guess for the estimation. Either:
  - "default": using the default values of the covariance model ("len_scale" will be mean of given bin centers; "var" and "nugget" will be mean of given variogram values (if in given bounds))
  - "current": using the current values of the covariance model
  - dict: dictionary with parameter names and given value (separate "default" can bet set to "default" or "current" for unspecified values to get same behavior as given above ("default" by default)) Example: `{"len_scale": 10, "default": "current"}`

  Default: "default"

- **weights** (str, numpy.ndarray, callable, optional) – Weights applied to each point in the estimation. Either:
  - 'inv': inverse distance `1 / (x_data + 1)`
  - list: weights given per bin
  - callable: function applied to x_data

  If callable, it must take a 1-d ndarray. Then `weights = f(x_data)`. Default: None

- **method** (*{'trf', 'dogbox'}, optional*) – Algorithm to perform minimization.
  - 'trf' : Trust Region Reflective algorithm, particularly suitable for large sparse problems with bounds. Generally robust method.
  - 'dogbox' : dogleg algorithm with rectangular trust regions, typical use case is small problems with bounds. Not recommended for problems with rank-deficient Jacobian.

  Default: 'trf'

- **loss** (str or callable, optional) – Determines the loss function in scipys curve_fit. The following keyword values are allowed:
  - 'linear' (default) : `rho(z) = z`. Gives a standard least-squares problem.
  - 'soft_l1' : `rho(z) = 2 * ((1 + z)**0.5 - 1)`. The smooth approximation of l1 (absolute value) loss. Usually a good choice for robust least squares.
  - 'huber' : `rho(z) = z if z <= 1 else 2*z**0.5 - 1`. Works similarly to 'soft_l1'.
  - 'cauchy' : `rho(z) = ln(1 + z)`. Severely weakens outliers influence, but may cause difficulties in optimization process.
  - 'arctan' : `rho(z) = arctan(z)`. Limits a maximum loss on a single residual, has properties similar to 'cauchy'.

  If callable, it must take a 1-d ndarray `z=f**2` and return an array_like with shape (3, m) where row 0 contains function values, row 1 contains first derivatives and row 2 contains second derivatives. Default: 'soft_l1'

- **max_eval** (int or None, optional) – Maximum number of function evaluations before the termination. If None (default), the value is chosen automatically: 100 * n.

- **return_r2** (bool, optional) – Whether to return the r2 score of the estimation. Default: False
- **curve_fit_kwargs** (dict, optional) – Other keyword arguments passed to scipys curve_fit. Default: None
- ****para_select** – You can deselect parameters from fitting, by setting them "False" using their names as keywords. You could also pass fixed values for each parameter. Then these values will be applied and the involved parameters wont be fitted. By default, all parameters are fitted.

Returns

- **fit_para** (dict) – Dictonary with the fitted parameter values
- **pcov** (numpy.ndarray) – The estimated covariance of *popt* from scipy.optimize.curve_fit. To compute one standard deviation errors on the parameters use `perr = np.sqrt(np.diag(pcov))`.
- **r2_score** (float, optional) – r2 score of the curve fitting results. Only if return_r2 is True.

Notes

You can set the bounds for each parameter by accessing *CovModel.set_arg_bounds*.

The fitted parameters will be instantly set in the model.

---

fix_dim()
Set a fix dimension for the model.

isometrize(*pos*)
Make a position tuple ready for isotropic operations.

ln_spectral_rad_pdf(*r*)
Log radial spectral density of the model.

main_axes()
Axes of the rotated coordinate-system.

percentile_scale(*per=0.9*)
Calculate the percentile scale of the isotrope model.

This is the distance, where the given percentile of the variance is reached by the variogram

plot(*func='variogram'*, ***kwargs*)
Plot a function of a the CovModel.

Parameters

- **func** (str, optional) – Function to be plotted. Could be one of:
  - "variogram"
  - "covariance"
  - "correlation"
  - "vario_spatial"
  - "cov_spatial"
  - "cor_spatial"
  - "vario_yadrenko"
  - "cov_yadrenko"
  - "cor_yadrenko"

- "vario_axis"
- "cov_axis"
- "cor_axis"
- "spectrum"
- "spectral_density"
- "spectral_rad_pdf"

- ****kwargs** – Keyword arguments forwarded to the plotting function *"plot_" + func* in `gstools.covmodel.plot`.

See also:

`gstools.covmodel.plot`

pykrige_vario(*args=None*, *r=0*)
Isotropic variogram of the model for pykrige.

set_arg_bounds(*check_args=True*, ***kwargs*)
Set bounds for the parameters of the model.

Parameters

- **check_args** (*bool, optional*) – Whether to check if the arguments are in their valid bounds. In case not, a propper default value will be determined. Default: True
- ****kwargs** – Parameter name as keyword ("var", "len_scale", "nugget", <opt_arg>) and a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

spectral_density(*k*)
Spectral density of the covariance model.

This is given by:

$$\tilde{S}(k) = \frac{S(k)}{\sigma^2}$$

Where $S(k)$ is the spectrum of the covariance model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

spectral_rad_pdf(*r*)
Radial spectral density of the model.

spectrum(*k*)
Spectrum of the covariance model.

This is given by:

$$S(\mathbf{k}) = \left(\frac{1}{2\pi}\right)^n \int C(r) e^{i\mathbf{k}\cdot\mathbf{r}} d^n\mathbf{r}$$

Internally, this is calculated by the hankel transformation:

$$S(k) = \left(\frac{1}{2\pi}\right)^n \cdot \frac{(2\pi)^{n/2}}{k^{n/2-1}} \int_0^\infty r^{n/2} C(r) J_{n/2-1}(kr) dr$$

Where $C(r)$ is the covariance function of the model.

Parameters **k** (`float`) – Radius of the phase: $k = \|\mathbf{k}\|$

var_factor()
Factor for the variance.

vario_axis(*r*, *axis=0*)
Variogram along axis of anisotropy.

vario_nugget(*r*)
Isotropic variogram of the model respecting the nugget at r=0.

vario_spatial(*pos*)
Spatial variogram respecting anisotropy and rotation.

vario_yadrenko(*zeta*)
Yadrenko variogram for great-circle distance from latlon-pos.

variogram(*r*)
Isotropic variogram of the model.

property angles
Rotation angles (in rad) of the model.

Type numpy.ndarray

property anis
The anisotropy factors of the model.

Type numpy.ndarray

property anis_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property arg
Names of all arguments.

Type list of str

property arg_bounds
Bounds for all parameters.

Notes

Keys are the arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property arg_list
Values of all arguments.

Type list of float

property dim
The dimension of the model.

Type int

property dist_func
pdf, cdf and ppf.

Spectral distribution info from the model.

Type tuple of callable

property do_rotation
State if a rotation is performed.

Type bool

property field_dim
The field dimension of the model.

Type int

property hankel_kw
hankel.SymmetricFourierTransform kwargs.

Type dict

property has_cdf
State if a cdf is defined by the user.

Type bool

property has_ppf
State if a ppf is defined by the user.

Type bool

property integral_scale
The main integral scale of the model.

Raises ValueError – If integral scale is not setable.

Type float

property integral_scale_vec
The integral scales in each direction.

Notes

This is calculated by:

- `integral_scale_vec[0] = integral_scale`
- `integral_scale_vec[1] = integral_scale*anis[0]`
- `integral_scale_vec[2] = integral_scale*anis[1]`

Type numpy.ndarray

property is_isotropic
State if a model is isotropic.

Type bool

property iso_arg
Names of isotropic arguments.

Type list of str

property iso_arg_list
Values of isotropic arguments.

Type list of float

property latlon
Whether the model depends on geographical coords.

Type bool

property len_rescaled
The rescaled main length scale of the model.

Type float

property len_scale
The main length scale of the model.

Type float

property len_scale_bounds
Bounds for the lenght scale.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property len_scale_vec
The length scales in each direction.

Notes

This is calculated by:

- `len_scale_vec[0] = len_scale`
- `len_scale_vec[1] = len_scale*anis[0]`
- `len_scale_vec[2] = len_scale*anis[1]`

Type numpy.ndarray

property name
The name of the CovModel class.

Type str

property nugget
The nugget of the model.

Type float

property nugget_bounds
Bounds for the nugget.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where <type> is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property opt_arg
Names of the optional arguments.

Type list of str

property opt_arg_bounds
Bounds for the optional arguments.

Notes

Keys are the opt-arg names and values are lists of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type dict

property pykrige_angle
2D rotation angle for pykrige.

property pykrige_angle_x
3D rotation angle around x for pykrige.

property pykrige_angle_y
3D rotation angle around y for pykrige.

property pykrige_angle_z
3D rotation angle around z for pykrige.

property pykrige_anis
2D anisotropy ratio for pykrige.

property pykrige_anis_y
3D anisotropy ratio in y direction for pykrige.

property pykrige_anis_z
3D anisotropy ratio in z direction for pykrige.

property pykrige_kwargs
Keyword arguments for pykrige routines.

property rescale
Rescale factor for the length scale of the model.

Type float

property sill
The sill of the variogram.

Notes

This is calculated by:

- `sill = variance + nugget`

Type float

property var
The variance of the model.

Type float

property var_bounds
Bounds for the variance.

Notes

Is a list of 2 or 3 values: `[a, b]` or `[a, b, <type>]` where `<type>` is one of `"oo"`, `"cc"`, `"oc"` or `"co"` to define if the bounds are open ("o") or closed ("c").

Type list

property var_raw
The raw variance of the model without factor.

(See. CovModel.var_factor)

Type float

gstools.covmodel.plot

GStools subpackage providing plotting routines for the covariance models.

The following classes and functions are provided

plot_variogram(model[, x_min, x_max, fig, ax])	Plot variogram of a given CovModel.
plot_covariance(model[, x_min, x_max, fig, ax])	Plot covariance of a given CovModel.
plot_correlation(model[, x_min, x_max, fig, ax])	Plot correlation function of a given CovModel.
plot_vario_yadrenko(model[, x_min, x_max, ...])	Plot Yadrenko variogram of a given CovModel.
plot_cov_yadrenko(model[, x_min, x_max, fig, ax])	Plot Yadrenko covariance of a given CovModel.
plot_cor_yadrenko(model[, x_min, x_max, fig, ax])	Plot Yadrenko correlation function of a given CovModel.
plot_vario_axis(model[, axis, x_min, x_max, ...])	Plot variogram of a given CovModel.
plot_cov_axis(model[, axis, x_min, x_max, ...])	Plot variogram of a given CovModel.
plot_cor_axis(model[, axis, x_min, x_max, ...])	Plot variogram of a given CovModel.
plot_vario_spatial(model[, x_min, x_max, ...])	Plot spatial variogram of a given CovModel.
plot_cov_spatial(model[, x_min, x_max, fig, ax])	Plot spatial covariance of a given CovModel.
plot_cor_spatial(model[, x_min, x_max, fig, ax])	Plot spatial correlation of a given CovModel.
plot_spectrum(model[, x_min, x_max, fig, ax])	Plot specturm of a given CovModel.
plot_spectral_density(model[, x_min, x_max, ...])	Plot spectral density of a given CovModel.
plot_spectral_rad_pdf(model[, x_min, x_max, ...])	Plot radial spectral pdf of a given CovModel.

gstools.covmodel.plot.**plot_cor_axis**(*model*, *axis=0*, *x_min=0.0*, *x_max=None*, *fig=None*, *ax=None*, ***kwargs*)

Plot variogram of a given CovModel.

gstools.covmodel.plot.**plot_cor_spatial**(*model*, *x_min=0.0*, *x_max=None*, *fig=None*, *ax=None*, ***kwargs*)

Plot spatial correlation of a given CovModel.

gstools.covmodel.plot.**plot_cor_yadrenko**(*model*, *x_min=0.0*, *x_max=None*, *fig=None*, *ax=None*, ***kwargs*)

Plot Yadrenko correlation function of a given CovModel.

gstools.covmodel.plot.**plot_correlation**(*model*, *x_min=0.0*, *x_max=None*, *fig=None*, *ax=None*, ***kwargs*)

Plot correlation function of a given CovModel.

gstools.covmodel.plot.**plot_cov_axis**(*model*, *axis=0*, *x_min=0.0*, *x_max=None*, *fig=None*, *ax=None*, ***kwargs*)

Plot variogram of a given CovModel.

gstools.covmodel.plot.**plot_cov_spatial**(*model*, *x_min=0.0*, *x_max=None*, *fig=None*, *ax=None*, ***kwargs*)

Plot spatial covariance of a given CovModel.

gstools.covmodel.plot.**plot_cov_yadrenko**(*model*, *x_min=0.0*, *x_max=None*, *fig=None*, *ax=None*, ***kwargs*)

Plot Yadrenko covariance of a given CovModel.

gstools.covmodel.plot.**plot_covariance**(*model*, *x_min=0.0*, *x_max=None*, *fig=None*, *ax=None*, ***kwargs*)

Plot covariance of a given CovModel.

gstools.covmodel.plot.**plot_spectral_density**(*model*, *x_min=0.0*, *x_max=None*, *fig=None*, *ax=None*, ***kwargs*)

Plot spectral density of a given CovModel.

gstools.covmodel.plot.**plot_spectral_rad_pdf**(*model*, *x_min=0.0*, *x_max=None*, *fig=None*, *ax=None*, ***kwargs*)

Plot radial spectral pdf of a given CovModel.

gstools.covmodel.plot.**plot_spectrum**(*model*, *x_min=0.0*, *x_max=None*, *fig=None*, *ax=None*, ***kwargs*)

Plot specturm of a given CovModel.

gstools.covmodel.plot.**plot_vario_axis**(*model*, *axis=0*, *x_min=0.0*, *x_max=None*, *fig=None*, *ax=None*, ***kwargs*)

Plot variogram of a given CovModel.

gstools.covmodel.plot.**plot_vario_spatial**(*model*, *x_min=0.0*, *x_max=None*, *fig=None*, *ax=None*, ***kwargs*)

Plot spatial variogram of a given CovModel.

gstools.covmodel.plot.**plot_vario_yadrenko**(*model*, *x_min=0.0*, *x_max=None*, *fig=None*, *ax=None*, ***kwargs*)

Plot Yadrenko variogram of a given CovModel.

gstools.covmodel.plot.**plot_variogram**(*model*, *x_min=0.0*, *x_max=None*, *fig=None*, *ax=None*, ***kwargs*)

Plot variogram of a given CovModel.

3.7 gstools.field

GStools subpackage providing tools for spatial random fields.

Subpackages

generator	GStools subpackage providing generators for spatial random fields.
upscaling	GStools subpackage providing upscaling routines for the spatial random field.

Spatial Random Field

SRF(model[, mean, normalizer, trend, ...])	A class to generate spatial random fields (SRF).
CondSRF(krige[, generator])	A class to generate conditioned spatial random fields (SRF).

gstools.field.SRF

class gstools.field.**SRF**(*model*, *mean=0.0*, *normalizer=None*, *trend=None*, *upscaling='no_scaling'*, *generator='RandMeth'*, ***generator_kwargs*)

Bases: *gstools.field.base.Field*

A class to generate spatial random fields (SRF).

Parameters

- **model** (*CovModel*) – Covariance Model of the spatial random field.
- **mean** (float or callable, optional) – Mean of the SRF (in normal form). Could also be a callable. The default is 0.0.
- **normalizer** (None or *Normalizer*, optional) – Normalizer to be applied to the SRF to transform the field values. The default is None.
- **trend** (None or float or callable, optional) – Trend of the SRF (in transformed form). If no normalizer is applied, this behaves equal to 'mean'. The default is None.
- **upscaling** (str, optional) – Method to be used for upscaling the variance at each point depending on the related element volume. See the `point_volumes` keyword in the *SRF.__call__* routine. At the moment, the following upscaling methods are provided:
  - "no_scaling" : No upscaling is applied to the variance. See: *var_no_scaling*
  - "coarse_graining" : A volume depended variance is calculated by the upscaling technique coarse graining. See: *var_coarse_graining*

  Default: "no_scaling"
- **generator** (str, optional) – Name of the field generator to be used. At the moment, the following generators are provided:
  - "RandMeth" : The Randomization Method. See: *RandMeth*
  - "IncomprRandMeth" : The incompressible Randomization Method. This is the original algorithm proposed by Kraichnan 1970 See: *IncomprRandMeth*
  - "VectorField" : an alias for "IncomprRandMeth"
  - "VelocityField" : an alias for "IncomprRandMeth"

Default: "RandMeth"

- ****generator_kwargs** – Keyword arguments that are forwarded to the generator in use. Have a look at the provided generators for further information.

Attributes

dim `int`: Dimension of the field.

generator `callable`: The generator of the field.

latlon `bool`: Whether the field depends on geographical coords.

mean `float` or `callable`: The mean of the field.

model *CovModel*: The covariance model of the field.

name `str`: The name of the class.

normalizer *Normalizer*: Normalizer of the field.

trend `float` or `callable`: The trend of the field.

upscaling `str`: Name of the upscaling method.

value_type `str`: Type of the field values (scalar, vector).

Methods

__call__(pos[, seed, point_volumes, mesh_type])	Generate the spatial random field.
mesh(mesh[, points, direction, name])	Generate a field on a given meshio, ogs5py or PyVista mesh.
plot([field, fig, ax])	Plot the spatial random field.
post_field(field[, name, process, save])	Postprocessing field values.
pre_pos(pos[, mesh_type])	Preprocessing positions and mesh_type.
set_generator(generator, **generator_kwargs)	Set the generator for the field.
structured(*args, **kwargs)	Generate a field on a structured mesh.
to_pyvista([field_select, fieldname])	Create a VTK/PyVista grid of the stored field.
unstructured(*args, **kwargs)	Generate a field on an unstructured mesh.
upscaling_func(*args, **kwargs)	Upscaling method applied to the field variance.
vtk_export(filename[, field_select, fieldname])	Export the stored field to vtk.

__call__(*pos*, *seed=nan*, *point_volumes=0.0*, *mesh_type='unstructured'*)

Generate the spatial random field.

The field is saved as *self.field* and is also returned.

Parameters

- **pos** (`list`) – the position tuple, containing main direction and transversal directions
- **seed** (`int`, optional) – seed for RNG for reseting. Default: keep seed from generator
- **point_volumes** (`float` or `numpy.ndarray`) – If your evaluation points for the field are coming from a mesh, they are probably representing a certain element volume. This volume can be passed by *point_volumes* to apply the given variance upscaling. If *point_volumes* is `0` nothing is changed. Default: `0`
- **mesh_type** (`str`) – 'structured' / 'unstructured'

Returns **field** – the SRF

Return type `numpy.ndarray`

mesh(*mesh*, *points='centroids'*, *direction='all'*, *name='field'*, ***kwargs*)
Generate a field on a given meshio, ogs5py or PyVista mesh.

Parameters

- **mesh** (*meshio.Mesh or ogs5py.MSH or PyVista mesh*) – The given mesh
- **points** (str, optional) – The points to evaluate the field at. Either the "centroids" of the mesh cells (calculated as mean of the cell vertices) or the "points" of the given mesh. Default: "centroids"
- **direction** (str or list, optional) – Here you can state which direction should be choosen for lower dimension. For example, if you got a 2D mesh in xz direction, you have to pass "xz". By default, all directions are used. One can also pass a list of indices. Default: "all"
- **name** (str or list of str, optional) – Name(s) to store the field(s) in the given mesh as point_data or cell_data. If to few names are given, digits will be appended. Default: "field"
- ****kwargs** – Keyword arguments forwarded to *__call__*.

Notes

This will store the field in the given mesh under the given name, if a meshio or PyVista mesh was given.

See:

- meshio: https://github.com/nschloe/meshio
- ogs5py: https://github.com/GeoStat-Framework/ogs5py
- PyVista: https://github.com/pyvista/pyvista

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plot(*field='field'*, *fig=None*, *ax=None*, ***kwargs*)
Plot the spatial random field.

Parameters

- **field** (str, optional) – Field that should be plotted. Default: "field"
- **fig** (Figure or None) – Figure to plot the axes on. If *None*, a new one will be created. Default: *None*
- **ax** (Axes or None) – Axes to plot on. If *None*, a new one will be added to the figure. Default: *None*
- ****kwargs** – Forwarded to the plotting routine.

post_field(*field*, *name='field'*, *process=True*, *save=True*)
Postprocessing field values.

Parameters

- **field** (numpy.ndarray) – Field values.
- **name** (str, optional) – Name. to store the field. The default is "field".
- **process** (bool, optional) – Whether to process field to apply mean, normalizer and trend. The default is True.
- **save** (bool, optional) – Whether to store the field under the given name. The default is True.

Returns **field** – Processed field values.

Return type numpy.ndarray

pre_pos(*pos*, *mesh_type='unstructured'*)
Preprocessing positions and mesh_type.

Parameters

- **pos** (iterable) – the position tuple, containing main direction and transversal directions
- **mesh_type** (str, optional) – 'structured' / 'unstructured' Default: *"unstructured"*

Returns

- **iso_pos** ((d, n), numpy.ndarray) – the isometrized position tuple
- **shape** (tuple) – Shape of the resulting field.

set_generator(*generator*, ***generator_kwargs*)
Set the generator for the field.

Parameters

- **generator** (str, optional) – Name of the generator to use for field generation. Default: "RandMeth"
- ****generator_kwargs** – keyword arguments that are forwarded to the generator in use.

structured(**args*, ***kwargs*)
Generate a field on a structured mesh.

See *__call__*

to_pyvista(*field_select='field'*, *fieldname='field'*)
Create a VTK/PyVista grid of the stored field.

Parameters

- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

unstructured(**args*, ***kwargs*)
Generate a field on an unstructured mesh.

See *__call__*

upscaling_func(**args*, ***kwargs*)
Upscaling method applied to the field variance.

vtk_export(*filename*, *field_select='field'*, *fieldname='field'*)
Export the stored field to vtk.

Parameters

- **filename** (str) – Filename of the file to be saved, including the path. Note that an ending (.vtr or .vtu) will be added to the name.
- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

property dim
Dimension of the field.

Type int

property generator
The generator of the field.

Default: *RandMeth*

Type callable

property latlon
Whether the field depends on geographical coords.

Type bool

property mean
The mean of the field.

Type float or callable

property model
The covariance model of the field.

Type *CovModel*

property name
The name of the class.

Type str

property normalizer
Normalizer of the field.

Type *Normalizer*

property trend
The trend of the field.

Type float or callable

property upscaling
Name of the upscaling method.

See the point_volumes keyword in the *SRF.__call__* routine. Default: "no_scaling"

Type str

property value_type
Type of the field values (scalar, vector).

Type str

gstools.field.CondSRF

class gstools.field.**CondSRF**(*krige*, *generator='RandMeth'*, ***generator_kwargs*)
Bases: *gstools.field.base.Field*

A class to generate conditioned spatial random fields (SRF).

Parameters

- **krige** (*Krige*) – Kriging setup to condition the spatial random field.
- **generator** (str, optional) – Name of the field generator to be used. At the moment, only the following generator is provided:
  - ”RandMeth” : The Randomization Method. See: *RandMeth*

  Default: “RandMeth”
- ****generator_kwargs** – Keyword arguments that are forwarded to the generator in use. Have a look at the provided generators for further information.

Attributes

dim int: Dimension of the field.

generator callable: The generator of the field.

krige *Krige*: The underlying kriging class.

latlon bool: Whether the field depends on geographical coords.

mean float or callable: The mean of the field.

model *CovModel*: The covariance model of the field.

name str: The name of the class.

normalizer *Normalizer*: Normalizer of the field.

trend float or callable: The trend of the field.

value_type str: Type of the field values (scalar, vector).

Methods

__call__(pos[, seed, mesh_type])	Generate the conditioned spatial random field.
get_scaling(krige_var, shape)	Get scaling coefficients for the random field.
mesh(mesh[, points, direction, name])	Generate a field on a given meshio, ogs5py or PyVista mesh.
plot([field, fig, ax])	Plot the spatial random field.
post_field(field[, name, process, save])	Postprocessing field values.
pre_pos(pos[, mesh_type])	Preprocessing positions and mesh_type.
set_generator(generator, **generator_kwargs)	Set the generator for the field.
structured(*args, **kwargs)	Generate a field on a structured mesh.
to_pyvista([field_select, fieldname])	Create a VTK/PyVista grid of the stored field.
unstructured(*args, **kwargs)	Generate a field on an unstructured mesh.
vtk_export(filename[, field_select, fieldname])	Export the stored field to vtk.

__call__(*pos*, *seed=nan*, *mesh_type='unstructured'*, ***kwargs*)
Generate the conditioned spatial random field.

The field is saved as *self.field* and is also returned.

Parameters

- **pos** (list) – the position tuple, containing main direction and transversal directions

- **seed** (int, optional) – seed for RNG for reseting. Default: keep seed from generator
- **mesh_type** (str) – 'structured' / 'unstructured'
- ****kwargs** – keyword arguments that are forwarded to the kriging routine in use.

Returns **field** – the conditioned SRF

Return type numpy.ndarray

get_scaling(*krige_var*, *shape*)
Get scaling coefficients for the random field.

Parameters

- **krige_var** (numpy.ndarray) – Kriging variance.
- **shape** (tuple of int) – Field shape.

Returns

- **var_scale** (numpy.ndarray) – Variance scaling factor for the random field.
- **nugget** (numpy.ndarray or int) – Nugget to be added to the field.

mesh(*mesh*, *points='centroids'*, *direction='all'*, *name='field'*, ***kwargs*)
Generate a field on a given meshio, ogs5py or PyVista mesh.

Parameters

- **mesh** (*meshio.Mesh or ogs5py.MSH or PyVista mesh*) – The given mesh
- **points** (str, optional) – The points to evaluate the field at. Either the "centroids" of the mesh cells (calculated as mean of the cell vertices) or the "points" of the given mesh. Default: "centroids"
- **direction** (str or list, optional) – Here you can state which direction should be choosen for lower dimension. For example, if you got a 2D mesh in xz direction, you have to pass "xz". By default, all directions are used. One can also pass a list of indices. Default: "all"
- **name** (str or list of str, optional) – Name(s) to store the field(s) in the given mesh as point_data or cell_data. If to few names are given, digits will be appended. Default: "field"
- ****kwargs** – Keyword arguments forwarded to *__call__*.

Notes

This will store the field in the given mesh under the given name, if a meshio or PyVista mesh was given.

See:

- meshio: https://github.com/nschloe/meshio
- ogs5py: https://github.com/GeoStat-Framework/ogs5py
- PyVista: https://github.com/pyvista/pyvista

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plot(*field='field'*, *fig=None*, *ax=None*, ***kwargs*)
Plot the spatial random field.

Parameters

- **field** (str, optional) – Field that should be plotted. Default: "field"
- **fig** (Figure or None) – Figure to plot the axes on. If *None*, a new one will be created. Default: *None*
- **ax** (Axes or None) – Axes to plot on. If *None*, a new one will be added to the figure. Default: *None*

- ****kwargs** – Forwarded to the plotting routine.

post_field(*field*, *name='field'*, *process=True*, *save=True*)
Postprocessing field values.

Parameters

- **field** (numpy.ndarray) – Field values.
- **name** (str, optional) – Name. to store the field. The default is "field".
- **process** (bool, optional) – Whether to process field to apply mean, normalizer and trend. The default is True.
- **save** (bool, optional) – Whether to store the field under the given name. The default is True.

Returns **field** – Processed field values.

Return type numpy.ndarray

pre_pos(*pos*, *mesh_type='unstructured'*)
Preprocessing positions and mesh_type.

Parameters

- **pos** (iterable) – the position tuple, containing main direction and transversal directions
- **mesh_type** (str, optional) – 'structured' / 'unstructured' Default: *"unstructured"*

Returns

- **iso_pos** ((d, n), numpy.ndarray) – the isometrized position tuple
- **shape** (tuple) – Shape of the resulting field.

set_generator(*generator*, ***generator_kwargs*)
Set the generator for the field.

Parameters

- **generator** (str, optional) – Name of the generator to use for field generation. Default: "RandMeth"
- ****generator_kwargs** – keyword arguments that are forwarded to the generator in use.

structured(**args*, ***kwargs*)
Generate a field on a structured mesh.

See *__call__*

to_pyvista(*field_select='field'*, *fieldname='field'*)
Create a VTK/PyVista grid of the stored field.

Parameters

- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

unstructured(**args*, ***kwargs*)
Generate a field on an unstructured mesh.

See *__call__*

vtk_export(*filename*, *field_select='field'*, *fieldname='field'*)
Export the stored field to vtk.

Parameters

- **filename** (str) – Filename of the file to be saved, including the path. Note that an ending (.vtr or .vtu) will be added to the name.
- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

property dim
Dimension of the field.

Type int

property generator
The generator of the field.

Type callable

property krige
The underlying kriging class.

Type *Krige*

property latlon
Whether the field depends on geographical coords.

Type bool

property mean
The mean of the field.

Type float or callable

property model
The covariance model of the field.

Type *CovModel*

property name
The name of the class.

Type str

property normalizer
Normalizer of the field.

Type *Normalizer*

property trend
The trend of the field.

Type float or callable

property value_type
Type of the field values (scalar, vector).

Type str

Field Base Class

Field([model, value_type, mean, normalizer, ...])	A base class for random fields, kriging fields, etc.

gstools.field.Field

class gstools.field.**Field**(*model=None*, *value_type='scalar'*, *mean=None*, *normalizer=None*, *trend=None*, *dim=None*)

Bases: `object`

A base class for random fields, kriging fields, etc.

Parameters

- **model** (*CovModel*, optional) – Covariance Model related to the field.
- **value_type** (`str`, optional) – Value type of the field. Either "scalar" or "vector". The default is "scalar".
- **mean** (`None` or `float` or `callable`, optional) – Mean of the field if wanted. Could also be a callable. The default is None.
- **normalizer** (`None` or *Normalizer*, optional) – Normalizer to be applied to the field. The default is None.
- **trend** (`None` or `float` or `callable`, optional) – Trend of the denormalized fields. If no normalizer is applied, this behaves equal to 'mean'. The default is None.
- **dim** (`None` or `int`, optional) – Dimension of the field if no model is given.

Attributes

dim `int`: Dimension of the field.

latlon `bool`: Whether the field depends on geographical coords.

mean `float` or `callable`: The mean of the field.

model *CovModel*: The covariance model of the field.

name `str`: The name of the class.

normalizer *Normalizer*: Normalizer of the field.

trend `float` or `callable`: The trend of the field.

value_type `str`: Type of the field values (scalar, vector).

Methods

__call__(pos[, field, mesh_type, post_process])	Generate the field.
mesh(mesh[, points, direction, name])	Generate a field on a given meshio, ogs5py or PyVista mesh.
plot([field, fig, ax])	Plot the spatial random field.
post_field(field[, name, process, save])	Postprocessing field values.
pre_pos(pos[, mesh_type])	Preprocessing positions and mesh_type.
structured(*args, **kwargs)	Generate a field on a structured mesh.
to_pyvista([field_select, fieldname])	Create a VTK/PyVista grid of the stored field.
unstructured(*args, **kwargs)	Generate a field on an unstructured mesh.
vtk_export(filename[, field_select, fieldname])	Export the stored field to vtk.

__call__(*pos*, *field=None*, *mesh_type='unstructured'*, *post_process=True*)

Generate the field.

Parameters

- **pos** (list) – the position tuple, containing main direction and transversal directions
- **field** (numpy.ndarray or None, optional) – the field values. Will be all zeros if None is given.
- **mesh_type** (str, optional) – 'structured' / 'unstructured'. Default: 'unstructured'
- **post_process** (bool, optional) – Whether to apply mean, normalizer and trend to the field. Default: *True*

Returns **field** – the field values.

Return type numpy.ndarray

mesh(*mesh*, *points='centroids'*, *direction='all'*, *name='field'*, ***kwargs*)
Generate a field on a given meshio, ogs5py or PyVista mesh.

Parameters

- **mesh** (*meshio.Mesh or ogs5py.MSH or PyVista mesh*) – The given mesh
- **points** (str, optional) – The points to evaluate the field at. Either the "centroids" of the mesh cells (calculated as mean of the cell vertices) or the "points" of the given mesh. Default: "centroids"
- **direction** (str or list, optional) – Here you can state which direction should be choosen for lower dimension. For example, if you got a 2D mesh in xz direction, you have to pass "xz". By default, all directions are used. One can also pass a list of indices. Default: "all"
- **name** (str or list of str, optional) – Name(s) to store the field(s) in the given mesh as point_data or cell_data. If to few names are given, digits will be appended. Default: "field"
- ****kwargs** – Keyword arguments forwarded to *__call__*.

Notes

This will store the field in the given mesh under the given name, if a meshio or PyVista mesh was given.

See:

- meshio: https://github.com/nschloe/meshio
- ogs5py: https://github.com/GeoStat-Framework/ogs5py
- PyVista: https://github.com/pyvista/pyvista

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plot(*field='field'*, *fig=None*, *ax=None*, ***kwargs*)
Plot the spatial random field.

Parameters

- **field** (str, optional) – Field that should be plotted. Default: "field"
- **fig** (Figure or None) – Figure to plot the axes on. If *None*, a new one will be created. Default: *None*
- **ax** (Axes or None) – Axes to plot on. If *None*, a new one will be added to the figure. Default: *None*
- ****kwargs** – Forwarded to the plotting routine.

post_field(*field*, *name='field'*, *process=True*, *save=True*)
Postprocessing field values.

Parameters

- **field** (numpy.ndarray) – Field values.
- **name** (str, optional) – Name. to store the field. The default is "field".
- **process** (bool, optional) – Whether to process field to apply mean, normalizer and trend. The default is True.
- **save** (bool, optional) – Whether to store the field under the given name. The default is True.

Returns **field** – Processed field values.

Return type numpy.ndarray

pre_pos(*pos*, *mesh_type='unstructured'*)
Preprocessing positions and mesh_type.

Parameters

- **pos** (iterable) – the position tuple, containing main direction and transversal directions
- **mesh_type** (str, optional) – 'structured' / 'unstructured' Default: *"unstructured"*

Returns

- **iso_pos** ((d, n), numpy.ndarray) – the isometrized position tuple
- **shape** (tuple) – Shape of the resulting field.

structured(**args*, ***kwargs*)
Generate a field on a structured mesh.

See *__call__*

to_pyvista(*field_select='field'*, *fieldname='field'*)
Create a VTK/PyVista grid of the stored field.

Parameters

- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

unstructured(**args*, ***kwargs*)
Generate a field on an unstructured mesh.

See *__call__*

vtk_export(*filename*, *field_select='field'*, *fieldname='field'*)
Export the stored field to vtk.

Parameters

- **filename** (str) – Filename of the file to be saved, including the path. Note that an ending (.vtr or .vtu) will be added to the name.
- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

property dim
Dimension of the field.

Type int

property latlon
Whether the field depends on geographical coords.

Type bool

property mean
The mean of the field.

Type float or callable

property model
The covariance model of the field.

Type *CovModel*

property name
The name of the class.

Type str

property normalizer
Normalizer of the field.

Type *Normalizer*

property trend
The trend of the field.

Type float or callable

property value_type
Type of the field values (scalar, vector).

Type str

gstools.field.generator

GStools subpackage providing generators for spatial random fields.

The following classes are provided

RandMeth(model[, mode_no, seed, verbose, ...])	Randomization method for calculating isotropic random fields.
IncomprRandMeth(model[, mean_velocity, ...])	RandMeth for incompressible random vector fields.

class gstools.field.generator.**IncomprRandMeth**(*model*, *mean_velocity=1.0*, *mode_no=1000*, *seed=None*, *verbose=False*, *sampling='auto'*, ***kwargs*)

Bases: *gstools.field.generator.RandMeth*

RandMeth for incompressible random vector fields.

Parameters

- **model** (*CovModel*) – covariance model
- **mean_velocity** (float, optional) – the mean velocity in x-direction
- **mode_no** (int, optional) – number of Fourier modes. Default: 1000
- **seed** (int or None, optional) – the seed of the random number generator. If "None", a random seed is used. Default: None
- **verbose** (bool, optional) – State if there should be output during the generation. Default: False
- **sampling** (str, optional) – Sampling strategy. Either
  - "auto": select best strategy depending on given model
  - "inversion": use inversion method
  - "mcmc": use mcmc sampling
- ****kwargs** – Placeholder for keyword-args

Notes

The Randomization method is used to generate isotropic spatial incompressible random vector fields characterized by a given covariance model. The equation is [Kraichnan1970]:

$$u_i(x) = \bar{u_i}\delta_{i1} + \bar{u_i}\sqrt{\frac{\sigma^2}{N}} \cdot \sum_{j=1}^{N} p_i(k_j)\left(Z_{1,j} \cdot \cos\left(\langle k_j, x\rangle\right) + Z_{2,j} \cdot \sin\left(\langle k_j, x\rangle\right)\right)$$

where:

- $\bar{u}$: mean velocity in e_1 direction
- N : fourier mode number
- $Z_{k,j}$: random samples from a normal distribution
- k_j : samples from the spectral density distribution of the covariance model
- $p_i(k_j) = e_1 - \frac{k_i k_1}{k^2}$: the projector ensuring the incompressibility

References

Attributes

mode_no `int`: Number of modes in the randomization method.

model *CovModel*: Covariance model of the spatial random field.

name `str`: Name of the generator.

sampling `str`: Sampling strategy.

seed `int`: Seed of the master RNG.

value_type `str`: Type of the field values (scalar, vector).

verbose `bool`: Verbosity of the generator.

Methods

__call__(pos)	Calculate the random modes for the randomization method.
get_nugget(shape)	Generate normal distributed values for the nugget simulation.
reset_seed([seed])	Recalculate the random amplitudes and wave numbers with the given seed.
update([model, seed])	Update the model and the seed.

__call__(*pos*)
Calculate the random modes for the randomization method.

This method calls the *summate_incompr_** Cython methods, which are the heart of the randomization method. In this class the method contains a projector to ensure the incompressibility of the vector field.

Parameters **pos** ((d, n), `numpy.ndarray`) – the position tuple with d dimensions and n points.

Returns the random modes

Return type `numpy.ndarray`

get_nugget(*shape*)
Generate normal distributed values for the nugget simulation.

Parameters **shape** (`tuple`) – the shape of the summed modes

Returns **nugget** – the nugget in the same shape as the summed modes

Return type `numpy.ndarray`

reset_seed(*seed=nan*)
Recalculate the random amplitudes and wave numbers with the given seed.

Parameters **seed** (`int` or `None` or `numpy.nan`, optional) – the seed of the random number generator. If `None`, a random seed is used. If `numpy.nan`, the actual seed will be kept. Default: `numpy.nan`

Notes

Even if the given seed is the present one, modes will be recalculated.

update(*model=None*, *seed=nan*)
Update the model and the seed.

If model and seed are not different, nothing will be done.

Parameters

- **model** (*CovModel* or None, optional) – covariance model. Default: None
- **seed** (int or None or numpy.nan, optional) – the seed of the random number generator. If None, a random seed is used. If numpy.nan, the actual seed will be kept. Default: numpy.nan

property mode_no
Number of modes in the randomization method.

Type int

property model
Covariance model of the spatial random field.

Type *CovModel*

property name
Name of the generator.

Type str

property sampling
Sampling strategy.

Type str

property seed
Seed of the master RNG.

Notes

If a new seed is given, the setter property not only saves the new seed, but also creates new random modes with the new seed.

Type int

property value_type
Type of the field values (scalar, vector).

Type str

property verbose
Verbosity of the generator.

Type bool

class gstools.field.generator.**RandMeth**(*model*, *mode_no=1000*, *seed=None*, *verbose=False*, *sampling='auto'*, ***kwargs*)

Bases: object

Randomization method for calculating isotropic random fields.

Parameters

- **model** (*CovModel*) – Covariance model
- **mode_no** (int, optional) – Number of Fourier modes. Default: 1000
- **seed** (int or None, optional) – The seed of the random number generator. If "None", a random seed is used. Default: None
- **verbose** (bool, optional) – Be chatty during the generation. Default: False
- **sampling** (str, optional) – Sampling strategy. Either
  - "auto": select best strategy depending on given model

  - "inversion": use inversion method
  - "mcmc": use mcmc sampling
- ****kwargs** – Placeholder for keyword-args

Notes

The Randomization method is used to generate isotropic spatial random fields characterized by a given covariance model. The calculation looks like [Hesse2014]:

$$u(x) = \sqrt{\frac{\sigma^2}{N}} \cdot \sum_{i=1}^{N} \left(Z_{1,i} \cdot \cos\left(\langle k_i, x \rangle\right) + Z_{2,i} \cdot \sin\left(\langle k_i, x \rangle\right) \right)$$

where:

- N : fourier mode number
- $Z_{j,i}$: random samples from a normal distribution
- k_i : samples from the spectral density distribution of the covariance model

---

References

Attributes

mode_no `int`: Number of modes in the randomization method.

model *CovModel*: Covariance model of the spatial random field.

name `str`: Name of the generator.

sampling `str`: Sampling strategy.

seed `int`: Seed of the master RNG.

value_type `str`: Type of the field values (scalar, vector).

verbose `bool`: Verbosity of the generator.

Methods

__call__(pos[, add_nugget])	Calculate the random modes for the randomization method.
get_nugget(shape)	Generate normal distributed values for the nugget simulation.
reset_seed([seed])	Recalculate the random amplitudes and wave numbers with the given seed.
update([model, seed])	Update the model and the seed.

__call__(*pos*, *add_nugget=True*)

Calculate the random modes for the randomization method.

This method calls the *summate_** Cython methods, which are the heart of the randomization method.

Parameters

- **pos** ((d, n), `numpy.ndarray`) – the position tuple with d dimensions and n points.
- **add_nugget** (`bool`) – Whether to add nugget noise to the field.

Returns the random modes

Return type numpy.ndarray

get_nugget(*shape*)
Generate normal distributed values for the nugget simulation.

Parameters **shape** (tuple) – the shape of the summed modes

Returns **nugget** – the nugget in the same shape as the summed modes

Return type numpy.ndarray

reset_seed(*seed=nan*)
Recalculate the random amplitudes and wave numbers with the given seed.

Parameters **seed** (int or None or numpy.nan, optional) – the seed of the random number generator. If None, a random seed is used. If numpy.nan, the actual seed will be kept. Default: numpy.nan

Notes

Even if the given seed is the present one, modes will be recalculated.

update(*model=None*, *seed=nan*)
Update the model and the seed.

If model and seed are not different, nothing will be done.

Parameters

- **model** (*CovModel* or None, optional) – covariance model. Default: None
- **seed** (int or None or numpy.nan, optional) – the seed of the random number generator. If None, a random seed is used. If numpy.nan, the actual seed will be kept. Default: numpy.nan

property mode_no
Number of modes in the randomization method.

Type int

property model
Covariance model of the spatial random field.

Type *CovModel*

property name
Name of the generator.

Type str

property sampling
Sampling strategy.

Type str

property seed
Seed of the master RNG.

Notes

If a new seed is given, the setter property not only saves the new seed, but also creates new random modes with the new seed.

Type int

property value_type
Type of the field values (scalar, vector).

Type str

property verbose
Verbosity of the generator.

Type bool

gstools.field.upscaling

GStools subpackage providing upscaling routines for the spatial random field.

The following functions are provided

var_coarse_graining(model[, point_volumes])	Coarse Graning procedure to upscale the variance for uniform flow.
var_no_scaling(model, *args, **kwargs)	Dummy function to bypass scaling.

gstools.field.upscaling.**var_coarse_graining**(*model*, *point_volumes=0.0*)

Coarse Graning procedure to upscale the variance for uniform flow.

Parameters

- **model** (*CovModel*) – Covariance Model used for the field.
- **point_volumes** (float or numpy.ndarray) – Volumes of the elements at the given points. Default: 0

Returns **scaled_var** – The upscaled variance

Return type float or numpy.ndarray

Notes

This procedure was presented in [Attinger03]. It applies the upscaling procedure 'Coarse Graining' to the Groundwater flow equation under uniform flow on a lognormal distributed conductivity field following a gaussian covariance function. A filter over a cube with a given edge-length λ is applied and an upscaled conductivity field is obtained. The upscaled field is again following a gaussian covariance function with scale dependent variance and length-scale:

$$\lambda = V^{\frac{1}{d}}$$

$$\sigma^2(\lambda) = \sigma^2 \cdot \left(\frac{\ell^2}{\ell^2 + \left(\frac{\lambda}{2}\right)^2} \right)^{\frac{d}{2}}$$

$$\ell(\lambda) = \left(\ell^2 + \left(\frac{\lambda}{2}\right)^2 \right)^{\frac{1}{2}}$$

Therby λ will be calculated from the given point_volumes V by assuming a cube with the given volume.

The upscaled length scale will be ignored by this routine.

References

gstools.field.upscaling.**var_no_scaling**(*model*, **args*, ***kwargs*)

Dummy function to bypass scaling.

Parameters **model** (*CovModel*) – Covariance Model used for the field.

Returns **var** – The model variance.

Return type float

3.8 gstools.variogram

GStools subpackage providing tools for estimating and fitting variograms.

Variogram estimation

vario_estimate(pos, field[, bin_edges, ...])	Estimates the empirical variogram.
vario_estimate_axis(field[, direction, ...])	Estimates the variogram along array axis.

gstools.variogram.vario_estimate

gstools.variogram.**vario_estimate**(*pos*, *field*, *bin_edges=None*, *sampling_size=None*, *sampling_seed=None*, *estimator='matheron'*, *latlon=False*, *direction=None*, *angles=None*, *angles_tol=0.39269908169872414*, *bandwidth=None*, *no_data=nan*, *mask=False*, *mesh_type='unstructured'*, *return_counts=False*, *mean=None*, *normalizer=None*, *trend=None*, *fit_normalizer=False*)

Estimates the empirical variogram.

The algorithm calculates following equation:

$$\gamma(r_k) = \frac{1}{2N(r_k)} \sum_{i=1}^{N(r_k)} (z(\mathbf{x}_i) - z(\mathbf{x}_i'))^2 \ ,$$

with $r_k \leq \|\mathbf{x}_i - \mathbf{x}_i'\| < r_{k+1}$ being the bins.

Or if the estimator "cressie" was chosen:

$$\gamma(r_k) = \frac{\frac{1}{2}\left(\frac{1}{N(r_k)} \sum_{i=1}^{N(r_k)} |z(\mathbf{x}_i) - z(\mathbf{x}_i')|^{0.5}\right)^4}{0.457 + 0.494/N(r_k) + 0.045/N^2(r_k)} \ ,$$

with $r_k \leq \|\mathbf{x}_i - \mathbf{x}_i'\| < r_{k+1}$ being the bins. The Cressie estimator is more robust to outliers [Webster2007].

By provding *direction* vector[s] or angles, a directional variogram can be calculated. If multiple directions are given, a set of variograms will be returned. Directional bining is controled by a given angle tolerance (*angles_tol*) and an optional *bandwidth*, that truncates the width of the search band around the given direction[s].

To reduce the calcuation time, *sampling_size* could be passed to sample down the number of field points.

Parameters

- **pos** (list) – the position tuple, containing either the point coordinates (x, y, ...) or the axes descriptions (for mesh_type='structured')
- **field** (numpy.ndarray or list of numpy.ndarray) – The spatially distributed data. Can also be of type numpy.ma.MaskedArray to use masked values. You can pass a list of fields, that will be used simultaneously. This could be helpful, when there are multiple realizations at the same points, with the same statistical properties.
- **bin_edges** (numpy.ndarray, optional) – the bins on which the variogram will be calculated. If None are given, standard bins provided by the *standard_bins* routine will be used. Default: None
- **sampling_size** (int or None, optional) – for large input data, this method can take a long time to compute the variogram, therefore this argument specifies the number of data points to sample randomly Default: None
- **sampling_seed** (int or None, optional) – seed for samples if sampling_size is given. Default: None

- **estimator** (str, optional) – the estimator function, possible choices:
  - "matheron": the standard method of moments of Matheron
  - "cressie": an estimator more robust to outliers

  Default: "matheron"
- **latlon** (bool, optional) – Whether the data is representing 2D fields on earths surface described by latitude and longitude. When using this, the estimator will use great-circle distance for variogram estimation. Note, that only an isotropic variogram can be estimated and a ValueError will be raised, if a direction was specified. Bin edges need to be given in radians in this case. Default: False
- **direction** (list of numpy.ndarray, optional) – directions to evaluate a directional variogram. Anglular tolerance is given by *angles_tol*. bandwidth to cut off how wide the search for point pairs should be is given by *bandwidth*. You can provide multiple directions at once to get one variogram for each direction. For a single direction you can also use the *angles* parameter, to provide the direction by its spherical coordianates. Default: None
- **angles** (numpy.ndarray, optional) – the angles of the main axis to calculate the variogram for in radians angle definitions from ISO standard 80000-2:2009 for 1d this parameter will have no effect at all for 2d supply one angle which is azimuth φ (ccw from +x in xy plane) for 3d supply two angles which are azimuth φ (ccw from +x in xy plane) and inclination θ (cw from +z). Can be used instead of direction. Default: None
- **angles_tol** (class:*float*, optional) – the tolerance around the variogram angle to count a point as being within this direction from another point (the angular tolerance around the directional vector given by angles) Default: *np.pi/8* = 22.5°
- **bandwidth** (class:*float*, optional) – bandwidth to cut off the angular tolerance for directional variograms. If None is given, only the *angles_tol* parameter will control the point selection. Default: None
- **no_data** (float, optional) – Value to identify missing data in the given field. Default: *numpy.nan*
- **mask** (numpy.ndarray of bool, optional) – Mask to deselect data in the given field. Default: numpy.ma.nomask
- **mesh_type** (str, optional) – 'structured' / 'unstructured', indicates whether the pos tuple describes the axis or the point coordinates. Default: *'unstructured'*
- **return_counts** (bool, optional) – if set to true, this function will also return the number of data points found at each lag distance as a third return value Default: False
- **mean** (float, optional) – mean value used to shift normalized input data. Can also be a callable. The default is None.
- **normalizer** (None or *Normalizer*, optional) – Normalizer to be applied to the input data to gain normality. The default is None.
- **trend** (None or float or callable, optional) – A callable trend function. Should have the signiture: f(x, [y, z, ...]) If no normalizer is applied, this behaves equal to 'mean'. The default is None.
- **fit_normalizer** (bool, optional) – Wheater to fit the data-normalizer to the given (detrended) field. Default: False

Returns

- **bin_center** ((n), numpy.ndarray) – The bin centers.
- **gamma** ((n) or (d, n), numpy.ndarray) – The estimated variogram values at bin centers. Is stacked if multiple *directions* (d>1) are given.

- **counts** ((n) or (d, n), numpy.ndarray, optional) – The number of point pairs found for each bin. Is stacked if multiple *directions* (d>1) are given. Only provided if *return_counts* is True.
- **normalizer** (*Normalizer*, optional) – The fitted normalizer for the given data. Only provided if *fit_normalizer* is True.

Notes

Internally uses double precision and also returns doubles.

References

gstools.variogram.vario_estimate_axis

gstools.variogram.**vario_estimate_axis**(*field*, *direction='x'*, *estimator='matheron'*, *no_data=nan*)

Estimates the variogram along array axis.

The indices of the given direction are used for the bins. Uniform spacings along the given axis are assumed.

The algorithm calculates following equation:

$$\gamma(r_k) = \frac{1}{2N(r_k)} \sum_{i=1}^{N(r_k)} (z(\mathbf{x}_i) - z(\mathbf{x}'_i))^2 \ ,$$

with $r_k \leq \|\mathbf{x}_i - \mathbf{x}'_i\| < r_{k+1}$ being the bins.

Or if the estimator "cressie" was chosen:

$$\gamma(r_k) = \frac{\frac{1}{2}\left(\frac{1}{N(r_k)} \sum_{i=1}^{N(r_k)} |z(\mathbf{x}_i) - z(\mathbf{x}'_i)|^{0.5}\right)^4}{0.457 + 0.494/N(r_k) + 0.045/N^2(r_k)} \ ,$$

with $r_k \leq \|\mathbf{x}_i - \mathbf{x}'_i\| < r_{k+1}$ being the bins. The Cressie estimator is more robust to outliers [Webster2007].

Parameters

- **field** (numpy.ndarray or numpy.ma.MaskedArray) – the spatially distributed data (can be masked)
- **direction** (str or int) – the axis over which the variogram will be estimated (x, y, z) or (0, 1, 2, ...)
- **estimator** (str, optional) – the estimator function, possible choices:
  - "matheron": the standard method of moments of Matheron
  - "cressie": an estimator more robust to outliers

  Default: "matheron"
- **no_data** (float, optional) – Value to identify missing data in the given field. Default: *numpy.nan*

Returns the estimated variogram along the given direction.

Return type numpy.ndarray

Warning: It is assumed that the field is defined on an equidistant Cartesian grid.

Notes

Internally uses double precision and also returns doubles.

References

Binning

standard_bins([pos, dim, latlon, mesh_type, ...])	Get standard binning.

gstools.variogram.standard_bins

gstools.variogram.**standard_bins**(*pos=None*, *dim=2*, *latlon=False*, *mesh_type='unstructured'*, *bin_no=None*, *max_dist=None*)

Get standard binning.

Parameters

- **pos** (list, optional) – the position tuple, containing either the point coordinates (x, y, ...) or the axes descriptions (for mesh_type='structured')
- **dim** (int, optional) – Field dimension.
- **latlon** (bool, optional) – Whether the data is representing 2D fields on earths surface described by latitude and longitude. When using this, the estimator will use great-circle distance for variogram estimation. Note, that only an isotropic variogram can be estimated and a ValueError will be raised, if a direction was specified. Bin edges need to be given in radians in this case. Default: False
- **mesh_type** (str, optional) – 'structured' / 'unstructured', indicates whether the pos tuple describes the axis or the point coordinates. Default: *'unstructured'*
- **bin_no** (int, optional) – number of bins to create. If None is given, will be determined by Sturges' rule from the number of points. Default: None
- **max_dist** (float, optional) – Cut of length for the bins. If None is given, it will be set to one third of the box-diameter from the given points. Default: None

Returns The generated bin edges.

Return type numpy.ndarray

Notes

Internally uses double precision and also returns doubles.

3.9 gstools.krige

GStools subpackage providing kriging.

Kriging Classes

Krige(model, cond_pos, cond_val[, ...])	A Swiss Army knife for kriging.
Simple(model, cond_pos, cond_val[, mean, ...])	Simple kriging.
Ordinary(model, cond_pos, cond_val[, ...])	Ordinary kriging.
Universal(model, cond_pos, cond_val, ...[, ...])	Universal kriging.
ExtDrift(model, cond_pos, cond_val, ext_drift)	External drift kriging (EDK).
Detrended(model, cond_pos, cond_val, trend)	Detrended simple kriging.

gstools.krige.Krige

class gstools.krige.**Krige**(*model*, *cond_pos*, *cond_val*, *drift_functions=None*, *ext_drift=None*, *mean=None*, *normalizer=None*, *trend=None*, *unbiased=True*, *exact=False*, *cond_err='nugget'*, *pseudo_inv=True*, *pseudo_inv_type='pinv'*, *fit_normalizer=False*, *fit_variogram=False*)

Bases: *gstools.field.base.Field*

A Swiss Army knife for kriging.

A Kriging class enabling the basic kriging routines: Simple-, Ordinary-, Univseral-, External Drift- and detrended/regression-Kriging as well as Kriging the Mean [Wackernagel2003].

Parameters

- **model** (*CovModel*) – Covariance Model used for kriging.
- **cond_pos** (list) – tuple, containing the given condition positions (x, [y, z])
- **cond_val** (numpy.ndarray) – the values of the conditions
- **drift_functions** (list of callable, str or int) – Either a list of callable functions, an integer representing the polynomial order of the drift or one of the following strings:
  - "linear" : regional linear drift (equals order=1)
  - "quadratic" : regional quadratic drift (equals order=2)
- **ext_drift** (numpy.ndarray or None, optional) – the external drift values at the given cond. positions.
- **mean** (float, optional) – mean value used to shift normalized conditioning data. Could also be a callable. The default is None.
- **normalizer** (None or *Normalizer*, optional) – Normalizer to be applied to the input data to gain normality. The default is None.
- **trend** (None or float or callable, optional) – A callable trend function. Should have the signiture: f(x, [y, z, ...]) This is used for detrended kriging, where the trended is subtracted from the conditions before kriging is applied. This can be used for regression kriging, where the trend function is determined by an external regression algorithm. If no normalizer is applied, this behaves equal to 'mean'. The default is None.
- **unbiased** (bool, optional) – Whether the kriging weights should sum up to 1, so the estimator is unbiased. If unbiased is *False* and no drifts are given, this results in simple kriging. Default: True

- **exact** (bool, optional) – Whether the interpolator should reproduce the exact input values. If *False*, *cond_err* is interpreted as measurement error at the conditioning points and the result will be more smooth. Default: False
- **cond_err** (str, :class float or list, optional) – The measurement error at the conditioning points. Either "nugget" to apply the model-nugget, a single value applied to all points or an array with individual values for each point. The "exact=True" variant only works with "cond_err='nugget'". Default: "nugget"
- **pseudo_inv** (bool, optional) – Whether the kriging system is solved with the pseudo inverted kriging matrix. If *True*, this leads to more numerical stability and redundant points are averaged. But it can take more time. Default: True
- **pseudo_inv_type** (str or callable, optional) – Here you can select the algorithm to compute the pseudo-inverse matrix:
  - *"pinv"*: use *pinv* from *scipy* which uses *lstsq*
  - *"pinv2"*: use *pinv2* from *scipy* which uses *SVD*
  - *"pinvh"*: use *pinvh* from *scipy* which uses eigen-values

  If you want to use another routine to invert the kriging matrix, you can pass a callable which takes a matrix and returns the inverse. Default: *"pinv"*
- **fit_normalizer** (bool, optional) – Wheater to fit the data-normalizer to the given conditioning data. Default: False
- **fit_variogram** (bool, optional) – Wheater to fit the given variogram model to the data. This is done by using isotropy settings of the given model, assuming the sill to be the data variance and with the standard bins provided by the *standard_bins* routine. Default: False

Notes

If you have changed any properties in the class, you can update the kriging setup by calling *Krige.set_condition* without any arguments.

References

Attributes

cond_err list: The measurement errors at the condition points.

cond_ext_drift numpy.ndarray: The ext. drift at the conditions.

cond_mean numpy.ndarray: Trend at the conditions.

cond_no int: The number of the conditions.

cond_pos list: The position tuple of the conditions.

cond_trend numpy.ndarray: Trend at the conditions.

cond_val list: The values of the conditions.

dim int: Dimension of the field.

drift_functions list of callable: The drift functions.

drift_no int: Number of drift values per point.

exact bool: Whether the interpolator is exact.

ext_drift_no int: Number of external drift values per point.

has_const_mean bool: Whether the field has a constant mean or not.

int_drift_no `int`: Number of internal drift values per point.

krige_size `int`: Size of the kriging system.

latlon `bool`: Whether the field depends on geographical coords.

mean `float` or `callable`: The mean of the field.

model *CovModel*: The covariance model of the field.

name `str`: The name of the kriging class.

normalizer *Normalizer*: Normalizer of the field.

pseudo_inv `bool`: Whether pseudo inverse matrix is used.

pseudo_inv_type `str`: Method selector for pseudo inverse calculation.

trend `float` or `callable`: The trend of the field.

unbiased `bool`: Whether the kriging is unbiased or not.

value_type `str`: Type of the field values (scalar, vector).

Methods

__call__(pos[, mesh_type, ext_drift, ...])	Generate the kriging field.
get_mean([post_process])	Calculate the estimated mean of the detrended field.
mesh(mesh[, points, direction, name])	Generate a field on a given meshio, ogs5py or PyVista mesh.
plot([field, fig, ax])	Plot the spatial random field.
post_field(field[, name, process, save])	Postprocessing field values.
pre_pos(pos[, mesh_type])	Preprocessing positions and mesh_type.
set_condition([cond_pos, cond_val, ...])	Set the conditions for kriging.
set_drift_functions([drift_functions])	Set the drift functions for universal kriging.
structured(*args, **kwargs)	Generate a field on a structured mesh.
to_pyvista([field_select, fieldname])	Create a VTK/PyVista grid of the stored field.
unstructured(*args, **kwargs)	Generate a field on an unstructured mesh.
vtk_export(filename[, field_select, fieldname])	Export the stored field to vtk.

__call__(*pos*, *mesh_type='unstructured'*, *ext_drift=None*, *chunk_size=None*, *only_mean=False*, *return_var=True*, *post_process=True*)

Generate the kriging field.

The field is saved as *self.field* and is also returned. The error variance is saved as *self.krige_var* and is also returned.

Parameters

- **pos** (`list`) – the position tuple, containing main direction and transversal directions (x, [y, z])
- **mesh_type** (`str`, optional) – 'structured' / 'unstructured'
- **ext_drift** (`numpy.ndarray` or `None`, optional) – the external drift values at the given positions (only for EDK)
- **chunk_size** (`int`, optional) – Chunk size to cut down the size of the kriging system to prevent memory errors. Default: None
- **only_mean** (`bool`, optional) – Whether to only calculate the mean of the kriging field. Default: *False*
- **return_var** (`bool`, optional) – Whether to return the variance along with the field. Default: *True*

- **post_process** (bool, optional) – Whether to apply mean, normalizer and trend to the field. Default: *True*

Returns

- **field** (numpy.ndarray) – the kriged field or mean_field
- **krige_var** (numpy.ndarray, optional) – the kriging error variance (if return_var is True and only_mean is False)

get_mean(*post_process=True*)
Calculate the estimated mean of the detrended field.

Parameters **post_process** (bool, optional) – Whether to apply field-mean and normalizer. Default: *True*

Returns **mean** – Mean of the Kriging System.

Return type float or None

Notes

Only not None if the Kriging System has a constant mean. This means, no drift is given and the given field-mean is constant. The result is neglecting a potential given trend.

---

mesh(*mesh*, *points='centroids'*, *direction='all'*, *name='field'*, ***kwargs*)
Generate a field on a given meshio, ogs5py or PyVista mesh.

Parameters

- **mesh** (*meshio.Mesh or ogs5py.MSH or PyVista mesh*) – The given mesh
- **points** (str, optional) – The points to evaluate the field at. Either the "centroids" of the mesh cells (calculated as mean of the cell vertices) or the "points" of the given mesh. Default: "centroids"
- **direction** (str or list, optional) – Here you can state which direction should be choosen for lower dimension. For example, if you got a 2D mesh in xz direction, you have to pass "xz". By default, all directions are used. One can also pass a list of indices. Default: "all"
- **name** (str or list of str, optional) – Name(s) to store the field(s) in the given mesh as point_data or cell_data. If to few names are given, digits will be appended. Default: "field"
- ****kwargs** – Keyword arguments forwarded to *__call__*.

Notes

This will store the field in the given mesh under the given name, if a meshio or PyVista mesh was given.

See:

- meshio: https://github.com/nschloe/meshio
- ogs5py: https://github.com/GeoStat-Framework/ogs5py
- PyVista: https://github.com/pyvista/pyvista

---

plot(*field='field'*, *fig=None*, *ax=None*, ***kwargs*)
Plot the spatial random field.

Parameters

- **field** (str, optional) – Field that should be plotted. Default: "field"

- **fig** (Figure or None) – Figure to plot the axes on. If *None*, a new one will be created. Default: *None*
- **ax** (Axes or None) – Axes to plot on. If *None*, a new one will be added to the figure. Default: *None*
- ****kwargs** – Forwarded to the plotting routine.

post_field(*field*, *name='field'*, *process=True*, *save=True*)
Postprocessing field values.

Parameters

- **field** (numpy.ndarray) – Field values.
- **name** (str, optional) – Name. to store the field. The default is "field".
- **process** (bool, optional) – Whether to process field to apply mean, normalizer and trend. The default is True.
- **save** (bool, optional) – Whether to store the field under the given name. The default is True.

Returns **field** – Processed field values.

Return type numpy.ndarray

pre_pos(*pos*, *mesh_type='unstructured'*)
Preprocessing positions and mesh_type.

Parameters

- **pos** (iterable) – the position tuple, containing main direction and transversal directions
- **mesh_type** (str, optional) – 'structured' / 'unstructured' Default: *"unstructured"*

Returns

- **iso_pos** ((d, n), numpy.ndarray) – the isometrized position tuple
- **shape** (tuple) – Shape of the resulting field.

set_condition(*cond_pos=None*, *cond_val=None*, *ext_drift=None*, *cond_err=None*, *fit_normalizer=False*, *fit_variogram=False*)
Set the conditions for kriging.

This method could also be used to update the kriging setup, when properties were changed. Then you can call it without arguments.

Parameters

- **cond_pos** (list, optional) – the position tuple of the conditions (x, [y, z]). Default: current.
- **cond_val** (numpy.ndarray, optional) – the values of the conditions. Default: current.
- **ext_drift** (numpy.ndarray or None, optional) – the external drift values at the given conditions (only for EDK) For multiple external drifts, the first dimension should be the index of the drift term. When passing *None*, the extisting external drift will be used.
- **cond_err** (str, :class float, list, optional) – The measurement error at the conditioning points. Either "nugget" to apply the model-nugget, a single value applied to all points or an array with individual values for each point. The measurement error has to be <= nugget. The "exact=True" variant only works with "cond_err='nugget'". Default: "nugget"
- **fit_normalizer** (bool, optional) – Wheater to fit the data-normalizer to the given conditioning data. Default: False

- **fit_variogram** (bool, optional) – Wheater to fit the given variogram model to the data. This is done by using isotropy settings of the given model, assuming the sill to be the data variance and with the standard bins provided by the *standard_bins* routine. Default: False

set_drift_functions(*drift_functions=None*)
Set the drift functions for universal kriging.

Parameters **drift_functions** (list of callable, str or int) – Either a list of callable functions, an integer representing the polynomial order of the drift or one of the following strings:

- "linear" : regional linear drift (equals order=1)
- "quadratic" : regional quadratic drift (equals order=2)

Raises ValueError – If the given drift functions are not callable.

structured(**args*, ***kwargs*)
Generate a field on a structured mesh.

See *__call__*

to_pyvista(*field_select='field'*, *fieldname='field'*)
Create a VTK/PyVista grid of the stored field.

Parameters

- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

unstructured(**args*, ***kwargs*)
Generate a field on an unstructured mesh.

See *__call__*

vtk_export(*filename*, *field_select='field'*, *fieldname='field'*)
Export the stored field to vtk.

Parameters

- **filename** (str) – Filename of the file to be saved, including the path. Note that an ending (.vtr or .vtu) will be added to the name.
- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

property cond_err
The measurement errors at the condition points.

Type list

property cond_ext_drift
The ext. drift at the conditions.

Type numpy.ndarray

property cond_mean
Trend at the conditions.

Type numpy.ndarray

property cond_no
The number of the conditions.

Type int

property cond_pos
The position tuple of the conditions.

Type list

property cond_trend
Trend at the conditions.

Type numpy.ndarray

property cond_val
The values of the conditions.

Type list

property dim
Dimension of the field.

Type int

property drift_functions
The drift functions.

Type list of callable

property drift_no
Number of drift values per point.

Type int

property exact
Whether the interpolator is exact.

Type bool

property ext_drift_no
Number of external drift values per point.

Type int

property has_const_mean
Whether the field has a constant mean or not.

Type bool

property int_drift_no
Number of internal drift values per point.

Type int

property krige_size
Size of the kriging system.

Type int

property latlon
Whether the field depends on geographical coords.

Type bool

property mean
The mean of the field.

Type float or callable

property model
The covariance model of the field.

Type *CovModel*

property name
The name of the kriging class.

Type str

property normalizer
Normalizer of the field.

Type *Normalizer*

property pseudo_inv
Whether pseudo inverse matrix is used.

Type bool

property pseudo_inv_type
Method selector for pseudo inverse calculation.

Type str

property trend
The trend of the field.

Type float or callable

property unbiased
Whether the kriging is unbiased or not.

Type bool

property value_type
Type of the field values (scalar, vector).

Type str

gstools.krige.Simple

class gstools.krige.**Simple**(*model*, *cond_pos*, *cond_val*, *mean=0.0*, *normalizer=None*, *trend=None*, *exact=False*, *cond_err='nugget'*, *pseudo_inv=True*, *pseudo_inv_type='pinv'*, *fit_normalizer=False*, *fit_variogram=False*)

Bases: *gstools.krige.base.Krige*

Simple kriging.

Simple kriging is used to interpolate data with a given mean.

Parameters

- **model** (*CovModel*) – Covariance Model used for kriging.
- **cond_pos** (list) – tuple, containing the given condition positions (x, [y, z])
- **cond_val** (numpy.ndarray) – the values of the conditions
- **mean** (float, optional) – mean value used to shift normalized conditioning data. Could also be a callable. The default is None.
- **normalizer** (None or *Normalizer*, optional) – Normalizer to be applied to the input data to gain normality. The default is None.
- **trend** (None or float or callable, optional) – A callable trend function. Should have the signiture: f(x, [y, z, ...]) This is used for detrended kriging, where the trended is subtracted from the conditions before kriging is applied. This can be used for regression kriging, where the trend function is determined by an external regression algorithm. If no normalizer is applied, this behaves equal to 'mean'. The default is None.
- **exact** (bool, optional) – Whether the interpolator should reproduce the exact input values. If *False*, *cond_err* is interpreted as measurement error at the conditioning points and the result will be more smooth. Default: False
- **cond_err** (str, :class float or list, optional) – The measurement error at the conditioning points. Either "nugget" to apply the model-nugget, a single value applied to all points or an array with individual values for each point. The measurement error has to be <= nugget. The "exact=True" variant only works with "cond_err='nugget'". Default: "nugget"
- **pseudo_inv** (bool, optional) – Whether the kriging system is solved with the pseudo inverted kriging matrix. If *True*, this leads to more numerical stability and redundant points are averaged. But it can take more time. Default: True
- **pseudo_inv_type** (str or callable, optional) – Here you can select the algorithm to compute the pseudo-inverse matrix:
  - *"pinv"*: use *pinv* from *scipy* which uses *lstsq*
  - *"pinv2"*: use *pinv2* from *scipy* which uses *SVD*
  - *"pinvh"*: use *pinvh* from *scipy* which uses eigen-values

  If you want to use another routine to invert the kriging matrix, you can pass a callable which takes a matrix and returns the inverse. Default: *"pinv"*
- **fit_normalizer** (bool, optional) – Wheater to fit the data-normalizer to the given conditioning data. Default: False
- **fit_variogram** (bool, optional) – Wheater to fit the given variogram model to the data. This is done by using isotropy settings of the given model, assuming the sill to be the data variance and with the standard bins provided by the *standard_bins* routine. Default: False

Attributes

cond_err list: The measurement errors at the condition points.

cond_ext_drift `numpy.ndarray`: The ext. drift at the conditions.

cond_mean `numpy.ndarray`: Trend at the conditions.

cond_no `int`: The number of the conditions.

cond_pos `list`: The position tuple of the conditions.

cond_trend `numpy.ndarray`: Trend at the conditions.

cond_val `list`: The values of the conditions.

dim `int`: Dimension of the field.

drift_functions `list` of `callable`: The drift functions.

drift_no `int`: Number of drift values per point.

exact `bool`: Whether the interpolator is exact.

ext_drift_no `int`: Number of external drift values per point.

has_const_mean `bool`: Whether the field has a constant mean or not.

int_drift_no `int`: Number of internal drift values per point.

krige_size `int`: Size of the kriging system.

latlon `bool`: Whether the field depends on geographical coords.

mean `float` or `callable`: The mean of the field.

model *CovModel*: The covariance model of the field.

name `str`: The name of the kriging class.

normalizer *Normalizer*: Normalizer of the field.

pseudo_inv `bool`: Whether pseudo inverse matrix is used.

pseudo_inv_type `str`: Method selector for pseudo inverse calculation.

trend `float` or `callable`: The trend of the field.

unbiased `bool`: Whether the kriging is unbiased or not.

value_type `str`: Type of the field values (scalar, vector).

Methods

__call__(pos[, mesh_type, ext_drift, ...])	Generate the kriging field.
get_mean([post_process])	Calculate the estimated mean of the detrended field.
mesh(mesh[, points, direction, name])	Generate a field on a given meshio, ogs5py or PyVista mesh.
plot([field, fig, ax])	Plot the spatial random field.
post_field(field[, name, process, save])	Postprocessing field values.
pre_pos(pos[, mesh_type])	Preprocessing positions and mesh_type.
set_condition([cond_pos, cond_val, ...])	Set the conditions for kriging.
set_drift_functions([drift_functions])	Set the drift functions for universal kriging.
structured(*args, **kwargs)	Generate a field on a structured mesh.
to_pyvista([field_select, fieldname])	Create a VTK/PyVista grid of the stored field.
unstructured(*args, **kwargs)	Generate a field on an unstructured mesh.
vtk_export(filename[, field_select, fieldname])	Export the stored field to vtk.

__call__(*pos*, *mesh_type='unstructured'*, *ext_drift=None*, *chunk_size=None*, *only_mean=False*, *return_var=True*, *post_process=True*)

Generate the kriging field.

The field is saved as *self.field* and is also returned. The error variance is saved as *self.krige_var* and is also returned.

Parameters

- **pos** (list) – the position tuple, containing main direction and transversal directions (x, [y, z])
- **mesh_type** (str, optional) – 'structured' / 'unstructured'
- **ext_drift** (numpy.ndarray or None, optional) – the external drift values at the given positions (only for EDK)
- **chunk_size** (int, optional) – Chunk size to cut down the size of the kriging system to prevent memory errors. Default: None
- **only_mean** (bool, optional) – Whether to only calculate the mean of the kriging field. Default: *False*
- **return_var** (bool, optional) – Whether to return the variance along with the field. Default: *True*
- **post_process** (bool, optional) – Whether to apply mean, normalizer and trend to the field. Default: *True*

Returns

- **field** (numpy.ndarray) – the kriged field or mean_field
- **krige_var** (numpy.ndarray, optional) – the kriging error variance (if return_var is True and only_mean is False)

get_mean(*post_process=True*)

Calculate the estimated mean of the detrended field.

Parameters **post_process** (bool, optional) – Whether to apply field-mean and normalizer. Default: *True*

Returns **mean** – Mean of the Kriging System.

Return type float or None

Notes

Only not None if the Kriging System has a constant mean. This means, no drift is given and the given field-mean is constant. The result is neglecting a potential given trend.

---

mesh(*mesh*, *points='centroids'*, *direction='all'*, *name='field'*, ***kwargs*)

Generate a field on a given meshio, ogs5py or PyVista mesh.

Parameters

- **mesh** (*meshio.Mesh or ogs5py.MSH or PyVista mesh*) – The given mesh
- **points** (str, optional) – The points to evaluate the field at. Either the "centroids" of the mesh cells (calculated as mean of the cell vertices) or the "points" of the given mesh. Default: "centroids"
- **direction** (str or list, optional) – Here you can state which direction should be choosen for lower dimension. For example, if you got a 2D mesh in xz direction, you have to pass "xz". By default, all directions are used. One can also pass a list of indices. Default: "all"

- **name** (str or list of str, optional) – Name(s) to store the field(s) in the given mesh as point_data or cell_data. If to few names are given, digits will be appended. Default: "field"
- ****kwargs** – Keyword arguments forwarded to *__call__*.

Notes

This will store the field in the given mesh under the given name, if a meshio or PyVista mesh was given.

See:

- meshio: https://github.com/nschloe/meshio
- ogs5py: https://github.com/GeoStat-Framework/ogs5py
- PyVista: https://github.com/pyvista/pyvista

---

plot(*field='field'*, *fig=None*, *ax=None*, ***kwargs*)
Plot the spatial random field.

Parameters

- **field** (str, optional) – Field that should be plotted. Default: "field"
- **fig** (Figure or None) – Figure to plot the axes on. If *None*, a new one will be created. Default: *None*
- **ax** (Axes or None) – Axes to plot on. If *None*, a new one will be added to the figure. Default: *None*
- ****kwargs** – Forwarded to the plotting routine.

post_field(*field*, *name='field'*, *process=True*, *save=True*)
Postprocessing field values.

Parameters

- **field** (numpy.ndarray) – Field values.
- **name** (str, optional) – Name. to store the field. The default is "field".
- **process** (bool, optional) – Whether to process field to apply mean, normalizer and trend. The default is True.
- **save** (bool, optional) – Whether to store the field under the given name. The default is True.

Returns **field** – Processed field values.

Return type numpy.ndarray

pre_pos(*pos*, *mesh_type='unstructured'*)
Preprocessing positions and mesh_type.

Parameters

- **pos** (iterable) – the position tuple, containing main direction and transversal directions
- **mesh_type** (str, optional) – 'structured' / 'unstructured' Default: *"unstructured"*

Returns

- **iso_pos** ((d, n), numpy.ndarray) – the isometrized position tuple
- **shape** (tuple) – Shape of the resulting field.

set_condition(*cond_pos=None*, *cond_val=None*, *ext_drift=None*, *cond_err=None*, *fit_normalizer=False*, *fit_variogram=False*)
Set the conditions for kriging.

This method could also be used to update the kriging setup, when properties were changed. Then you can call it without arguments.

Parameters

- **cond_pos** (list, optional) – the position tuple of the conditions (x, [y, z]). Default: current.
- **cond_val** (numpy.ndarray, optional) – the values of the conditions. Default: current.
- **ext_drift** (numpy.ndarray or None, optional) – the external drift values at the given conditions (only for EDK) For multiple external drifts, the first dimension should be the index of the drift term. When passing *None*, the extisting external drift will be used.
- **cond_err** (str, :class float, list, optional) – The measurement error at the conditioning points. Either "nugget" to apply the model-nugget, a single value applied to all points or an array with individual values for each point. The measurement error has to be <= nugget. The "exact=True" variant only works with "cond_err='nugget'". Default: "nugget"
- **fit_normalizer** (bool, optional) – Wheater to fit the data-normalizer to the given conditioning data. Default: False
- **fit_variogram** (bool, optional) – Wheater to fit the given variogram model to the data. This is done by using isotropy settings of the given model, assuming the sill to be the data variance and with the standard bins provided by the *standard_bins* routine. Default: False

set_drift_functions(*drift_functions=None*)
Set the drift functions for universal kriging.

Parameters **drift_functions** (list of callable, str or int) – Either a list of callable functions, an integer representing the polynomial order of the drift or one of the following strings:

- "linear" : regional linear drift (equals order=1)
- "quadratic" : regional quadratic drift (equals order=2)

Raises ValueError – If the given drift functions are not callable.

structured(**args*, ***kwargs*)
Generate a field on a structured mesh.

See *__call__*

to_pyvista(*field_select='field'*, *fieldname='field'*)
Create a VTK/PyVista grid of the stored field.

Parameters

- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

unstructured(**args*, ***kwargs*)
Generate a field on an unstructured mesh.

See *__call__*

vtk_export(*filename*, *field_select='field'*, *fieldname='field'*)
Export the stored field to vtk.

Parameters

- **filename** (str) – Filename of the file to be saved, including the path. Note that an ending (.vtr or .vtu) will be added to the name.
- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

property cond_err
The measurement errors at the condition points.

Type list

property cond_ext_drift
The ext. drift at the conditions.

Type numpy.ndarray

property cond_mean
Trend at the conditions.

Type numpy.ndarray

property cond_no
The number of the conditions.

Type int

property cond_pos
The position tuple of the conditions.

Type list

property cond_trend
Trend at the conditions.

Type numpy.ndarray

property cond_val
The values of the conditions.

Type list

property dim
Dimension of the field.

Type int

property drift_functions
The drift functions.

Type list of callable

property drift_no
Number of drift values per point.

Type int

property exact
Whether the interpolator is exact.

Type bool

property ext_drift_no
Number of external drift values per point.

Type int

property has_const_mean
Whether the field has a constant mean or not.

Type bool

property int_drift_no
Number of internal drift values per point.

Type int

property krige_size
Size of the kriging system.

Type int

property latlon
Whether the field depends on geographical coords.

Type bool

property mean
The mean of the field.

Type float or callable

property model
The covariance model of the field.

Type *CovModel*

property name
The name of the kriging class.

Type str

property normalizer
Normalizer of the field.

Type *Normalizer*

property pseudo_inv
Whether pseudo inverse matrix is used.

Type bool

property pseudo_inv_type
Method selector for pseudo inverse calculation.

Type str

property trend
The trend of the field.

Type float or callable

property unbiased
Whether the kriging is unbiased or not.

Type bool

property value_type
Type of the field values (scalar, vector).

Type str

gstools.krige.Ordinary

class gstools.krige.**Ordinary**(*model*, *cond_pos*, *cond_val*, *normalizer=None*, *trend=None*, *exact=False*, *cond_err='nugget'*, *pseudo_inv=True*, *pseudo_inv_type='pinv'*, *fit_normalizer=False*, *fit_variogram=False*)

Bases: *gstools.krige.base.Krige*

Ordinary kriging.

Ordinary kriging is used to interpolate data and estimate a proper mean.

Parameters

- **model** (*CovModel*) – Covariance Model used for kriging.
- **cond_pos** (list) – tuple, containing the given condition positions (x, [y, z])
- **cond_val** (numpy.ndarray) – the values of the conditions
- **normalizer** (None or *Normalizer*, optional) – Normalizer to be applied to the input data to gain normality. The default is None.
- **trend** (None or float or callable, optional) – A callable trend function. Should have the signiture: f(x, [y, z, ...]) This is used for detrended kriging, where the trended is subtracted from the conditions before kriging is applied. This can be used for regression kriging, where the trend function is determined by an external regression algorithm. If no normalizer is applied, this behaves equal to 'mean'. The default is None.
- **exact** (bool, optional) – Whether the interpolator should reproduce the exact input values. If *False*, *cond_err* is interpreted as measurement error at the conditioning points and the result will be more smooth. Default: False
- **cond_err** (str, :class float or list, optional) – The measurement error at the conditioning points. Either "nugget" to apply the model-nugget, a single value applied to all points or an array with individual values for each point. The measurement error has to be <= nugget. The "exact=True" variant only works with "cond_err='nugget'". Default: "nugget"
- **pseudo_inv** (bool, optional) – Whether the kriging system is solved with the pseudo inverted kriging matrix. If *True*, this leads to more numerical stability and redundant points are averaged. But it can take more time. Default: True
- **pseudo_inv_type** (str or callable, optional) – Here you can select the algorithm to compute the pseudo-inverse matrix:
  - *"pinv"*: use *pinv* from *scipy* which uses *lstsq*
  - *"pinv2"*: use *pinv2* from *scipy* which uses *SVD*
  - *"pinvh"*: use *pinvh* from *scipy* which uses eigen-values

  If you want to use another routine to invert the kriging matrix, you can pass a callable which takes a matrix and returns the inverse. Default: *"pinv"*
- **fit_normalizer** (bool, optional) – Wheater to fit the data-normalizer to the given conditioning data. Default: False
- **fit_variogram** (bool, optional) – Wheater to fit the given variogram model to the data. This is done by using isotropy settings of the given model, assuming the sill to be the data variance and with the standard bins provided by the *standard_bins* routine. Default: False

Attributes

cond_err list: The measurement errors at the condition points.

cond_ext_drift numpy.ndarray: The ext. drift at the conditions.

cond_mean numpy.ndarray: Trend at the conditions.

cond_no `int`: The number of the conditions.

cond_pos `list`: The position tuple of the conditions.

cond_trend `numpy.ndarray`: Trend at the conditions.

cond_val `list`: The values of the conditions.

dim `int`: Dimension of the field.

drift_functions `list` of `callable`: The drift functions.

drift_no `int`: Number of drift values per point.

exact `bool`: Whether the interpolator is exact.

ext_drift_no `int`: Number of external drift values per point.

has_const_mean `bool`: Whether the field has a constant mean or not.

int_drift_no `int`: Number of internal drift values per point.

krige_size `int`: Size of the kriging system.

latlon `bool`: Whether the field depends on geographical coords.

mean `float` or `callable`: The mean of the field.

model *CovModel*: The covariance model of the field.

name `str`: The name of the kriging class.

normalizer *Normalizer*: Normalizer of the field.

pseudo_inv `bool`: Whether pseudo inverse matrix is used.

pseudo_inv_type `str`: Method selector for pseudo inverse calculation.

trend `float` or `callable`: The trend of the field.

unbiased `bool`: Whether the kriging is unbiased or not.

value_type `str`: Type of the field values (scalar, vector).

Methods

__call__(pos[, mesh_type, ext_drift, ...])	Generate the kriging field.
get_mean([post_process])	Calculate the estimated mean of the detrended field.
mesh(mesh[, points, direction, name])	Generate a field on a given meshio, ogs5py or PyVista mesh.
plot([field, fig, ax])	Plot the spatial random field.
post_field(field[, name, process, save])	Postprocessing field values.
pre_pos(pos[, mesh_type])	Preprocessing positions and mesh_type.
set_condition([cond_pos, cond_val, ...])	Set the conditions for kriging.
set_drift_functions([drift_functions])	Set the drift functions for universal kriging.
structured(*args, **kwargs)	Generate a field on a structured mesh.
to_pyvista([field_select, fieldname])	Create a VTK/PyVista grid of the stored field.
unstructured(*args, **kwargs)	Generate a field on an unstructured mesh.
vtk_export(filename[, field_select, fieldname])	Export the stored field to vtk.

__call__(*pos*, *mesh_type='unstructured'*, *ext_drift=None*, *chunk_size=None*, *only_mean=False*, *return_var=True*, *post_process=True*)

Generate the kriging field.

The field is saved as *self.field* and is also returned. The error variance is saved as *self.krige_var* and is also returned.

Parameters

- **pos** (list) – the position tuple, containing main direction and transversal directions (x, [y, z])
- **mesh_type** (str, optional) – 'structured' / 'unstructured'
- **ext_drift** (numpy.ndarray or None, optional) – the external drift values at the given positions (only for EDK)
- **chunk_size** (int, optional) – Chunk size to cut down the size of the kriging system to prevent memory errors. Default: None
- **only_mean** (bool, optional) – Whether to only calculate the mean of the kriging field. Default: *False*
- **return_var** (bool, optional) – Whether to return the variance along with the field. Default: *True*
- **post_process** (bool, optional) – Whether to apply mean, normalizer and trend to the field. Default: *True*

Returns

- **field** (numpy.ndarray) – the kriged field or mean_field
- **krige_var** (numpy.ndarray, optional) – the kriging error variance (if return_var is True and only_mean is False)

get_mean(*post_process=True*)

Calculate the estimated mean of the detrended field.

Parameters **post_process** (bool, optional) – Whether to apply field-mean and normalizer. Default: *True*

Returns **mean** – Mean of the Kriging System.

Return type float or None

Notes

Only not None if the Kriging System has a constant mean. This means, no drift is given and the given field-mean is constant. The result is neglecting a potential given trend.

---

mesh(*mesh*, *points='centroids'*, *direction='all'*, *name='field'*, ***kwargs*)

Generate a field on a given meshio, ogs5py or PyVista mesh.

Parameters

- **mesh** (*meshio.Mesh or ogs5py.MSH or PyVista mesh*) – The given mesh
- **points** (str, optional) – The points to evaluate the field at. Either the "centroids" of the mesh cells (calculated as mean of the cell vertices) or the "points" of the given mesh. Default: "centroids"
- **direction** (str or list, optional) – Here you can state which direction should be choosen for lower dimension. For example, if you got a 2D mesh in xz direction, you have to pass "xz". By default, all directions are used. One can also pass a list of indices. Default: "all"
- **name** (str or list of str, optional) – Name(s) to store the field(s) in the given mesh as point_data or cell_data. If to few names are given, digits will be appended. Default: "field"
- ****kwargs** – Keyword arguments forwarded to *__call__*.

Notes

This will store the field in the given mesh under the given name, if a meshio or PyVista mesh was given.

See:

- meshio: https://github.com/nschloe/meshio
- ogs5py: https://github.com/GeoStat-Framework/ogs5py
- PyVista: https://github.com/pyvista/pyvista

plot(*field='field'*, *fig=None*, *ax=None*, ***kwargs*)
Plot the spatial random field.

Parameters

- **field** (str, optional) – Field that should be plotted. Default: "field"
- **fig** (Figure or None) – Figure to plot the axes on. If *None*, a new one will be created. Default: *None*
- **ax** (Axes or None) – Axes to plot on. If *None*, a new one will be added to the figure. Default: *None*
- ****kwargs** – Forwarded to the plotting routine.

post_field(*field*, *name='field'*, *process=True*, *save=True*)
Postprocessing field values.

Parameters

- **field** (numpy.ndarray) – Field values.
- **name** (str, optional) – Name. to store the field. The default is "field".
- **process** (bool, optional) – Whether to process field to apply mean, normalizer and trend. The default is True.
- **save** (bool, optional) – Whether to store the field under the given name. The default is True.

Returns **field** – Processed field values.

Return type numpy.ndarray

pre_pos(*pos*, *mesh_type='unstructured'*)
Preprocessing positions and mesh_type.

Parameters

- **pos** (iterable) – the position tuple, containing main direction and transversal directions
- **mesh_type** (str, optional) – 'structured' / 'unstructured' Default: *"unstructured"*

Returns

- **iso_pos** ((d, n), numpy.ndarray) – the isometrized position tuple
- **shape** (tuple) – Shape of the resulting field.

set_condition(*cond_pos=None*, *cond_val=None*, *ext_drift=None*, *cond_err=None*, *fit_normalizer=False*, *fit_variogram=False*)
Set the conditions for kriging.

This method could also be used to update the kriging setup, when properties were changed. Then you can call it without arguments.

Parameters

- **cond_pos** (list, optional) – the position tuple of the conditions (x, [y, z]). Default: current.
- **cond_val** (numpy.ndarray, optional) – the values of the conditions. Default: current.
- **ext_drift** (numpy.ndarray or None, optional) – the external drift values at the given conditions (only for EDK) For multiple external drifts, the first dimension should be the index of the drift term. When passing *None*, the extisting external drift will be used.
- **cond_err** (str, :class float, list, optional) – The measurement error at the conditioning points. Either "nugget" to apply the model-nugget, a single value applied to all points or an array with individual values for each point. The measurement error has to be <= nugget. The "exact=True" variant only works with "cond_err='nugget'". Default: "nugget"
- **fit_normalizer** (bool, optional) – Wheater to fit the data-normalizer to the given conditioning data. Default: False
- **fit_variogram** (bool, optional) – Wheater to fit the given variogram model to the data. This is done by using isotropy settings of the given model, assuming the sill to be the data variance and with the standard bins provided by the *standard_bins* routine. Default: False

set_drift_functions(*drift_functions=None*)
Set the drift functions for universal kriging.

Parameters **drift_functions** (list of callable, str or int) – Either a list of callable functions, an integer representing the polynomial order of the drift or one of the following strings:

- "linear" : regional linear drift (equals order=1)
- "quadratic" : regional quadratic drift (equals order=2)

Raises ValueError – If the given drift functions are not callable.

structured(**args*, ***kwargs*)
Generate a field on a structured mesh.

See *__call__*

to_pyvista(*field_select='field'*, *fieldname='field'*)
Create a VTK/PyVista grid of the stored field.

Parameters

- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

unstructured(**args*, ***kwargs*)
Generate a field on an unstructured mesh.

See *__call__*

vtk_export(*filename*, *field_select='field'*, *fieldname='field'*)
Export the stored field to vtk.

Parameters

- **filename** (str) – Filename of the file to be saved, including the path. Note that an ending (.vtr or .vtu) will be added to the name.
- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

property cond_err
The measurement errors at the condition points.

Type list

property cond_ext_drift
The ext. drift at the conditions.

Type numpy.ndarray

property cond_mean
Trend at the conditions.

Type numpy.ndarray

property cond_no
The number of the conditions.

Type int

property cond_pos
The position tuple of the conditions.

Type list

property cond_trend
Trend at the conditions.

Type numpy.ndarray

property cond_val
The values of the conditions.

Type list

property dim
Dimension of the field.

Type int

property drift_functions
The drift functions.

Type list of callable

property drift_no
Number of drift values per point.

Type int

property exact
Whether the interpolator is exact.

Type bool

property ext_drift_no
Number of external drift values per point.

Type int

property has_const_mean
Whether the field has a constant mean or not.

Type bool

property int_drift_no
Number of internal drift values per point.

Type int

property krige_size
Size of the kriging system.

Type int

property latlon
Whether the field depends on geographical coords.

Type bool

property mean
The mean of the field.

Type float or callable

property model
The covariance model of the field.

Type *CovModel*

property name
The name of the kriging class.

Type str

property normalizer
Normalizer of the field.

Type *Normalizer*

property pseudo_inv
Whether pseudo inverse matrix is used.

Type bool

property pseudo_inv_type
Method selector for pseudo inverse calculation.

Type str

property trend
The trend of the field.

Type float or callable

property unbiased
Whether the kriging is unbiased or not.

Type bool

property value_type
Type of the field values (scalar, vector).

Type str

gstools.krige.Universal

class gstools.krige.**Universal**(*model*, *cond_pos*, *cond_val*, *drift_functions*, *normalizer=None*, *trend=None*, *exact=False*, *cond_err='nugget'*, *pseudo_inv=True*, *pseudo_inv_type='pinv'*, *fit_normalizer=False*, *fit_variogram=False*)

Bases: *gstools.krige.base.Krige*

Universal kriging.

Universal kriging is used to interpolate given data with a variable mean, that is determined by a functional drift.

This estimator is set to be unbiased by default. This means, that the weights in the kriging equation sum up to 1. Consequently no constant function needs to be given for a constant drift, since the unbiased condition is applied to all given drift functions.

Parameters

- **model** (*CovModel*) – Covariance Model used for kriging.
- **cond_pos** (list) – tuple, containing the given condition positions (x, [y, z])
- **cond_val** (numpy.ndarray) – the values of the conditions
- **drift_functions** (list of callable, str or int) – Either a list of callable functions, an integer representing the polynomial order of the drift or one of the following strings:
  - ”linear” : regional linear drift (equals order=1)
  - ”quadratic” : regional quadratic drift (equals order=2)
- **normalizer** (None or *Normalizer*, optional) – Normalizer to be applied to the input data to gain normality. The default is None.
- **trend** (None or float or callable, optional) – A callable trend function. Should have the signiture: f(x, [y, z, ...]) This is used for detrended kriging, where the trended is subtracted from the conditions before kriging is applied. This can be used for regression kriging, where the trend function is determined by an external regression algorithm. If no normalizer is applied, this behaves equal to ‘mean’. The default is None.
- **exact** (bool, optional) – Whether the interpolator should reproduce the exact input values. If *False*, *cond_err* is interpreted as measurement error at the conditioning points and the result will be more smooth. Default: False
- **cond_err** (str, :class float or list, optional) – The measurement error at the conditioning points. Either “nugget” to apply the model-nugget, a single value applied to all points or an array with individual values for each point. The measurement error has to be <= nugget. The “exact=True” variant only works with “cond_err=’nugget’”. Default: “nugget”
- **pseudo_inv** (bool, optional) – Whether the kriging system is solved with the pseudo inverted kriging matrix. If *True*, this leads to more numerical stability and redundant points are averaged. But it can take more time. Default: True
- **pseudo_inv_type** (str or callable, optional) – Here you can select the algorithm to compute the pseudo-inverse matrix:
  - *”pinv”*: use *pinv* from *scipy* which uses *lstsq*
  - *”pinv2”*: use *pinv2* from *scipy* which uses *SVD*
  - *”pinvh”*: use *pinvh* from *scipy* which uses eigen-values

  If you want to use another routine to invert the kriging matrix, you can pass a callable which takes a matrix and returns the inverse. Default: *“pinv”*
- **fit_normalizer** (bool, optional) – Wheater to fit the data-normalizer to the given conditioning data. Default: False

- **fit_variogram** (bool, optional) – Wheater to fit the given variogram model to the data. This is done by using isotropy settings of the given model, assuming the sill to be the data variance and with the standard bins provided by the *standard_bins* routine. Default: False

Attributes

cond_err list: The measurement errors at the condition points.

cond_ext_drift numpy.ndarray: The ext. drift at the conditions.

cond_mean numpy.ndarray: Trend at the conditions.

cond_no int: The number of the conditions.

cond_pos list: The position tuple of the conditions.

cond_trend numpy.ndarray: Trend at the conditions.

cond_val list: The values of the conditions.

dim int: Dimension of the field.

drift_functions list of callable: The drift functions.

drift_no int: Number of drift values per point.

exact bool: Whether the interpolator is exact.

ext_drift_no int: Number of external drift values per point.

has_const_mean bool: Whether the field has a constant mean or not.

int_drift_no int: Number of internal drift values per point.

krige_size int: Size of the kriging system.

latlon bool: Whether the field depends on geographical coords.

mean float or callable: The mean of the field.

model *CovModel*: The covariance model of the field.

name str: The name of the kriging class.

normalizer *Normalizer*: Normalizer of the field.

pseudo_inv bool: Whether pseudo inverse matrix is used.

pseudo_inv_type str: Method selector for pseudo inverse calculation.

trend float or callable: The trend of the field.

unbiased bool: Whether the kriging is unbiased or not.

value_type str: Type of the field values (scalar, vector).

Methods

__call__(pos[, mesh_type, ext_drift, ...])	Generate the kriging field.
get_mean([post_process])	Calculate the estimated mean of the detrended field.
mesh(mesh[, points, direction, name])	Generate a field on a given meshio, ogs5py or PyVista mesh.
plot([field, fig, ax])	Plot the spatial random field.
post_field(field[, name, process, save])	Postprocessing field values.
pre_pos(pos[, mesh_type])	Preprocessing positions and mesh_type.
set_condition([cond_pos, cond_val, ...])	Set the conditions for kriging.

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Table 49 – continued from previous page

set_drift_functions([drift_functions])	Set the drift functions for universal kriging.
structured(*args, **kwargs)	Generate a field on a structured mesh.
to_pyvista([field_select, fieldname])	Create a VTK/PyVista grid of the stored field.
unstructured(*args, **kwargs)	Generate a field on an unstructured mesh.
vtk_export(filename[, field_select, fieldname])	Export the stored field to vtk.

__call__(*pos*, *mesh_type='unstructured'*, *ext_drift=None*, *chunk_size=None*, *only_mean=False*, *return_var=True*, *post_process=True*)

Generate the kriging field.

The field is saved as *self.field* and is also returned. The error variance is saved as *self.krige_var* and is also returned.

Parameters

- **pos** (list) – the position tuple, containing main direction and transversal directions (x, [y, z])
- **mesh_type** (str, optional) – 'structured' / 'unstructured'
- **ext_drift** (numpy.ndarray or None, optional) – the external drift values at the given positions (only for EDK)
- **chunk_size** (int, optional) – Chunk size to cut down the size of the kriging system to prevent memory errors. Default: None
- **only_mean** (bool, optional) – Whether to only calculate the mean of the kriging field. Default: *False*
- **return_var** (bool, optional) – Whether to return the variance along with the field. Default: *True*
- **post_process** (bool, optional) – Whether to apply mean, normalizer and trend to the field. Default: *True*

Returns

- **field** (numpy.ndarray) – the kriged field or mean_field
- **krige_var** (numpy.ndarray, optional) – the kriging error variance (if return_var is True and only_mean is False)

get_mean(*post_process=True*)

Calculate the estimated mean of the detrended field.

Parameters **post_process** (bool, optional) – Whether to apply field-mean and normalizer. Default: *True*

Returns **mean** – Mean of the Kriging System.

Return type float or None

Notes

Only not None if the Kriging System has a constant mean. This means, no drift is given and the given field-mean is constant. The result is neglecting a potential given trend.

---

mesh(*mesh*, *points='centroids'*, *direction='all'*, *name='field'*, ***kwargs*)

Generate a field on a given meshio, ogs5py or PyVista mesh.

Parameters

- **mesh** (*meshio.Mesh or ogs5py.MSH or PyVista mesh*) – The given mesh

- **points** (str, optional) – The points to evaluate the field at. Either the "centroids" of the mesh cells (calculated as mean of the cell vertices) or the "points" of the given mesh. Default: "centroids"
- **direction** (str or list, optional) – Here you can state which direction should be choosen for lower dimension. For example, if you got a 2D mesh in xz direction, you have to pass "xz". By default, all directions are used. One can also pass a list of indices. Default: "all"
- **name** (str or list of str, optional) – Name(s) to store the field(s) in the given mesh as point_data or cell_data. If to few names are given, digits will be appended. Default: "field"
- ****kwargs** – Keyword arguments forwarded to *__call__*.

Notes

This will store the field in the given mesh under the given name, if a meshio or PyVista mesh was given.

See:

- meshio: https://github.com/nschloe/meshio
- ogs5py: https://github.com/GeoStat-Framework/ogs5py
- PyVista: https://github.com/pyvista/pyvista

---

plot(*field='field'*, *fig=None*, *ax=None*, ***kwargs*)
Plot the spatial random field.

Parameters

- **field** (str, optional) – Field that should be plotted. Default: "field"
- **fig** (Figure or None) – Figure to plot the axes on. If *None*, a new one will be created. Default: *None*
- **ax** (Axes or None) – Axes to plot on. If *None*, a new one will be added to the figure. Default: *None*
- ****kwargs** – Forwarded to the plotting routine.

post_field(*field*, *name='field'*, *process=True*, *save=True*)
Postprocessing field values.

Parameters

- **field** (numpy.ndarray) – Field values.
- **name** (str, optional) – Name. to store the field. The default is "field".
- **process** (bool, optional) – Whether to process field to apply mean, normalizer and trend. The default is True.
- **save** (bool, optional) – Whether to store the field under the given name. The default is True.

Returns **field** – Processed field values.

Return type numpy.ndarray

pre_pos(*pos*, *mesh_type='unstructured'*)
Preprocessing positions and mesh_type.

Parameters

- **pos** (iterable) – the position tuple, containing main direction and transversal directions
- **mesh_type** (str, optional) – 'structured' / 'unstructured' Default: *"unstructured"*

Returns

- **iso_pos** ((d, n), numpy.ndarray) – the isometrized position tuple
- **shape** (tuple) – Shape of the resulting field.

set_condition(*cond_pos=None*, *cond_val=None*, *ext_drift=None*, *cond_err=None*, *fit_normalizer=False*, *fit_variogram=False*)

Set the conditions for kriging.

This method could also be used to update the kriging setup, when properties were changed. Then you can call it without arguments.

Parameters

- **cond_pos** (list, optional) – the position tuple of the conditions (x, [y, z]). Default: current.
- **cond_val** (numpy.ndarray, optional) – the values of the conditions. Default: current.
- **ext_drift** (numpy.ndarray or None, optional) – the external drift values at the given conditions (only for EDK) For multiple external drifts, the first dimension should be the index of the drift term. When passing *None*, the extisting external drift will be used.
- **cond_err** (str, :class float, list, optional) – The measurement error at the conditioning points. Either "nugget" to apply the model-nugget, a single value applied to all points or an array with individual values for each point. The measurement error has to be <= nugget. The "exact=True" variant only works with "cond_err='nugget'". Default: "nugget"
- **fit_normalizer** (bool, optional) – Wheater to fit the data-normalizer to the given conditioning data. Default: False
- **fit_variogram** (bool, optional) – Wheater to fit the given variogram model to the data. This is done by using isotropy settings of the given model, assuming the sill to be the data variance and with the standard bins provided by the *standard_bins* routine. Default: False

set_drift_functions(*drift_functions=None*)

Set the drift functions for universal kriging.

Parameters **drift_functions** (list of callable, str or int) – Either a list of callable functions, an integer representing the polynomial order of the drift or one of the following strings:

- "linear" : regional linear drift (equals order=1)
- "quadratic" : regional quadratic drift (equals order=2)

Raises **ValueError** – If the given drift functions are not callable.

structured(**args*, ***kwargs*)

Generate a field on a structured mesh.

See *__call__*

to_pyvista(*field_select='field'*, *fieldname='field'*)

Create a VTK/PyVista grid of the stored field.

Parameters

- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

unstructured(**args*, ***kwargs*)

Generate a field on an unstructured mesh.

See *__call__*

vtk_export(*filename*, *field_select='field'*, *fieldname='field'*)
Export the stored field to vtk.

Parameters

- **filename** (str) – Filename of the file to be saved, including the path. Note that an ending (.vtr or .vtu) will be added to the name.
- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

property cond_err
The measurement errors at the condition points.

Type list

property cond_ext_drift
The ext. drift at the conditions.

Type numpy.ndarray

property cond_mean
Trend at the conditions.

Type numpy.ndarray

property cond_no
The number of the conditions.

Type int

property cond_pos
The position tuple of the conditions.

Type list

property cond_trend
Trend at the conditions.

Type numpy.ndarray

property cond_val
The values of the conditions.

Type list

property dim
Dimension of the field.

Type int

property drift_functions
The drift functions.

Type list of callable

property drift_no
Number of drift values per point.

Type int

property exact
Whether the interpolator is exact.

Type bool

property ext_drift_no
Number of external drift values per point.

Type int

property has_const_mean
Whether the field has a constant mean or not.

Type bool

property int_drift_no
Number of internal drift values per point.

Type int

property krige_size
Size of the kriging system.

Type int

property latlon
Whether the field depends on geographical coords.

Type bool

property mean
The mean of the field.

Type float or callable

property model
The covariance model of the field.

Type *CovModel*

property name
The name of the kriging class.

Type str

property normalizer
Normalizer of the field.

Type *Normalizer*

property pseudo_inv
Whether pseudo inverse matrix is used.

Type bool

property pseudo_inv_type
Method selector for pseudo inverse calculation.

Type str

property trend
The trend of the field.

Type float or callable

property unbiased
Whether the kriging is unbiased or not.

Type bool

property value_type
Type of the field values (scalar, vector).

Type str

gstools.krige.ExtDrift

class gstools.krige.**ExtDrift**(*model*, *cond_pos*, *cond_val*, *ext_drift*, *normalizer=None*, *trend=None*, *exact=False*, *cond_err='nugget'*, *pseudo_inv=True*, *pseudo_inv_type='pinv'*, *fit_normalizer=False*, *fit_variogram=False*)

Bases: *gstools.krige.base.Krige*

External drift kriging (EDK).

External drift kriging is used to interpolate given data with a variable mean, that is determined by an external drift.

This estimator is set to be unbiased by default. This means, that the weights in the kriging equation sum up to 1. Consequently no constant external drift needs to be given to estimate a proper mean.

Parameters

- **model** (*CovModel*) – Covariance Model used for kriging.
- **cond_pos** (list) – tuple, containing the given condition positions (x, [y, z])
- **cond_val** (numpy.ndarray) – the values of the conditions
- **ext_drift** (numpy.ndarray) – the external drift values at the given condition positions.
- **normalizer** (None or *Normalizer*, optional) – Normalizer to be applied to the input data to gain normality. The default is None.
- **trend** (None or float or callable, optional) – A callable trend function. Should have the signiture: f(x, [y, z, ...]) This is used for detrended kriging, where the trended is subtracted from the conditions before kriging is applied. This can be used for regression kriging, where the trend function is determined by an external regression algorithm. If no normalizer is applied, this behaves equal to 'mean'. The default is None.
- **exact** (bool, optional) – Whether the interpolator should reproduce the exact input values. If *False*, *cond_err* is interpreted as measurement error at the conditioning points and the result will be more smooth. Default: False
- **cond_err** (str, :class float or list, optional) – The measurement error at the conditioning points. Either "nugget" to apply the model-nugget, a single value applied to all points or an array with individual values for each point. The measurement error has to be <= nugget. The "exact=True" variant only works with "cond_err='nugget'". Default: "nugget"
- **pseudo_inv** (bool, optional) – Whether the kriging system is solved with the pseudo inverted kriging matrix. If *True*, this leads to more numerical stability and redundant points are averaged. But it can take more time. Default: True
- **pseudo_inv_type** (str or callable, optional) – Here you can select the algorithm to compute the pseudo-inverse matrix:
  - *"pinv"*: use *pinv* from *scipy* which uses *lstsq*
  - *"pinv2"*: use *pinv2* from *scipy* which uses *SVD*
  - *"pinvh"*: use *pinvh* from *scipy* which uses eigen-values

  If you want to use another routine to invert the kriging matrix, you can pass a callable which takes a matrix and returns the inverse. Default: *"pinv"*
- **fit_normalizer** (bool, optional) – Wheater to fit the data-normalizer to the given conditioning data. Default: False
- **fit_variogram** (bool, optional) – Wheater to fit the given variogram model to the data. This is done by using isotropy settings of the given model, assuming the sill to be the data variance and with the standard bins provided by the *standard_bins* routine. Default: False

Attributes

cond_err `list`: The measurement errors at the condition points.

cond_ext_drift `numpy.ndarray`: The ext. drift at the conditions.

cond_mean `numpy.ndarray`: Trend at the conditions.

cond_no `int`: The number of the conditions.

cond_pos `list`: The position tuple of the conditions.

cond_trend `numpy.ndarray`: Trend at the conditions.

cond_val `list`: The values of the conditions.

dim `int`: Dimension of the field.

drift_functions `list` of `callable`: The drift functions.

drift_no `int`: Number of drift values per point.

exact `bool`: Whether the interpolator is exact.

ext_drift_no `int`: Number of external drift values per point.

has_const_mean `bool`: Whether the field has a constant mean or not.

int_drift_no `int`: Number of internal drift values per point.

krige_size `int`: Size of the kriging system.

latlon `bool`: Whether the field depends on geographical coords.

mean `float` or `callable`: The mean of the field.

model *CovModel*: The covariance model of the field.

name `str`: The name of the kriging class.

normalizer *Normalizer*: Normalizer of the field.

pseudo_inv `bool`: Whether pseudo inverse matrix is used.

pseudo_inv_type `str`: Method selector for pseudo inverse calculation.

trend `float` or `callable`: The trend of the field.

unbiased `bool`: Whether the kriging is unbiased or not.

value_type `str`: Type of the field values (scalar, vector).

Methods

__call__(pos[, mesh_type, ext_drift, ...])	Generate the kriging field.
get_mean([post_process])	Calculate the estimated mean of the detrended field.
mesh(mesh[, points, direction, name])	Generate a field on a given meshio, ogs5py or PyVista mesh.
plot([field, fig, ax])	Plot the spatial random field.
post_field(field[, name, process, save])	Postprocessing field values.
pre_pos(pos[, mesh_type])	Preprocessing positions and mesh_type.
set_condition([cond_pos, cond_val, ...])	Set the conditions for kriging.
set_drift_functions([drift_functions])	Set the drift functions for universal kriging.
structured(*args, **kwargs)	Generate a field on a structured mesh.
to_pyvista([field_select, fieldname])	Create a VTK/PyVista grid of the stored field.
unstructured(*args, **kwargs)	Generate a field on an unstructured mesh.
vtk_export(filename[, field_select, fieldname])	Export the stored field to vtk.

__call__(*pos*, *mesh_type='unstructured'*, *ext_drift=None*, *chunk_size=None*, *only_mean=False*, *return_var=True*, *post_process=True*)

Generate the kriging field.

The field is saved as *self.field* and is also returned. The error variance is saved as *self.krige_var* and is also returned.

Parameters

- **pos** (list) – the position tuple, containing main direction and transversal directions (x, [y, z])
- **mesh_type** (str, optional) – 'structured' / 'unstructured'
- **ext_drift** (numpy.ndarray or None, optional) – the external drift values at the given positions (only for EDK)
- **chunk_size** (int, optional) – Chunk size to cut down the size of the kriging system to prevent memory errors. Default: None
- **only_mean** (bool, optional) – Whether to only calculate the mean of the kriging field. Default: *False*
- **return_var** (bool, optional) – Whether to return the variance along with the field. Default: *True*
- **post_process** (bool, optional) – Whether to apply mean, normalizer and trend to the field. Default: *True*

Returns

- **field** (numpy.ndarray) – the kriged field or mean_field
- **krige_var** (numpy.ndarray, optional) – the kriging error variance (if return_var is True and only_mean is False)

get_mean(*post_process=True*)

Calculate the estimated mean of the detrended field.

Parameters **post_process** (bool, optional) – Whether to apply field-mean and normalizer. Default: *True*

Returns **mean** – Mean of the Kriging System.

Return type float or None

Notes

Only not None if the Kriging System has a constant mean. This means, no drift is given and the given field-mean is constant. The result is neglecting a potential given trend.

---

mesh(*mesh*, *points='centroids'*, *direction='all'*, *name='field'*, ***kwargs*)

Generate a field on a given meshio, ogs5py or PyVista mesh.

Parameters

- **mesh** (*meshio.Mesh or ogs5py.MSH or PyVista mesh*) – The given mesh
- **points** (str, optional) – The points to evaluate the field at. Either the "centroids" of the mesh cells (calculated as mean of the cell vertices) or the "points" of the given mesh. Default: "centroids"
- **direction** (str or list, optional) – Here you can state which direction should be choosen for lower dimension. For example, if you got a 2D mesh in xz direction, you have to pass "xz". By default, all directions are used. One can also pass a list of indices. Default: "all"

- **name** (str or list of str, optional) – Name(s) to store the field(s) in the given mesh as point_data or cell_data. If to few names are given, digits will be appended. Default: "field"
- ****kwargs** – Keyword arguments forwarded to *__call__*.

Notes

This will store the field in the given mesh under the given name, if a meshio or PyVista mesh was given.

See:

- meshio: https://github.com/nschloe/meshio
- ogs5py: https://github.com/GeoStat-Framework/ogs5py
- PyVista: https://github.com/pyvista/pyvista

---

plot(*field='field'*, *fig=None*, *ax=None*, ***kwargs*)
Plot the spatial random field.

Parameters

- **field** (str, optional) – Field that should be plotted. Default: "field"
- **fig** (Figure or None) – Figure to plot the axes on. If *None*, a new one will be created. Default: *None*
- **ax** (Axes or None) – Axes to plot on. If *None*, a new one will be added to the figure. Default: *None*
- ****kwargs** – Forwarded to the plotting routine.

post_field(*field*, *name='field'*, *process=True*, *save=True*)
Postprocessing field values.

Parameters

- **field** (numpy.ndarray) – Field values.
- **name** (str, optional) – Name. to store the field. The default is "field".
- **process** (bool, optional) – Whether to process field to apply mean, normalizer and trend. The default is True.
- **save** (bool, optional) – Whether to store the field under the given name. The default is True.

Returns **field** – Processed field values.

Return type numpy.ndarray

pre_pos(*pos*, *mesh_type='unstructured'*)
Preprocessing positions and mesh_type.

Parameters

- **pos** (iterable) – the position tuple, containing main direction and transversal directions
- **mesh_type** (str, optional) – 'structured' / 'unstructured' Default: *"unstructured"*

Returns

- **iso_pos** ((d, n), numpy.ndarray) – the isometrized position tuple
- **shape** (tuple) – Shape of the resulting field.

set_condition(*cond_pos=None*, *cond_val=None*, *ext_drift=None*, *cond_err=None*, *fit_normalizer=False*, *fit_variogram=False*)
Set the conditions for kriging.

This method could also be used to update the kriging setup, when properties were changed. Then you can call it without arguments.

Parameters

- **cond_pos** (list, optional) – the position tuple of the conditions (x, [y, z]). Default: current.
- **cond_val** (numpy.ndarray, optional) – the values of the conditions. Default: current.
- **ext_drift** (numpy.ndarray or None, optional) – the external drift values at the given conditions (only for EDK) For multiple external drifts, the first dimension should be the index of the drift term. When passing *None*, the extisting external drift will be used.
- **cond_err** (str, :class float, list, optional) – The measurement error at the conditioning points. Either "nugget" to apply the model-nugget, a single value applied to all points or an array with individual values for each point. The measurement error has to be <= nugget. The "exact=True" variant only works with "cond_err='nugget'". Default: "nugget"
- **fit_normalizer** (bool, optional) – Wheater to fit the data-normalizer to the given conditioning data. Default: False
- **fit_variogram** (bool, optional) – Wheater to fit the given variogram model to the data. This is done by using isotropy settings of the given model, assuming the sill to be the data variance and with the standard bins provided by the *standard_bins* routine. Default: False

set_drift_functions(*drift_functions=None*)
Set the drift functions for universal kriging.

Parameters **drift_functions** (list of callable, str or int) – Either a list of callable functions, an integer representing the polynomial order of the drift or one of the following strings:

- "linear" : regional linear drift (equals order=1)
- "quadratic" : regional quadratic drift (equals order=2)

Raises ValueError – If the given drift functions are not callable.

structured(**args*, ***kwargs*)
Generate a field on a structured mesh.

See *__call__*

to_pyvista(*field_select='field'*, *fieldname='field'*)
Create a VTK/PyVista grid of the stored field.

Parameters

- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

unstructured(**args*, ***kwargs*)
Generate a field on an unstructured mesh.

See *__call__*

vtk_export(*filename*, *field_select='field'*, *fieldname='field'*)
Export the stored field to vtk.

Parameters

- **filename** (str) – Filename of the file to be saved, including the path. Note that an ending (.vtr or .vtu) will be added to the name.
- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

property cond_err
The measurement errors at the condition points.

Type list

property cond_ext_drift
The ext. drift at the conditions.

Type numpy.ndarray

property cond_mean
Trend at the conditions.

Type numpy.ndarray

property cond_no
The number of the conditions.

Type int

property cond_pos
The position tuple of the conditions.

Type list

property cond_trend
Trend at the conditions.

Type numpy.ndarray

property cond_val
The values of the conditions.

Type list

property dim
Dimension of the field.

Type int

property drift_functions
The drift functions.

Type list of callable

property drift_no
Number of drift values per point.

Type int

property exact
Whether the interpolator is exact.

Type bool

property ext_drift_no
Number of external drift values per point.

Type int

property has_const_mean
Whether the field has a constant mean or not.

Type bool

property int_drift_no
Number of internal drift values per point.

Type int

property krige_size
Size of the kriging system.

Type int

property latlon
Whether the field depends on geographical coords.

Type bool

property mean
The mean of the field.

Type float or callable

property model
The covariance model of the field.

Type *CovModel*

property name
The name of the kriging class.

Type str

property normalizer
Normalizer of the field.

Type *Normalizer*

property pseudo_inv
Whether pseudo inverse matrix is used.

Type bool

property pseudo_inv_type
Method selector for pseudo inverse calculation.

Type str

property trend
The trend of the field.

Type float or callable

property unbiased
Whether the kriging is unbiased or not.

Type bool

property value_type
Type of the field values (scalar, vector).

Type str

gstools.krige.Detrended

class gstools.krige.**Detrended**(*model*, *cond_pos*, *cond_val*, *trend*, *exact=False*, *cond_err='nugget'*, *pseudo_inv=True*, *pseudo_inv_type='pinv'*, *fit_variogram=False*)

Bases: *gstools.krige.base.Krige*

Detrended simple kriging.

In detrended kriging, the data is detrended before interpolation by simple kriging with zero mean.

The trend needs to be a callable function the user has to provide. This can be used for regression kriging, where the trend function is determined by an external regression algorithm.

This is just a shortcut for simple kriging with a given trend function, zero mean and no normalizer.

A trend can be given with EVERY provided kriging routine.

Parameters

- **model** (*CovModel*) – Covariance Model used for kriging.
- **cond_pos** (list) – tuple, containing the given condition positions (x, [y, z])
- **cond_val** (numpy.ndarray) – the values of the conditions
- **trend_function** (callable) – The callable trend function. Should have the signiture: f(x, [y, z])
- **exact** (bool, optional) – Whether the interpolator should reproduce the exact input values. If *False*, *cond_err* is interpreted as measurement error at the conditioning points and the result will be more smooth. Default: False
- **cond_err** (str, :class float or list, optional) – The measurement error at the conditioning points. Either "nugget" to apply the model-nugget, a single value applied to all points or an array with individual values for each point. The measurement error has to be <= nugget. The "exact=True" variant only works with "cond_err='nugget'". Default: "nugget"
- **pseudo_inv** (bool, optional) – Whether the kriging system is solved with the pseudo inverted kriging matrix. If *True*, this leads to more numerical stability and redundant points are averaged. But it can take more time. Default: True
- **pseudo_inv_type** (str or callable, optional) – Here you can select the algorithm to compute the pseudo-inverse matrix:
  - *"pinv"*: use *pinv* from *scipy* which uses *lstsq*
  - *"pinv2"*: use *pinv2* from *scipy* which uses *SVD*
  - *"pinvh"*: use *pinvh* from *scipy* which uses eigen-values

  If you want to use another routine to invert the kriging matrix, you can pass a callable which takes a matrix and returns the inverse. Default: *"pinv"*
- **fit_variogram** (bool, optional) – Wheater to fit the given variogram model to the data. This is done by using isotropy settings of the given model, assuming the sill to be the data variance and with the standard bins provided by the *standard_bins* routine. Default: False

Attributes

cond_err list: The measurement errors at the condition points.

cond_ext_drift numpy.ndarray: The ext. drift at the conditions.

cond_mean numpy.ndarray: Trend at the conditions.

cond_no int: The number of the conditions.

cond_pos list: The position tuple of the conditions.

cond_trend `numpy.ndarray`: Trend at the conditions.

cond_val `list`: The values of the conditions.

dim `int`: Dimension of the field.

drift_functions `list` of `callable`: The drift functions.

drift_no `int`: Number of drift values per point.

exact `bool`: Whether the interpolator is exact.

ext_drift_no `int`: Number of external drift values per point.

has_const_mean `bool`: Whether the field has a constant mean or not.

int_drift_no `int`: Number of internal drift values per point.

krige_size `int`: Size of the kriging system.

latlon `bool`: Whether the field depends on geographical coords.

mean `float` or `callable`: The mean of the field.

model *CovModel*: The covariance model of the field.

name `str`: The name of the kriging class.

normalizer *Normalizer*: Normalizer of the field.

pseudo_inv `bool`: Whether pseudo inverse matrix is used.

pseudo_inv_type `str`: Method selector for pseudo inverse calculation.

trend `float` or `callable`: The trend of the field.

unbiased `bool`: Whether the kriging is unbiased or not.

value_type `str`: Type of the field values (scalar, vector).

Methods

__call__(pos[, mesh_type, ext_drift, ...])	Generate the kriging field.
get_mean([post_process])	Calculate the estimated mean of the detrended field.
mesh(mesh[, points, direction, name])	Generate a field on a given meshio, ogs5py or PyVista mesh.
plot([field, fig, ax])	Plot the spatial random field.
post_field(field[, name, process, save])	Postprocessing field values.
pre_pos(pos[, mesh_type])	Preprocessing positions and mesh_type.
set_condition([cond_pos, cond_val, ...])	Set the conditions for kriging.
set_drift_functions([drift_functions])	Set the drift functions for universal kriging.
structured(*args, **kwargs)	Generate a field on a structured mesh.
to_pyvista([field_select, fieldname])	Create a VTK/PyVista grid of the stored field.
unstructured(*args, **kwargs)	Generate a field on an unstructured mesh.
vtk_export(filename[, field_select, fieldname])	Export the stored field to vtk.

__call__(*pos*, *mesh_type='unstructured'*, *ext_drift=None*, *chunk_size=None*, *only_mean=False*, *return_var=True*, *post_process=True*)

Generate the kriging field.

The field is saved as *self.field* and is also returned. The error variance is saved as *self.krige_var* and is also returned.

Parameters

- **pos** (`list`) – the position tuple, containing main direction and transversal directions

(x, [y, z])

- **mesh_type** (str, optional) – 'structured' / 'unstructured'
- **ext_drift** (numpy.ndarray or None, optional) – the external drift values at the given positions (only for EDK)
- **chunk_size** (int, optional) – Chunk size to cut down the size of the kriging system to prevent memory errors. Default: None
- **only_mean** (bool, optional) – Whether to only calculate the mean of the kriging field. Default: *False*
- **return_var** (bool, optional) – Whether to return the variance along with the field. Default: *True*
- **post_process** (bool, optional) – Whether to apply mean, normalizer and trend to the field. Default: *True*

Returns

- **field** (numpy.ndarray) – the kriged field or mean_field
- **krige_var** (numpy.ndarray, optional) – the kriging error variance (if return_var is True and only_mean is False)

get_mean(*post_process=True*)

Calculate the estimated mean of the detrended field.

Parameters **post_process** (bool, optional) – Whether to apply field-mean and normalizer. Default: *True*

Returns **mean** – Mean of the Kriging System.

Return type float or None

Notes

Only not None if the Kriging System has a constant mean. This means, no drift is given and the given field-mean is constant. The result is neglecting a potential given trend.

---

mesh(*mesh*, *points='centroids'*, *direction='all'*, *name='field'*, ***kwargs*)

Generate a field on a given meshio, ogs5py or PyVista mesh.

Parameters

- **mesh** (*meshio.Mesh or ogs5py.MSH or PyVista mesh*) – The given mesh
- **points** (str, optional) – The points to evaluate the field at. Either the "centroids" of the mesh cells (calculated as mean of the cell vertices) or the "points" of the given mesh. Default: "centroids"
- **direction** (str or list, optional) – Here you can state which direction should be choosen for lower dimension. For example, if you got a 2D mesh in xz direction, you have to pass "xz". By default, all directions are used. One can also pass a list of indices. Default: "all"
- **name** (str or list of str, optional) – Name(s) to store the field(s) in the given mesh as point_data or cell_data. If to few names are given, digits will be appended. Default: "field"
- ****kwargs** – Keyword arguments forwarded to *__call__*.

Notes

This will store the field in the given mesh under the given name, if a meshio or PyVista mesh was given.

See:

- meshio: https://github.com/nschloe/meshio
- ogs5py: https://github.com/GeoStat-Framework/ogs5py
- PyVista: https://github.com/pyvista/pyvista

---

plot(*field='field'*, *fig=None*, *ax=None*, ***kwargs*)
Plot the spatial random field.

Parameters

- **field** (str, optional) – Field that should be plotted. Default: "field"
- **fig** (Figure or None) – Figure to plot the axes on. If *None*, a new one will be created. Default: *None*
- **ax** (Axes or None) – Axes to plot on. If *None*, a new one will be added to the figure. Default: *None*
- ****kwargs** – Forwarded to the plotting routine.

post_field(*field*, *name='field'*, *process=True*, *save=True*)
Postprocessing field values.

Parameters

- **field** (numpy.ndarray) – Field values.
- **name** (str, optional) – Name. to store the field. The default is "field".
- **process** (bool, optional) – Whether to process field to apply mean, normalizer and trend. The default is True.
- **save** (bool, optional) – Whether to store the field under the given name. The default is True.

Returns **field** – Processed field values.

Return type numpy.ndarray

pre_pos(*pos*, *mesh_type='unstructured'*)
Preprocessing positions and mesh_type.

Parameters

- **pos** (iterable) – the position tuple, containing main direction and transversal directions
- **mesh_type** (str, optional) – 'structured' / 'unstructured' Default: *"unstructured"*

Returns

- **iso_pos** ((d, n), numpy.ndarray) – the isometrized position tuple
- **shape** (tuple) – Shape of the resulting field.

set_condition(*cond_pos=None*, *cond_val=None*, *ext_drift=None*, *cond_err=None*, *fit_normalizer=False*, *fit_variogram=False*)
Set the conditions for kriging.

This method could also be used to update the kriging setup, when properties were changed. Then you can call it without arguments.

Parameters

- **cond_pos** (list, optional) – the position tuple of the conditions (x, [y, z]). Default: current.
- **cond_val** (numpy.ndarray, optional) – the values of the conditions. Default: current.

- **ext_drift** (numpy.ndarray or None, optional) – the external drift values at the given conditions (only for EDK) For multiple external drifts, the first dimension should be the index of the drift term. When passing *None*, the extisting external drift will be used.
- **cond_err** (str, :class float, list, optional) – The measurement error at the conditioning points. Either "nugget" to apply the model-nugget, a single value applied to all points or an array with individual values for each point. The measurement error has to be <= nugget. The "exact=True" variant only works with "cond_err='nugget'". Default: "nugget"
- **fit_normalizer** (bool, optional) – Wheater to fit the data-normalizer to the given conditioning data. Default: False
- **fit_variogram** (bool, optional) – Wheater to fit the given variogram model to the data. This is done by using isotropy settings of the given model, assuming the sill to be the data variance and with the standard bins provided by the *standard_bins* routine. Default: False

set_drift_functions(*drift_functions=None*)
Set the drift functions for universal kriging.

Parameters **drift_functions** (list of callable, str or int) – Either a list of callable functions, an integer representing the polynomial order of the drift or one of the following strings:

- "linear" : regional linear drift (equals order=1)
- "quadratic" : regional quadratic drift (equals order=2)

Raises ValueError – If the given drift functions are not callable.

structured(**args*, ***kwargs*)
Generate a field on a structured mesh.

See *__call__*

to_pyvista(*field_select='field'*, *fieldname='field'*)
Create a VTK/PyVista grid of the stored field.

Parameters

- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

unstructured(**args*, ***kwargs*)
Generate a field on an unstructured mesh.

See *__call__*

vtk_export(*filename*, *field_select='field'*, *fieldname='field'*)
Export the stored field to vtk.

Parameters

- **filename** (str) – Filename of the file to be saved, including the path. Note that an ending (.vtr or .vtu) will be added to the name.
- **field_select** (str, optional) – Field that should be stored. Can be: "field", "raw_field", "krige_field", "err_field" or "krige_var". Default: "field"
- **fieldname** (str, optional) – Name of the field in the VTK file. Default: "field"

property cond_err
The measurement errors at the condition points.

Type list

property cond_ext_drift
The ext. drift at the conditions.

Type numpy.ndarray

property cond_mean
Trend at the conditions.

Type numpy.ndarray

property cond_no
The number of the conditions.

Type int

property cond_pos
The position tuple of the conditions.

Type list

property cond_trend
Trend at the conditions.

Type numpy.ndarray

property cond_val
The values of the conditions.

Type list

property dim
Dimension of the field.

Type int

property drift_functions
The drift functions.

Type list of callable

property drift_no
Number of drift values per point.

Type int

property exact
Whether the interpolator is exact.

Type bool

property ext_drift_no
Number of external drift values per point.

Type int

property has_const_mean
Whether the field has a constant mean or not.

Type bool

property int_drift_no
Number of internal drift values per point.

Type int

property krige_size
Size of the kriging system.

Type int

property latlon
Whether the field depends on geographical coords.

Type bool

property mean
The mean of the field.

Type float or callable

property model
The covariance model of the field.

Type *CovModel*

property name
The name of the kriging class.

Type str

property normalizer
Normalizer of the field.

Type *Normalizer*

property pseudo_inv
Whether pseudo inverse matrix is used.

Type bool

property pseudo_inv_type
Method selector for pseudo inverse calculation.

Type str

property trend
The trend of the field.

Type float or callable

property unbiased
Whether the kriging is unbiased or not.

Type bool

property value_type
Type of the field values (scalar, vector).

Type str

3.10 gstools.random

GStools subpackage for random number generation.

Random Number Generator

RNG([seed])	A random number generator for different distributions and multiple streams.

Seed Generator

MasterRNG(seed)	Master random number generator for generating seeds.

Distribution factory

dist_gen([pdf_in, cdf_in, ppf_in])	Distribution Factory.

class gstools.random.**MasterRNG**(*seed*)

Master random number generator for generating seeds.

Parameters **seed** (int or None, optional) – The seed of the master RNG, if None, a random seed is used. Default: None

Attributes

seed int: Seed of the master RNG.

Methods

__call__()	Return a random seed.

__call__()

Return a random seed.

property seed

Seed of the master RNG.

The setter property not only saves the new seed, but also creates a new master RNG function with the new seed.

Type int

class gstools.random.**RNG**(*seed=None*)

A random number generator for different distributions and multiple streams.

Parameters **seed** (int or None, optional) – The seed of the master RNG, if None, a random seed is used. Default: None

Attributes

random numpy.random.RandomState: Randomstate.

seed int: Seed of the master RNG.

Methods

sample_dist([pdf, cdf, ppf, size])	Sample from a distribution given by pdf, cdf and/or ppf.
sample_ln_pdf(ln_pdf[, size, sample_around, ...])	Sample from a distribution given by ln(pdf).
sample_sphere(dim[, size])	Uniform sampling on a d-dimensional sphere.

sample_dist(*pdf=None*, *cdf=None*, *ppf=None*, *size=None*, ***kwargs*)
Sample from a distribution given by pdf, cdf and/or ppf.

Parameters

- **pdf** (callable or None, optional) – Probability density function of the given distribution, that takes a single argument Default: None
- **cdf** (callable or None, optional) – Cumulative distribution function of the given distribution, that takes a single argument Default: None
- **ppf** (callable or None, optional) – Percent point function of the given distribution, that takes a single argument Default: None
- **size** (int or None, optional) – sample size. Default: None
- ****kwargs** – Keyword-arguments that are forwarded to scipy.stats.rv_continuous.

Returns **samples** – the samples from the given distribution

Return type float or numpy.ndarray

Notes

At least pdf or cdf needs to be given.

sample_ln_pdf(*ln_pdf*, *size=None*, *sample_around=1.0*, *nwalkers=50*, *burn_in=20*, *oversampling_factor=10*)
Sample from a distribution given by ln(pdf).

This algorithm uses the emcee.EnsembleSampler

Parameters

- **ln_pdf** (callable) – The logarithm of the Probability density function of the given distribution, that takes a single argument
- **size** (int or None, optional) – sample size. Default: None
- **sample_around** (float, optional) – Starting point for initial guess Default: 1.
- **nwalkers** (int, optional) – The number of walkers in the mcmc sampler. Used for the emcee.EnsembleSampler class. Default: 50
- **burn_in** (int, optional) – Number of burn-in runs in the mcmc algorithm. Default: 20
- **oversampling_factor** (int, optional) – To guess the sample number needed for proper results, we use a factor for oversampling. The intern used sample-size is calculated by

  `sample_size = max(burn_in, (size/nwalkers)*oversampling_factor)`

  So at least, as much as the burn-in runs. Default: 10

sample_sphere(*dim*, *size=None*)
Uniform sampling on a d-dimensional sphere.

Parameters

- **dim** (int) – Dimension of the sphere. Just 1, 2, and 3 supported.
- **size** (int, optional) – sample size

Returns **coord** – x[, y[, z]] coordinates on the sphere with shape (dim, size)

Return type numpy.ndarray

property random

Randomstate.

Get a stream to the numpy Random number generator. You can use this, to call any provided distribution from numpy.random.RandomState.

Type numpy.random.RandomState

property seed

Seed of the master RNG.

The setter property not only saves the new seed, but also creates a new master RNG function with the new seed.

Type int

gstools.random.**dist_gen**(*pdf_in=None*, *cdf_in=None*, *ppf_in=None*, ***kwargs*)

Distribution Factory.

Parameters

- **pdf_in** (callable or None, optional) – Proprobability distribution function of the given distribution, that takes a single argument Default: None
- **cdf_in** (callable or None, optional) – Cumulative distribution function of the given distribution, that takes a single argument Default: None
- **ppf_in** (callable or None, optional) – Percent point function of the given distribution, that takes a single argument Default: None
- ****kwargs** – Keyword-arguments forwarded to scipy.stats.rv_continuous.

Returns **dist** – The constructed distribution.

Return type scipy.stats.rv_continuous

Notes

At least pdf or cdf needs to be given.

3.11 gstools.tools

GStools subpackage providing miscellaneous tools.

Export

vtk_export(filename, pos, fields[, mesh_type])	Export a field to vtk.
vtk_export_structured(filename, pos, fields)	Export a field to vtk structured rectilinear grid file.
vtk_export_unstructured(filename, pos, fields)	Export a field to vtk unstructured grid file.
to_vtk(pos, fields[, mesh_type])	Create a VTK/PyVista grid.
to_vtk_structured(pos, fields)	Create a vtk structured rectilinear grid from a field.
to_vtk_unstructured(pos, fields)	Export a field to vtk structured rectilinear grid file.

Special functions

confidence_scaling([per])	Scaling of standard deviation to get the desired confidence interval.
inc_gamma(s, x)	Calculate the (upper) incomplete gamma function.
exp_int(s, x)	Calculate the exponential integral $E_s(x)$.
inc_beta(a, b, x)	Calculate the incomplete Beta function.
tplstable_cor(r, len_scale, hurst, alpha)	Calculate the correlation function of the TPLStable model.
tpl_exp_spec_dens(k, dim, len_scale, hurst)	Spectal density of the TPLExponential covariance model.
tpl_gau_spec_dens(k, dim, len_scale, hurst)	Spectal density of the TPLGaussian covariance model.

Geometric

rotated_main_axes(dim, angles)	Create list of the main axis defined by the given system rotations.
set_angles(dim, angles)	Set the angles for the given dimension.
set_anis(dim, anis)	Set the anisotropy ratios for the given dimension.
no_of_angles(dim)	Calculate number of rotation angles depending on the dimension.
rotation_planes(dim)	Get all 2D sub-planes for rotation.
givens_rotation(dim, plane, angle)	Givens rotation matrix in arbitrary dimensions.
matrix_rotate(dim, angles)	Create a matrix to rotate points to the target coordinate-system.
matrix_derotate(dim, angles)	Create a matrix to derotate points to the initial coordinate-system.
matrix_isotropify(dim, anis)	Create a stretching matrix to make things isotrope.
matrix_anisotropify(dim, anis)	Create a stretching matrix to make things anisotrope.
matrix_isometrize(dim, angles, anis)	Create a matrix to derotate points and make them isotrope.
matrix_anisometrize(dim, angles, anis)	Create a matrix to rotate points and make them anisotrope.
ang2dir(angles[, dtype, dim])	Convert n-D spherical coordinates to Euclidean direction vectors.
generate_grid(pos)	Generate grid from a structured position tuple.

continues on next page

Table 59 – continued from previous page

generate_st_grid(pos, time[, mesh_type])	Generate spatio-temporal grid from a position tuple and time array.

Misc

EARTH_RADIUS	earth radius for WGS84 ellipsoid in km

gstools.tools.**ang2dir**(*angles*, *dtype=<class 'numpy.float64'>*, *dim=None*)
Convert n-D spherical coordinates to Euclidean direction vectors.

Parameters

- **angles** (list of numpy.ndarray) – spherical coordinates given as angles.
- **dtype** (*data-type, optional*) – The desired data-type for the array. If not given, then the type will be determined as the minimum type required to hold the objects in the sequence. Default: None
- **dim** (int, optional) – Cut of information above the given dimension. Otherwise, dimension is determined by number of angles Default: None

Returns the array of direction vectors

Return type numpy.ndarray

gstools.tools.**confidence_scaling**(*per=0.95*)
Scaling of standard deviation to get the desired confidence interval.

Parameters **per** (float, optional) – Confidence level. The default is 0.95.

Returns Scale to multiply the standard deviation with.

Return type float

gstools.tools.**exp_int**(*s*, *x*)
Calculate the exponential integral $E_s(x)$.

Given by: $E_s(x) = \int_1^\infty \frac{e^{-xt}}{t^s}\,\mathrm{d}t$

Parameters

- **s** (float) – exponent in the integral (should be > -100)
- **x** (numpy.ndarray) – input values

gstools.tools.**generate_grid**(*pos*)
Generate grid from a structured position tuple.

Parameters **pos** (tuple of numpy.ndarray) – The structured position tuple.

Returns Unstructured position tuple.

Return type numpy.ndarray

gstools.tools.**generate_st_grid**(*pos*, *time*, *mesh_type='unstructured'*)
Generate spatio-temporal grid from a position tuple and time array.

Parameters

- **pos** (tuple of numpy.ndarray) – The (un-)structured position tuple.
- **time** (iterable) – The time array.
- **mesh_type** (str, optional) – 'structured' / 'unstructured' Default: *"unstructured"*

Returns Unstructured spatio-temporal point tuple.

Return type numpy.ndarray

Notes

Time dimension will be the last one.

gstools.tools.**givens_rotation**(*dim*, *plane*, *angle*)
Givens rotation matrix in arbitrary dimensions.

Parameters

- **dim** (int) – spatial dimension
- **plane** (list of int) – the plane to rotate in, given by the indices of the two defining axes. For example the xy plane is defined by *(0,1)*
- **angle** (float or list) – the rotation angle in the given plane

Returns Rotation matrix.

Return type numpy.ndarray

gstools.tools.**inc_beta**(*a*, *b*, *x*)
Calculate the incomplete Beta function.

Given by: $B(a, b;\, x) = \int_0^x t^{a-1}\,(1-t)^{b-1}\,dt$

Parameters

- **a** (float) – first exponent in the integral
- **b** (float) – second exponent in the integral
- **x** (numpy.ndarray) – input values

gstools.tools.**inc_gamma**(*s*, *x*)
Calculate the (upper) incomplete gamma function.

Given by: $\Gamma(s, x) = \int_x^\infty t^{s-1}\,e^{-t}\,\mathrm{d}t$

Parameters

- **s** (float) – exponent in the integral
- **x** (numpy.ndarray) – input values

gstools.tools.**matrix_anisometrize**(*dim*, *angles*, *anis*)
Create a matrix to rotate points and make them anisotrope.

Parameters

- **dim** (int) – spatial dimension
- **angles** (float or list) – the rotation angles of the target coordinate-system
- **anis** (list of float) – the anisotropy of length scales along the transversal directions

Returns Transformation matrix.

Return type numpy.ndarray

gstools.tools.**matrix_anisotropify**(*dim*, *anis*)
Create a stretching matrix to make things anisotrope.

Parameters

- **dim** (int) – spatial dimension
- **anis** (list of float) – the anisotropy of length scales along the transversal directions

Returns Stretching matrix.

Return type numpy.ndarray

gstools.tools.**matrix_derotate**(*dim*, *angles*)
Create a matrix to derotate points to the initial coordinate-system.

Parameters

- **dim** (int) – spatial dimension
- **angles** (float or list) – the rotation angles of the target coordinate-system

Returns Rotation matrix.

Return type numpy.ndarray

gstools.tools.**matrix_isometrize**(*dim*, *angles*, *anis*)
Create a matrix to derotate points and make them isotrope.

Parameters

- **dim** (int) – spatial dimension
- **angles** (float or list) – the rotation angles of the target coordinate-system
- **anis** (list of float) – the anisotropy of length scales along the transversal directions

Returns Transformation matrix.

Return type numpy.ndarray

gstools.tools.**matrix_isotropify**(*dim*, *anis*)
Create a stretching matrix to make things isotrope.

Parameters

- **dim** (int) – spatial dimension
- **anis** (list of float) – the anisotropy of length scales along the transversal directions

Returns Stretching matrix.

Return type numpy.ndarray

gstools.tools.**matrix_rotate**(*dim*, *angles*)
Create a matrix to rotate points to the target coordinate-system.

Parameters

- **dim** (int) – spatial dimension
- **angles** (float or list) – the rotation angles of the target coordinate-system

Returns Rotation matrix.

Return type numpy.ndarray

gstools.tools.**no_of_angles**(*dim*)
Calculate number of rotation angles depending on the dimension.

Parameters **dim** (int) – spatial dimension

Returns Number of angles.

Return type int

gstools.tools.**rotated_main_axes**(*dim*, *angles*)
Create list of the main axis defined by the given system rotations.

Parameters

- **dim** (int) – spatial dimension
- **angles** (float or list) – the rotation angles of the target coordinate-system

Returns Main axes of the target coordinate-system.

Return type numpy.ndarray

gstools.tools.**rotation_planes**(*dim*)
Get all 2D sub-planes for rotation.

Parameters **dim** (int) – spatial dimension

Returns All 2D sub-planes for rotation.

Return type list of tuple of int

gstools.tools.**set_angles**(*dim*, *angles*)
Set the angles for the given dimension.

Parameters

- **dim** (int) – spatial dimension
- **angles** (float or list) – the angles of the SRF

Returns **angles** – the angles fitting to the dimension

Return type float

Notes

If too few angles are given, they are filled up with *0*.

---

gstools.tools.**set_anis**(*dim*, *anis*)
Set the anisotropy ratios for the given dimension.

Parameters

- **dim** (int) – spatial dimension
- **anis** (list of float) – the anisotropy of length scales along the transversal directions

Returns **anis** – the anisotropy of length scales fitting the dimensions

Return type list of float

Notes

If too few anisotropy ratios are given, they are filled up with *1*.

---

gstools.tools.**to_vtk**(*pos*, *fields*, *mesh_type='unstructured'*)
Create a VTK/PyVista grid.

Parameters

- **pos** (list) – the position tuple, containing main direction and transversal directions
- **fields** (dict or numpy.ndarray) – [Un]structured fields to be saved. Either a single numpy array as returned by SRF, or a dictionary of fields with theirs names as keys.
- **mesh_type** (str, optional) – 'structured' / 'unstructured'. Default: structured

Returns This will return a PyVista object for the given field data in its appropriate type. Structured meshes will return a pyvista.RectilinearGrid and unstructured meshes will return an pyvista.UnstructuredGrid object.

Return type pyvista.RectilinearGrid or pyvista.UnstructuredGrid

gstools.tools.**to_vtk_structured**(*pos*, *fields*)
Create a vtk structured rectilinear grid from a field.

Parameters

- **pos** (list) – the position tuple, containing main direction and transversal directions

- **fields** (dict or numpy.ndarray) – Structured fields to be saved. Either a single numpy array as returned by SRF, or a dictionary of fields with theirs names as keys.

Returns A PyVista rectilinear grid of the structured field data. Data arrays live on the point data of this PyVista dataset.

Return type pyvista.RectilinearGrid

gstools.tools.**to_vtk_unstructured**(*pos*, *fields*)
Export a field to vtk structured rectilinear grid file.

Parameters

- **pos** (list) – the position tuple, containing main direction and transversal directions
- **fields** (dict or numpy.ndarray) – Unstructured fields to be saved. Either a single numpy array as returned by SRF, or a dictionary of fields with theirs names as keys.

Returns A PyVista unstructured grid of the unstructured field data. Data arrays live on the point data of this PyVista dataset. This is essentially a point cloud with no topology.

Return type pyvista.UnstructuredGrid

gstools.tools.**tpl_exp_spec_dens**(*k*, *dim*, *len_scale*, *hurst*, *len_low=0.0*)
Spectal density of the TPLExponential covariance model.

Parameters

- **k** (float) – Radius of the phase: $k = \|\mathbf{k}\|$
- **dim** (int) – Dimension of the model.
- **len_scale** (float) – Length scale of the model.
- **hurst** (float) – Hurst coefficient of the power law.
- **len_low** (float, optional) – The lower length scale truncation of the model. Default: 0.0

Returns spectal density of the TPLExponential model

Return type float

gstools.tools.**tpl_gau_spec_dens**(*k*, *dim*, *len_scale*, *hurst*, *len_low=0.0*)
Spectal density of the TPLGaussian covariance model.

Parameters

- **k** (float) – Radius of the phase: $k = \|\mathbf{k}\|$
- **dim** (int) – Dimension of the model.
- **len_scale** (float) – Length scale of the model.
- **hurst** (float) – Hurst coefficient of the power law.
- **len_low** (float, optional) – The lower length scale truncation of the model. Default: 0.0

Returns spectal density of the TPLExponential model

Return type float

gstools.tools.**tplstable_cor**(*r*, *len_scale*, *hurst*, *alpha*)
Calculate the correlation function of the TPLStable model.

Given by the following correlation function:

$$\rho(r) = \frac{2H}{\alpha} \cdot E_{1+\frac{2H}{\alpha}}\left(\left(\frac{r}{\ell}\right)^{\alpha}\right)$$

Parameters

- **r** (numpy.ndarray) – input values
- **len_scale** (float) – length-scale of the model.
- **hurst** (float) – Hurst coefficient of the power law.
- **alpha** (float, optional) – Shape parameter of the stable model.

gstools.tools.**vtk_export**(*filename*, *pos*, *fields*, *mesh_type='unstructured'*)
Export a field to vtk.

Parameters

- **filename** (str) – Filename of the file to be saved, including the path. Note that an ending (.vtr or .vtu) will be added to the name.
- **pos** (list) – the position tuple, containing main direction and transversal directions
- **fields** (dict or numpy.ndarray) – [Un]structured fields to be saved. Either a single numpy array as returned by SRF, or a dictionary of fields with theirs names as keys.
- **mesh_type** (str, optional) – 'structured' / 'unstructured'. Default: structured

gstools.tools.**vtk_export_structured**(*filename*, *pos*, *fields*)
Export a field to vtk structured rectilinear grid file.

Parameters

- **filename** (str) – Filename of the file to be saved, including the path. Note that an ending (.vtr) will be added to the name.
- **pos** (list) – the position tuple, containing main direction and transversal directions
- **fields** (dict or numpy.ndarray) – Structured fields to be saved. Either a single numpy array as returned by SRF, or a dictionary of fields with theirs names as keys.

gstools.tools.**vtk_export_unstructured**(*filename*, *pos*, *fields*)
Export a field to vtk unstructured grid file.

Parameters

- **filename** (str) – Filename of the file to be saved, including the path. Note that an ending (.vtu) will be added to the name.
- **pos** (list) – the position tuple, containing main direction and transversal directions
- **fields** (dict or numpy.ndarray) – Unstructured fields to be saved. Either a single numpy array as returned by SRF, or a dictionary of fields with theirs names as keys.

gstools.tools.**EARTH_RADIUS = 6371.0**
earth radius for WGS84 ellipsoid in km

Type float

3.12 gstools.transform

GStools subpackage providing transformations to post-process normal fields.

Field-Transformations

binary(fld[, divide, upper, lower])	Binary transformation.
discrete(fld, values[, thresholds])	Discrete transformation.
boxcox(fld[, lmbda, shift])	(Inverse) Box-Cox transformation to denormalize data.
zinnharvey(fld[, conn])	Zinn and Harvey transformation to connect low or high values.
normal_force_moments(fld)	Force moments of a normal distributed field.
normal_to_lognormal(fld)	Transform normal distribution to log-normal distribution.
normal_to_uniform(fld)	Transform normal distribution to uniform distribution on [0, 1].
normal_to_arcsin(fld[, a, b])	Transform normal distribution to the bimodal arcsin distribution.
normal_to_uquad(fld[, a, b])	Transform normal distribution to U-quadratic distribution.

gstools.transform.**binary**(*fld*, *divide=None*, *upper=None*, *lower=None*)

Binary transformation.

After this transformation, the field only has two values.

Parameters

- **fld** (*Field*) – Spatial Random Field class containing a generated field. Field will be transformed inplace.
- **divide** (float, optional) – The dividing value. Default: `fld.mean`
- **upper** (float, optional) – The resulting upper value of the field. Default: `mean + sqrt(fld.model.sill)`
- **lower** (float, optional) – The resulting lower value of the field. Default: `mean - sqrt(fld.model.sill)`

gstools.transform.**boxcox**(*fld*, *lmbda=1*, *shift=0*)

(Inverse) Box-Cox transformation to denormalize data.

After this transformation, the again Box-Cox transformed field is normal distributed.

See: https://en.wikipedia.org/wiki/Power_transform#Box%E2%80%93Cox_transformation

Parameters

- **fld** (*Field*) – Spatial Random Field class containing a generated field. Field will be transformed inplace.
- **lmbda** (float, optional) – The lambda parameter of the Box-Cox transformation. For `lmbda=0` one obtains the log-normal transformation. Default: `1`
- **shift** (float, optional) – The shift parameter from the two-parametric Box-Cox transformation. The field will be shifted by that value before transformation. Default: `0`

gstools.transform.**discrete**(*fld*, *values*, *thresholds='arithmetic'*)

Discrete transformation.

After this transformation, the field has only *len(values)* discrete values.

Parameters

- **fld** (*Field*) – Spatial Random Field class containing a generated field. Field will be transformed inplace.
- **values** (numpy.ndarray) – The discrete values the field will take
- **thresholds** (str or numpy.ndarray, optional) – the thresholds, where the value classes are separated possible values are: * "arithmetic": the mean of the 2 neighbouring values * "equal": devide the field into equal parts * an array of explicitly given thresholds Default: "arithmetic"

gstools.transform.**normal_force_moments**(*fld*)
Force moments of a normal distributed field.

After this transformation, the field is still normal distributed.

Parameters **fld** (*Field*) – Spatial Random Field class containing a generated field. Field will be transformed inplace.

gstools.transform.**normal_to_arcsin**(*fld*, *a=None*, *b=None*)
Transform normal distribution to the bimodal arcsin distribution.

See: https://en.wikipedia.org/wiki/Arcsine_distribution

After this transformation, the field is arcsin-distributed on [a, b].

Parameters

- **fld** (*Field*) – Spatial Random Field class containing a generated field. Field will be transformed inplace.
- **a** (float, optional) – Parameter a of the arcsin distribution (lower bound). Default: keep mean and variance
- **b** (float, optional) – Parameter b of the arcsin distribution (upper bound). Default: keep mean and variance

gstools.transform.**normal_to_lognormal**(*fld*)
Transform normal distribution to log-normal distribution.

After this transformation, the field is log-normal distributed.

Parameters **fld** (*Field*) – Spatial Random Field class containing a generated field. Field will be transformed inplace.

gstools.transform.**normal_to_uniform**(*fld*)
Transform normal distribution to uniform distribution on [0, 1].

After this transformation, the field is uniformly distributed on [0, 1].

Parameters **fld** (*Field*) – Spatial Random Field class containing a generated field. Field will be transformed inplace.

gstools.transform.**normal_to_uquad**(*fld*, *a=None*, *b=None*)
Transform normal distribution to U-quadratic distribution.

See: https://en.wikipedia.org/wiki/U-quadratic_distribution

After this transformation, the field is U-quadratic-distributed on [a, b].

Parameters

- **fld** (*Field*) – Spatial Random Field class containing a generated field. Field will be transformed inplace.
- **a** (float, optional) – Parameter a of the U-quadratic distribution (lower bound). Default: keep mean and variance

- **b** (float, optional) – Parameter b of the U-quadratic distribution (upper bound). Default: keep mean and variance

gstools.transform.**zinnharvey**(*fld*, *conn='high'*)

Zinn and Harvey transformation to connect low or high values.

After this transformation, the field is still normal distributed.

Parameters

- **fld** (*Field*) – Spatial Random Field class containing a generated field. Field will be transformed inplace.
- **conn** (str, optional) – Desired connectivity. Either "low" or "high". Default: "high"

3.13 gstools.normalizer

GStools subpackage providing normalization routines.

Base-Normalizer

Normalizer([data])	Normalizer class.

gstools.normalizer.Normalizer

class gstools.normalizer.**Normalizer**(*data=None*, ***parameter*)

Bases: object

Normalizer class.

Parameters

- **data** (*array_like, optional*) – Input data to fit the transformation to in order to gain normality. The default is None.
- ****parameter** – Specified parameters given by name. If not given, default parameters will be used.

Attributes

name str: The name of the normalizer class.

Methods

denormalize(data)	Transform to input distribution.
derivative(data)	Factor for normal PDF to gain target PDF.
fit(data[, skip])	Fitting the transformation to data by maximizing Log-Likelihood.
kernel_loglikelihood(data)	Kernel Log-Likelihood for given data with current parameters.
likelihood(data)	Likelihood for given data with current parameters.
loglikelihood(data)	Log-Likelihood for given data with current parameters.
normalize(data)	Transform to normal distribution.

denormalize(*data*)

Transform to input distribution.

Parameters **data** (*array_like*) – Input data (normal distributed).

Returns Denormalized data.

Return type numpy.ndarray

derivative(*data*)

Factor for normal PDF to gain target PDF.

Parameters **data** (*array_like*) – Input data (not normal distributed).

Returns Derivative of the normalization transformation function.

Return type numpy.ndarray

fit(*data*, *skip=None*, ***kwargs*)

Fitting the transformation to data by maximizing Log-Likelihood.

Parameters

- **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.
- **skip** (list of str or None, optional) – Names of parameters to be skiped in fitting. The default is None.
- ****kwargs** – Keyword arguments passed to scipy.optimize.minimize_scalar when only one parameter present or scipy.optimize.minimize.

Returns Optimal paramters given by names.

Return type dict

kernel_loglikelihood(*data*)
Kernel Log-Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Kernel Log-Likelihood of the given data.

Return type float

Notes

This loglikelihood function is neglecting additive constants, that are not needed for optimization.

---

likelihood(*data*)
Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Likelihood of the given data.

Return type float

loglikelihood(*data*)
Log-Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Log-Likelihood of the given data.

Return type float

normalize(*data*)
Transform to normal distribution.

Parameters **data** (*array_like*) – Input data (not normal distributed).

Returns Normalized data.

Return type numpy.ndarray

default_parameter = {}
Default parameters of the Normalizer.

Type dict

denormalize_range = (-inf, inf)
Valid range for output/normal data.

Type tuple

property name
The name of the normalizer class.

Type str

normalize_range = (-inf, inf)
Valid range for input data.

Type tuple

Field-Normalizer

LogNormal([data])	Log-normal fields.
BoxCox([data])	Box-Cox (1964) transformed fields.
BoxCoxShift([data])	Box-Cox (1964) transformed fields including shifting.
YeoJohnson([data])	Yeo-Johnson (2000) transformed fields.
Modulus([data])	Modulus or John-Draper (1980) transformed fields.
Manly([data])	Manly (1971) transformed fields.

gstools.normalizer.LogNormal

class gstools.normalizer.**LogNormal**(*data=None*, ***parameter*)

Bases: *gstools.normalizer.base.Normalizer*

Log-normal fields.

Notes

This parameter-free transformation is given by:

$$y = \log(x)$$

Attributes

name `str`: The name of the normalizer class.

Methods

denormalize(data)	Transform to input distribution.
derivative(data)	Factor for normal PDF to gain target PDF.
fit(data[, skip])	Fitting the transformation to data by maximizing Log-Likelihood.
kernel_loglikelihood(data)	Kernel Log-Likelihood for given data with current parameters.
likelihood(data)	Likelihood for given data with current parameters.
loglikelihood(data)	Log-Likelihood for given data with current parameters.
normalize(data)	Transform to normal distribution.

denormalize(*data*)

Transform to input distribution.

Parameters **data** (*array_like*) – Input data (normal distributed).

Returns Denormalized data.

Return type `numpy.ndarray`

derivative(*data*)

Factor for normal PDF to gain target PDF.

Parameters **data** (*array_like*) – Input data (not normal distributed).

Returns Derivative of the normalization transformation function.

Return type `numpy.ndarray`

fit(*data*, *skip=None*, ***kwargs*)
Fitting the transformation to data by maximizing Log-Likelihood.

Parameters

- **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.
- **skip** (list of str or None, optional) – Names of parameters to be skiped in fitting. The default is None.
- ****kwargs** – Keyword arguments passed to scipy.optimize.minimize_scalar when only one parameter present or scipy.optimize.minimize.

Returns Optimal paramters given by names.

Return type dict

kernel_loglikelihood(*data*)
Kernel Log-Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Kernel Log-Likelihood of the given data.

Return type float

Notes

This loglikelihood function is neglecting additive constants, that are not needed for optimization.

---

likelihood(*data*)
Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Likelihood of the given data.

Return type float

loglikelihood(*data*)
Log-Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Log-Likelihood of the given data.

Return type float

normalize(*data*)
Transform to normal distribution.

Parameters **data** (*array_like*) – Input data (not normal distributed).

Returns Normalized data.

Return type numpy.ndarray

default_parameter = {}
Default parameters of the Normalizer.

Type dict

denormalize_range = (-inf, inf)
Valid range for output/normal data.

Type tuple

property name
The name of the normalizer class.

Type str

normalize_range = (0.0, inf)
Valid range for input data.

gstools.normalizer.BoxCox

class gstools.normalizer.**BoxCox**(*data=None*, ***parameter*)

Bases: *gstools.normalizer.base.Normalizer*

Box-Cox (1964) transformed fields.

Parameters

- **data** (*array_like, optional*) – Input data to fit the transformation in order to gain normality. The default is None.
- **lmbda** (float, optional) – Shape parameter. Default: 1

Notes

This transformation is given by [Box1964]:

$$y = \begin{cases} \frac{x^\lambda - 1}{\lambda} & \lambda \neq 0 \\ \log(x) & \lambda = 0 \end{cases}$$

References

Attributes

denormalize_range tuple: Valid range for output data depending on lmbda.

name str: The name of the normalizer class.

Methods

denormalize(data)	Transform to input distribution.
derivative(data)	Factor for normal PDF to gain target PDF.
fit(data[, skip])	Fitting the transformation to data by maximizing Log-Likelihood.
kernel_loglikelihood(data)	Kernel Log-Likelihood for given data with current parameters.
likelihood(data)	Likelihood for given data with current parameters.
loglikelihood(data)	Log-Likelihood for given data with current parameters.
normalize(data)	Transform to normal distribution.

denormalize(*data*)

Transform to input distribution.

Parameters **data** (*array_like*) – Input data (normal distributed).

Returns Denormalized data.

Return type numpy.ndarray

derivative(*data*)

Factor for normal PDF to gain target PDF.

Parameters **data** (*array_like*) – Input data (not normal distributed).

Returns Derivative of the normalization transformation function.

Return type numpy.ndarray

fit(*data*, *skip=None*, ***kwargs*)
Fitting the transformation to data by maximizing Log-Likelihood.

Parameters

- **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.
- **skip** (list of str or None, optional) – Names of parameters to be skiped in fitting. The default is None.
- ****kwargs** – Keyword arguments passed to scipy.optimize.minimize_scalar when only one parameter present or scipy.optimize.minimize.

Returns Optimal paramters given by names.

Return type dict

kernel_loglikelihood(*data*)
Kernel Log-Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Kernel Log-Likelihood of the given data.

Return type float

Notes

This loglikelihood function is neglecting additive constants, that are not needed for optimization.

---

likelihood(*data*)
Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Likelihood of the given data.

Return type float

loglikelihood(*data*)
Log-Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Log-Likelihood of the given data.

Return type float

normalize(*data*)
Transform to normal distribution.

Parameters **data** (*array_like*) – Input data (not normal distributed).

Returns Normalized data.

Return type numpy.ndarray

default_parameter = {'lmbda': 1}
Default parameter of the BoxCox-Normalizer.

Type dict

property denormalize_range
Valid range for output data depending on lmbda.

(-1/lmbda, inf) or *(-inf, -1/lmbda)*

Type tuple

property name
The name of the normalizer class.

Type str

normalize_range = (0.0, inf)
Valid range for input data.

Type tuple

gstools.normalizer.BoxCoxShift

class gstools.normalizer.**BoxCoxShift**(*data=None*, ***parameter*)

Bases: *gstools.normalizer.base.Normalizer*

Box-Cox (1964) transformed fields including shifting.

Parameters

- **data** (*array_like, optional*) – Input data to fit the transformation in order to gain normality. The default is None.
- **lmbda** (float, optional) – Shape parameter. Default: 1
- **shift** (float, optional) – Shift parameter. Default: 0

Notes

This transformation is given by [Box1964]:

$$y = \begin{cases} \frac{(x+s)^\lambda - 1}{\lambda} & \lambda \neq 0 \\ \log(x+s) & \lambda = 0 \end{cases}$$

Fitting the shift parameter is rather hard. You should consider skipping "shift" during fitting:

```
>>> data = range(5)
>>> norm = BoxCoxShift(shift=0.5)
>>> norm.fit(data, skip=["shift"])
{'shift': 0.5, 'lmbda': 0.6747515267420799}
```

References

Attributes

denormalize_range tuple: Valid range for output data depending on lmbda.

name str: The name of the normalizer class.

normalize_range tuple: Valid range for input data depending on shift.

Methods

denormalize(data)	Transform to input distribution.
derivative(data)	Factor for normal PDF to gain target PDF.
fit(data[, skip])	Fitting the transformation to data by maximizing Log-Likelihood.
kernel_loglikelihood(data)	Kernel Log-Likelihood for given data with current parameters.
likelihood(data)	Likelihood for given data with current parameters.
loglikelihood(data)	Log-Likelihood for given data with current parameters.
normalize(data)	Transform to normal distribution.

denormalize(*data*)

Transform to input distribution.

Parameters **data** (*array_like*) – Input data (normal distributed).

Returns Denormalized data.

Return type numpy.ndarray

derivative(*data*)
Factor for normal PDF to gain target PDF.

Parameters **data** (*array_like*) – Input data (not normal distributed).

Returns Derivative of the normalization transformation function.

Return type numpy.ndarray

fit(*data*, *skip=None*, ***kwargs*)
Fitting the transformation to data by maximizing Log-Likelihood.

Parameters

- **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.
- **skip** (list of str or None, optional) – Names of parameters to be skiped in fitting. The default is None.
- ****kwargs** – Keyword arguments passed to scipy.optimize.minimize_scalar when only one parameter present or scipy.optimize.minimize.

Returns Optimal paramters given by names.

Return type dict

kernel_loglikelihood(*data*)
Kernel Log-Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Kernel Log-Likelihood of the given data.

Return type float

Notes

This loglikelihood function is neglecting additive constants, that are not needed for optimization.

---

likelihood(*data*)
Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Likelihood of the given data.

Return type float

loglikelihood(*data*)
Log-Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Log-Likelihood of the given data.

Return type float

normalize(*data*)
Transform to normal distribution.

Parameters **data** (*array_like*) – Input data (not normal distributed).

Returns Normalized data.

Return type numpy.ndarray

default_parameter = {'lmbda': 1, 'shift': 0}
Default parameters of the BoxCoxShift-Normalizer.

Type dict

property denormalize_range
Valid range for output data depending on lmbda.

(-1/lmbda, inf) or *(-inf, -1/lmbda)*

Type tuple

property name
The name of the normalizer class.

Type str

property normalize_range
Valid range for input data depending on shift.

(-shift, inf)

Type tuple

gstools.normalizer.YeoJohnson

class gstools.normalizer.**YeoJohnson**(*data=None*, ***parameter*)

Bases: *gstools.normalizer.base.Normalizer*

Yeo-Johnson (2000) transformed fields.

Parameters

- **data** (*array_like, optional*) – Input data to fit the transformation in order to gain normality. The default is None.
- **lmbda** (float, optional) – Shape parameter. Default: 1

Notes

This transformation is given by [Yeo2000]:

$$y = \begin{cases} \frac{(x+1)^\lambda - 1}{\lambda} & x \geq 0,\ \lambda \neq 0 \\ \log(x+1) & x \geq 0,\ \lambda = 0 \\ -\frac{(|x|+1)^{2-\lambda} - 1}{2-\lambda} & x < 0,\ \lambda \neq 2 \\ -\log(|x|+1) & x < 0,\ \lambda = 2 \end{cases}$$

References

Attributes

name str: The name of the normalizer class.

Methods

denormalize(data)	Transform to input distribution.
derivative(data)	Factor for normal PDF to gain target PDF.
fit(data[, skip])	Fitting the transformation to data by maximizing Log-Likelihood.
kernel_loglikelihood(data)	Kernel Log-Likelihood for given data with current parameters.
likelihood(data)	Likelihood for given data with current parameters.
loglikelihood(data)	Log-Likelihood for given data with current parameters.
normalize(data)	Transform to normal distribution.

denormalize(*data*)

Transform to input distribution.

Parameters **data** (*array_like*) – Input data (normal distributed).

Returns Denormalized data.

Return type numpy.ndarray

derivative(*data*)

Factor for normal PDF to gain target PDF.

Parameters **data** (*array_like*) – Input data (not normal distributed).

Returns Derivative of the normalization transformation function.

Return type numpy.ndarray

fit(*data*, *skip=None*, ***kwargs*)
Fitting the transformation to data by maximizing Log-Likelihood.

Parameters

- **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.
- **skip** (list of str or None, optional) – Names of parameters to be skiped in fitting. The default is None.
- ****kwargs** – Keyword arguments passed to scipy.optimize.minimize_scalar when only one parameter present or scipy.optimize.minimize.

Returns Optimal paramters given by names.

Return type dict

kernel_loglikelihood(*data*)
Kernel Log-Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Kernel Log-Likelihood of the given data.

Return type float

Notes

This loglikelihood function is neglecting additive constants, that are not needed for optimization.

likelihood(*data*)
Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Likelihood of the given data.

Return type float

loglikelihood(*data*)
Log-Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Log-Likelihood of the given data.

Return type float

normalize(*data*)
Transform to normal distribution.

Parameters **data** (*array_like*) – Input data (not normal distributed).

Returns Normalized data.

Return type numpy.ndarray

default_parameter = {'lmbda': 1}
Default parameter of the YeoJohnson-Normalizer.

Type dict

denormalize_range = (-inf, inf)
Valid range for output/normal data.

Type tuple

property name
The name of the normalizer class.

Type str

normalize_range = (-inf, inf)
Valid range for input data.

Type tuple

gstools.normalizer.Modulus

class gstools.normalizer.**Modulus**(*data=None*, ***parameter*)

Bases: *gstools.normalizer.base.Normalizer*

Modulus or John-Draper (1980) transformed fields.

Parameters

- **data** (*array_like, optional*) – Input data to fit the transformation in order to gain normality. The default is None.
- **lmbda** (float, optional) – Shape parameter. Default: 1

Notes

This transformation is given by [John1980]:

$$y = \begin{cases} \operatorname{sgn}(x)\frac{(|x|+1)^{\lambda}-1}{\lambda} & \lambda \neq 0 \\ \operatorname{sgn}(x)\log(|x|+1) & \lambda = 0 \end{cases}$$

References

Attributes

name str: The name of the normalizer class.

Methods

denormalize(data)	Transform to input distribution.
derivative(data)	Factor for normal PDF to gain target PDF.
fit(data[, skip])	Fitting the transformation to data by maximizing Log-Likelihood.
kernel_loglikelihood(data)	Kernel Log-Likelihood for given data with current parameters.
likelihood(data)	Likelihood for given data with current parameters.
loglikelihood(data)	Log-Likelihood for given data with current parameters.
normalize(data)	Transform to normal distribution.

denormalize(*data*)

Transform to input distribution.

Parameters **data** (*array_like*) – Input data (normal distributed).

Returns Denormalized data.

Return type numpy.ndarray

derivative(*data*)

Factor for normal PDF to gain target PDF.

Parameters **data** (*array_like*) – Input data (not normal distributed).

Returns Derivative of the normalization transformation function.

Return type numpy.ndarray

fit(*data*, *skip=None*, ***kwargs*)

Fitting the transformation to data by maximizing Log-Likelihood.

Parameters

- **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.
- **skip** (list of str or None, optional) – Names of parameters to be skiped in fitting. The default is None.
- ****kwargs** – Keyword arguments passed to scipy.optimize.minimize_scalar when only one parameter present or scipy.optimize.minimize.

Returns Optimal paramters given by names.

Return type dict

kernel_loglikelihood(*data*)
Kernel Log-Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Kernel Log-Likelihood of the given data.

Return type float

Notes

This loglikelihood function is neglecting additive constants, that are not needed for optimization.

---

likelihood(*data*)
Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Likelihood of the given data.

Return type float

loglikelihood(*data*)
Log-Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Log-Likelihood of the given data.

Return type float

normalize(*data*)
Transform to normal distribution.

Parameters **data** (*array_like*) – Input data (not normal distributed).

Returns Normalized data.

Return type numpy.ndarray

default_parameter = {'lmbda': 1}
Default parameter of the Modulus-Normalizer.

Type dict

denormalize_range = (-inf, inf)
Valid range for output/normal data.

Type tuple

property name
The name of the normalizer class.

Type str

normalize_range = (-inf, inf)
Valid range for input data.

Type tuple

gstools.normalizer.Manly

class gstools.normalizer.**Manly**(*data=None*, ***parameter*)

Bases: *gstools.normalizer.base.Normalizer*

Manly (1971) transformed fields.

Parameters

- **data** (*array_like, optional*) – Input data to fit the transformation in order to gain normality. The default is None.
- **lmbda** (float, optional) – Shape parameter. Default: 1

Notes

This transformation is given by [Manly1976]:

$$y = \begin{cases} \frac{\exp(\lambda x)-1}{\lambda} & \lambda \neq 0 \\ x & \lambda = 0 \end{cases}$$

References

Attributes

denormalize_range tuple: Valid range for output data depending on lmbda.

name str: The name of the normalizer class.

Methods

denormalize(data)	Transform to input distribution.
derivative(data)	Factor for normal PDF to gain target PDF.
fit(data[, skip])	Fitting the transformation to data by maximizing Log-Likelihood.
kernel_loglikelihood(data)	Kernel Log-Likelihood for given data with current parameters.
likelihood(data)	Likelihood for given data with current parameters.
loglikelihood(data)	Log-Likelihood for given data with current parameters.
normalize(data)	Transform to normal distribution.

denormalize(*data*)

Transform to input distribution.

Parameters **data** (*array_like*) – Input data (normal distributed).

Returns Denormalized data.

Return type numpy.ndarray

derivative(*data*)

Factor for normal PDF to gain target PDF.

Parameters **data** (*array_like*) – Input data (not normal distributed).

Returns Derivative of the normalization transformation function.

Return type numpy.ndarray

fit(*data*, *skip=None*, ***kwargs*)
Fitting the transformation to data by maximizing Log-Likelihood.

Parameters

- **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.
- **skip** (list of str or None, optional) – Names of parameters to be skiped in fitting. The default is None.
- ****kwargs** – Keyword arguments passed to scipy.optimize.minimize_scalar when only one parameter present or scipy.optimize.minimize.

Returns Optimal paramters given by names.

Return type dict

kernel_loglikelihood(*data*)
Kernel Log-Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Kernel Log-Likelihood of the given data.

Return type float

Notes

This loglikelihood function is neglecting additive constants, that are not needed for optimization.

---

likelihood(*data*)
Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Likelihood of the given data.

Return type float

loglikelihood(*data*)
Log-Likelihood for given data with current parameters.

Parameters **data** (*array_like*) – Input data to fit the transformation to in order to gain normality.

Returns Log-Likelihood of the given data.

Return type float

normalize(*data*)
Transform to normal distribution.

Parameters **data** (*array_like*) – Input data (not normal distributed).

Returns Normalized data.

Return type numpy.ndarray

default_parameter = {'lmbda': 1}
Default parameter of the Manly-Normalizer.

Type dict

property denormalize_range
Valid range for output data depending on lmbda.

(-1/lmbda, inf) or *(-inf, -1/lmbda)*

Type tuple

property name
The name of the normalizer class.

Type str

normalize_range = (-inf, inf)
Valid range for input data.

Type tuple

Convenience Routines

apply_mean_norm_trend(pos, field[, mean, ...])	Apply mean, de-normalization and trend to given field.
remove_trend_norm_mean(pos, field[, mean, ...])	Remove trend, de-normalization and mean from given field.

gstools.normalizer.apply_mean_norm_trend

gstools.normalizer.**apply_mean_norm_trend**(*pos*, *field*, *mean=None*, *normalizer=None*, *trend=None*, *mesh_type='unstructured'*, *value_type='scalar'*, *check_shape=True*, *stacked=False*)

Apply mean, de-normalization and trend to given field.

Parameters

- **pos** (iterable) – Position tuple, containing main direction and transversal directions.
- **field** (numpy.ndarray or list of numpy.ndarray) – The spatially distributed data. You can pass a list of fields, that will be used simultaneously. Then you need to set stacked=True.
- **mean** (None or float or callable, optional) – Mean of the field if wanted. Could also be a callable. The default is None.
- **normalizer** (None or *Normalizer*, optional) – Normalizer to be applied to the field. The default is None.
- **trend** (None or float or callable, optional) – Trend of the denormalized fields. If no normalizer is applied, this behaves equal to 'mean'. The default is None.
- **mesh_type** (str, optional) – 'structured' / 'unstructured' Default: 'unstructured'
- **value_type** (str, optional) – Value type of the field. Either "scalar" or "vector". The default is "scalar".
- **check_shape** (bool, optional) – Wheather to check pos and field shapes. The default is True.
- **stacked** (bool, optional) – Wheather the field is stacked or not. The default is False.

Returns **field** – The transformed field.

Return type numpy.ndarray

gstools.normalizer.remove_trend_norm_mean

gstools.normalizer.**remove_trend_norm_mean**(*pos*, *field*, *mean=None*, *normalizer=None*, *trend=None*, *mesh_type='unstructured'*, *value_type='scalar'*, *check_shape=True*, *stacked=False*, *fit_normalizer=False*)

Remove trend, de-normalization and mean from given field.

Parameters

- **pos** (iterable) – Position tuple, containing main direction and transversal directions.
- **field** (numpy.ndarray or list of numpy.ndarray) – The spatially distributed data. You can pass a list of fields, that will be used simultaneously. Then you need to set stacked=True.
- **mean** (None or float or callable, optional) – Mean of the field if wanted. Could also be a callable. The default is None.

- **normalizer** (None or *Normalizer*, optional) – Normalizer to be applied to the field. The default is None.
- **trend** (None or float or callable, optional) – Trend of the denormalized fields. If no normalizer is applied, this behaves equal to 'mean'. The default is None.
- **mesh_type** (str, optional) – 'structured' / 'unstructured' Default: 'unstructured'
- **value_type** (str, optional) – Value type of the field. Either "scalar" or "vector". The default is "scalar".
- **check_shape** (bool, optional) – Wheather to check pos and field shapes. The default is True.
- **stacked** (bool, optional) – Wheather the field is stacked or not. The default is False.
- **fit_normalizer** (bool, optional) – Wheater to fit the data-normalizer to the given (detrended) field. Default: False

Returns

- **field** (numpy.ndarray) – The cleaned field.
- **normalizer** (*Normalizer*, optional) – The fitted normalizer for the given data. Only provided if *fit_normalizer* is True.

CHAPTER 4

CHANGELOG

All notable changes to **GSTools** will be documented in this file.

4.1 1.3.1 - Pure Pink - 2021-06

Enhancements

- Standalone use of Field class #166
- add social badges in README #169, #170

Bugfixes

- use `oldest-supported-numpy` to build cython extensions #165

4.2 1.3.0 - Pure Pink - 2021-04

Topics

Geographical Coordinates Support (#113)

- added boolean init parameter `latlon` to indicate a geographic model. When given, spatial dimension is fixed to `dim=3`, `anis` and `angles` will be ignored, since anisotropy is not well-defined on a sphere.
- add property `field_dim` to indicate the dimension of the resulting field. Will be 2 if `latlon=True`
- added yadrenko variogram, covariance and correlation method, since the geographic models are derived from standard models in 3D by plugging in the chordal distance of two points on a sphere derived from there great-circle distance `zeta`:
  - `vario_yadrenko`: given by `variogram(2 * np.sin(zeta / 2))`
  - `cov_yadrenko`: given by `covariance(2 * np.sin(zeta / 2))`
  - `cor_yadrenko`: given by `correlation(2 * np.sin(zeta / 2))`
- added plotting routines for yadrenko methods described above

- the `isometrize` and `anisometrize` methods will convert `latlon` tuples (given in degree) to points on the unit-sphere in 3D and vice versa
- representation of geographical models don't display the `dim`, `anis` and `angles` parameters, but `latlon=True`
- `fit_variogram` will expect an estimated variogram with great-circle distances given in radians
- **Variogram estimation**
  - `latlon` switch implemented in `estimate_vario` routine
  - will return a variogram estimated by the great-circle distance (haversine formula) given in radians
- **Field**
  - added plotting routines for latlon fields
  - no vector fields possible on latlon fields
  - corretly handle pos tuple for latlon fields

Krige Unification (#97)

- Swiss Army Knife for kriging: The `Krige` class now provides everything in one place
- "Kriging the mean" is now possible with the switch `only_mean` in the call routine
- `Simple/Ordinary/Universal/ExtDrift/Detrended` are only shortcuts to `Krige` with limited input parameter list
- We now use the `covariance` function to build up the kriging matrix (instead of variogram)
- An `unbiased` switch was added to enable simple kriging (where the unbiased condition is not given)
- An `exact` switch was added to allow smother results, if a `nugget` is present in the model
- An `cond_err` parameter was added, where measurement error variances can be given for each conditional point
- pseudo-inverse matrix is now used to solve the kriging system (can be disabled by the new switch `pseudo_inv`), this is equal to solving the system with least-squares and prevents numerical errors
- added options `fit_normalizer` and `fit_variogram` to automatically fit normalizer and variogram to given data

Directional Variograms and Auto-binning (#87, #106, #131)

- new routine name `vario_estimate` instead of `vario_estimate_unstructured` (old kept for legacy code) for simplicity
- new routine name `vario_estimate_axis` instead of `vario_estimate_structured` (old kept for legacy code) for simplicity
- **vario_estimate**
  - added simple automatic binning routine to determine bins from given data (one third of box diameter as max bin distance, sturges rule for number of bins)
  - allow to pass multiple fields for joint variogram estimation (e.g. for daily precipitation) on same mesh
  - `no_data` option added to allow missing values
  - **masked fields**
    - user can now pass a masked array (or a list of masked arrays) to deselect data points.
    - in addition, a `mask` keyword was added to provide an external mask
  - **directional variograms**

        * diretional variograms can now be estimated
        * either provide a list of direction vectors or angles for directions (spherical coordinates)
        * can be controlled by given angle tolerance and (optional) bandwidth
        * prepared for nD
    - structured fields (pos tuple describes axes) can now be passed to estimate an isotropic or directional variogram
    - distance calculation in cython routines in now independent of dimension
- **vario_estimate_axis**
    - estimation along array axis now possible in arbitrary dimensions
    - `no_data` option added to allow missing values (sovles #83)
    - axis can be given by name (`"x"`, `"y"`, `"z"`) or axis number (`0`, `1`, `2`, `3`, ...)

Better Variogram fitting (#78, #145)

- fixing sill possible now
- `loss` is now selectable for smoother handling of outliers
- r2 score can now be returned to get an impression of the goodness of fitting
- weights can be passed
- instead of deselecting parameters, one can also give fix values for each parameter
- default init guess for `len_scale` is now mean of given bin-centers
- default init guess for `var` and `nugget` is now mean of given variogram values

CovModel update (#109, #122, #157)

- add new `rescale` argument and attribute to the `CovModel` class to be able to rescale the `len_scale` (usefull for unit conversion or rescaling `len_scale` to coincide with the `integral_scale` like it's the case with the Gaussian model) See: #90, GeoStat-Framework/PyKrige#119
- added new `len_rescaled` attribute to the `CovModel` class, which is the rescaled `len_scale`: `len_rescaled = len_scale / rescale`
- new method `default_rescale` to provide default rescale factor (can be overridden)
- remove `doctest` calls
- docstring updates in CovModel and derived models
- updated all models to use the `cor` routine and make use of the `rescale` argument (See: #90)
- TPL models got a separate base class to not repeat code
- added **new models** (See: #88):
    - `HyperSpherical`: (Replaces the old `Intersection` model) Derived from the intersection of hyperspheres in arbitrary dimensions. Coincides with the linear model in 1D, the circular model in 2D and the classical spherical model in 3D
    - `SuperSpherical`: like the HyperSpherical, but the shape parameter derived from dimension can be set by the user. Coincides with the HyperSpherical model by default
    - `JBessel`: a hole model valid in all dimensions. The shape parameter controls the dimension it was derived from. For `nu=0.5` this model coincides with the well known `wave` hole model.
    - `TPLSimple`: a simple truncated power law controlled by a shape parameter `nu`. Coincides with the truncated linear model for `nu=1`

  - `Cubic`: to be compatible with scikit-gstat in the future
- all arguments are now stored as float internally (#157)
- string representation of the `CovModel` class is now using a float precision (`CovModel._prec=3`) to truncate longish output
- string representation of the `CovModel` class now only shows `anis` and `angles` if model is anisotropic resp. rotated
- dimension validity check: raise a warning, if given model is not valid in the desired dimension (See: #86)

Normalizer, Trend and Mean (#124)

- new `normalize` submodule containing power-transforms for data to gain normality
- Base-Class: `Normalizer` providing basic functionality including maximum likelihood fitting
- added: `LogNormal`, `BoxCox`, `BoxCoxShift`, `YeoJohnson`, `Modulus` and `Manly`
- normalizer, trend and mean can be passed to SRF, Krige and variogram estimation routines
  - A trend can be a callable function, that represents a trend in input data. For example a linear decrease of temperature with height.
  - The normalizer will be applied after the data was detrended, i.e. the trend was substracted from the data, in order to gain normality.
  - The mean is now interpreted as the mean of the normalized data. The user could also provide a callable mean, but it is mostly meant to be constant.

Arbitrary dimensions (#112)

- allow arbitrary dimensions in all routines (CovModel, Krige, SRF, variogram)
- anisotropy and rotation following a generalization of tait-bryan angles
- CovModel provides `isometrize` and `anisometrize` routines to convert points

New Class for Conditioned Random Fields (#130)

- **THIS BREAKS BACKWARD COMPATIBILITY**
- `CondSRF` replaces the conditioning feature of the SRF class, which was cumbersome and limited to Ordinary and Simple kriging
- `CondSRF` behaves similar to the `SRF` class, but instead of a covariance model, it takes a kriging class as input. With this kriging class, all conditioning related settings are defined.

Enhancements

- Python 3.9 Support #107
- add routines to format struct. pos tuple by given `dim` or `shape`
- add routine to format struct. pos tuple by given `shape` (variogram helper)
- remove `field.tools` subpackage
- support `meshio>=4.0` and add as dependency
- PyVista mesh support #59
- added `EARTH_RADIUS` as constant providing earths radius in km (can be used to rescale models)

- add routines `latlon2pos` and `pos2latlon` to convert lat-lon coordinates to points on unit-sphere and vice versa
- a lot of new examples and tutorials
- `RandMeth` class got a switch to select the sampling strategy
- plotter for n-D fields added #141
- antialias for contour plots of 2D fields #141
- building from source is now configured with `pyproject.toml` to care about build dependencies, see #154

Changes

- drop support for Python 3.5 #146
- added a finit limit for shape-parameters in some CovModels #147
- drop usage of `pos2xyz` and `xyz2pos`
- remove structured option from generators (structured pos need to be converted first)
- explicitly assert dim=2,3 when generating vector fields
- simplify `pre_pos` routine to save pos tuple and reformat it an unstructured tuple
- simplify field shaping
- simplify plotting routines
- only the `"unstructured"` keyword is recognized everywhere, everything else is interpreted as `"structured"` (e.g. `"rectilinear"`)
- use GitHub-Actions instead of TravisCI
- parallel build now controlled by env-var `GSTOOLS_BUILD_PARALLEL=1`, see #154
- install extra target for `[dev]` dropped, can be reproduced by `pip install gstools[test, doc]`, see #154

Bugfixes

- typo in keyword argument for vario_estimate_structured #80
- isotropic rotation of SRF was not possible #100
- `CovModel.opt_arg` now sorted #103
- CovModel.fit: check if weights are given as a string (numpy comparison error) #111
- several pylint fixes (#159)

4.3 1.2.1 - Volatile Violet - 2020-04-14

Bugfixes

- `ModuleNotFoundError` is not present in py35
- Fixing Cressie-Bug #76
- Adding analytical formula for integral scales of rational and stable model
- remove prange from IncomprRandMeth summators to prevent errors on Win and macOS

4.4 1.2.0 - Volatile Violet - 2020-03-20

Enhancements

- different variogram estimator functions can now be used #51
- the TPLGaussian and TPLExponential now have analytical spectra #67
- added property `is_isotropic` to CovModel #67
- reworked the whole krige sub-module to provide multiple kriging methods #67
  - Simple
  - Ordinary
  - Universal
  - External Drift Kriging
  - Detrended Kriging
- a new transformation function for discrete fields has been added #70
- reworked tutorial section in the documentation #63
- pyvista interface #29

Changes

- Python versions 2.7 and 3.4 are no longer supported #40 #43
- CovModel: in 3D the input of anisotropy is now treated slightly different: #67
  - single given anisotropy value [e] is converted to [1, e] (it was [e, e] before)
  - two given length-scales [l_1, l_2] are converted to [l_1, l_2, l_2] (it was [l_1, l_2, l_1] before)

Bugfixes

- a race condition in the structured variogram estimation has been fixed #51

4.5 1.1.1 - Reverberating Red - 2019-11-08

Enhancements

- added a changelog. See: commit fbea883

Changes

- deprecation warnings are now printed if Python versions 2.7 or 3.4 are used #40 #41

Bugfixes

- define spectral_density instead of spectrum in covariance models since Cov-base derives spectrum. See: commit 00f2747
- better boundaries for CovModel parameters. See: https://github.com/GeoStat-Framework/GSTools/issues/37

4.6 1.1.0 - Reverberating Red - 2019-10-01

Enhancements

- by using Cython for all the heavy computations, we could achieve quite some speed ups and reduce the memory consumption significantly #16
- parallel computation in Cython is now supported with the help of OpenMP and the performance increase is nearly linear with increasing cores #16
- new submodule `krige` providing simple (known mean) and ordinary (estimated mean) kriging working analogous to the srf class
- interface to pykrige to use the gstools CovModel with the pykrige routines (https://github.com/bsmurphy/PyKrige/issues/124)
- the srf class now provides a `plot` and a `vtk_export` routine
- incompressible flow fields can now be generated #14
- new submodule providing several field transformations like: Zinn&Harvey, log-normal, bimodal, … #13
- Python 3.4 and 3.7 wheel support #19
- field can now be generated directly on meshes from meshio and ogs5py, see: commit f4a3439
- the srf and kriging classes now store the last `pos`, `mesh_type` and `field` values to keep them accessible, see: commit 29f7f1b
- tutorials on all important features of GSTools have been written for you guys #20
- a new interface to pyvista is provided to export fields to python vtk representation, which can be used for plotting, exploring and exporting fields #29

Changes

- the license was changed from GPL to LGPL in order to promote the use of this library #25
- the rotation angles are now interpreted in positive direction (counter clock wise)
- the `force_moments` keyword was removed from the SRF call method, it is now in provided as a field transformation #13
- drop support of python implementations of the variogram estimators #18
- the `variogram_normed` method was removed from the `CovModel` class due to redundance commit 25b1647
- the position vector of 1D fields does not have to be provided in a list-like object with length 1 commit a6f5be8

Bugfixes

- several minor bugfixes

4.7 1.0.1 - Bouncy Blue - 2019-01-18

Bugfixes

- fixed Numpy and Cython version during build process

4.8 1.0.0 - Bouncy Blue - 2019-01-16

Enhancements

- added a new covariance class, which allows the easy usage of arbitrary covariance models
- added many predefined covariance models, including truncated power law models
- added tutorials and examples, showing and explaining the main features of GSTools
- variogram models can be fitted to data
- prebuilt binaries for many Linux distributions, Mac OS and Windows, making the installation, especially of the Cython code, much easier
- the generated fields can now easily be exported to vtk files
- variance scaling is supported for coarser grids
- added pure Python versions of the variogram estimators, in case somebody has problems compiling Cython code
- the documentation is now a lot cleaner and easier to use
- the code is a lot cleaner and more consistent now
- unit tests are now automatically tested when new code is pushed
- test coverage of code is shown
- GeoStat Framework now has a website, visit us: https://geostat-framework.github.io/

Changes

- release is not downwards compatible with release v0.4.0
- SRF creation has been adapted for the CovModel
- a tuple `pos` is now used instead of `x`, `y`, and `z` for the axes
- renamed `estimate_unstructured` and `estimate_structured` to `vario_estimate_unstructured` and `vario_estimate_structured` for less ambiguity

Bugfixes

- several minor bugfixes

4.9 0.4.0 - Glorious Green - 2018-07-17

Bugfixes

- import of cython functions put into a try-block

4.10 0.3.6 - Original Orange - 2018-07-17

First release of GSTools.

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PYTHON MODULE INDEX

g

INDEX

Symbols

A

B

C

D

E

F

G

H

I

J

K

L

M

N

O

P

R

S

T

U

V

Y

Z