



PyKrige Documentation

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PyKrige developers

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CONTENTS

1	PyKrige	1
1.1	Purpose	1
1.2	Installation	1
1.3	Features	2
1.3.1	Kriging algorithms	2
1.3.2	Wrappers	2
1.3.3	Tools	2
1.3.4	Kriging Parameters Tuning	2
1.3.5	Regression Kriging	2
1.3.6	Classification Kriging	2
1.4	License	3
2	Variogram Models	5
2.1	References	6
3	API Reference	7
3.1	Krigging algorithms	7
3.1.1	pykrige.ok.OrdinaryKriging	7
3.1.2	pykrige.uk.UniversalKriging	12
3.1.3	pykrige.ok3d.OrdinaryKriging3D	18
3.1.4	pykrige.uk3d.UniversalKriging3D	23
3.1.5	pykrige.rk.RegressionKriging	29
3.2	Wrappers	32
3.2.1	pykrige.rk.Krige	32
3.3	Tools	35
3.3.1	pykrige.kriging_tools.write_asc_grid	35
3.3.2	pykrige.kriging_tools.read_asc_grid	35
4	Examples	37
4.1	Universal Kriging Example	37
4.2	Ordinary Kriging Example	38
4.3	GSTools Interface	40
4.4	Exact Values	42
4.5	Regression kriging	43
4.6	Classification kriging	45
4.7	Geometric example	46
4.8	Three-Dimensional Kriging Example	49
4.9	Krige CV	51
4.10	1D Kriging	55

5	Changelog	57
5.1	Version 1.7.0	57
5.2	Version 1.6.1	57
5.3	Version 1.6.0	58
5.4	Version 1.5.1	58
5.5	Version 1.5.0	58
5.6	Version 1.4.1	59
5.7	Version 1.4.0	59
5.8	Version 1.3.1	59
5.9	Version 1.3.0	60
5.10	Version 1.2.0	60
5.11	Version 1.1.0	61
5.12	Version 1.0.3	61
5.13	Version 1.0	61
5.14	Version 0.2.0	62
5.15	Version 0.1.2	62
	Bibliography	63
	Index	65

Kriging Toolkit for Python.

1.1 Purpose

The code supports 2D and 3D ordinary and universal kriging. Standard variogram models (linear, power, spherical, gaussian, exponential) are built in, but custom variogram models can also be used. The 2D universal kriging code currently supports regional-linear, point-logarithmic, and external drift terms, while the 3D universal kriging code supports a regional-linear drift term in all three spatial dimensions. Both universal kriging classes also support generic ‘specified’ and ‘functional’ drift capabilities. With the ‘specified’ drift capability, the user may manually specify the values of the drift(s) at each data point and all grid points. With the ‘functional’ drift capability, the user may provide callable function(s) of the spatial coordinates that define the drift(s). The package includes a module that contains functions that should be useful in working with ASCII grid files (*.asc).

See the documentation at <http://pykrige.readthedocs.io/> for more details and examples.

1.2 Installation

PyKriging requires Python 3.5+ as well as numpy, scipy. It can be installed from PyPi with,

```
pip install pykrige
```

scikit-learn is an optional dependency needed for parameter tuning and regression kriging. matplotlib is an optional dependency needed for plotting.

If you use conda, PyKriging can be installed from the channel with,

```
conda install -c conda-forge pykrige
```

1.3 Features

1.3.1 Kriging algorithms

- `OrdinaryKriging`: 2D ordinary kriging with estimated mean
- `UniversalKriging`: 2D universal kriging providing drift terms
- `OrdinaryKriging3D`: 3D ordinary kriging
- `UniversalKriging3D`: 3D universal kriging
- `RegressionKriging`: An implementation of Regression-Kriging
- `ClassificationKriging`: An implementation of Simplicial Indicator Kriging

1.3.2 Wrappers

- `rk.Krige`: A scikit-learn wrapper class for Ordinary and Universal Kriging

1.3.3 Tools

- `kriging_tools.write_asc_grid`: Writes gridded data to ASCII grid file (`*.asc`)
- `kriging_tools.read_asc_grid`: Reads ASCII grid file (`*.asc`)
- `kriging_tools.write_zmap_grid`: Writes gridded data to zmap file (`*.zmap`)
- `kriging_tools.read_zmap_grid`: Reads zmap file (`*.zmap`)

1.3.4 Kriging Parameters Tuning

A scikit-learn compatible API for parameter tuning by cross-validation is exposed in `sklearn.model_selection.GridSearchCV`. See the [Krige CV](#) example for a more practical illustration.

1.3.5 Regression Kriging

[Regression kriging](#) can be performed with `pykrige.rk.RegressionKriging`. This class takes as parameters a scikit-learn regression model, and details of either the `OrdinaryKriging` or the `UniversalKriging` class, and performs a correction step on the ML regression prediction.

A demonstration of the regression kriging is provided in the [corresponding example](#).

1.3.6 Classification Kriging

[Simplifical Indicator kriging](#) can be performed with `pykrige.ck.ClassificationKriging`. This class takes as parameters a scikit-learn classification model, and details of either the `OrdinaryKriging` or the `UniversalKriging` class, and performs a correction step on the ML classification prediction.

A demonstration of the classification kriging is provided in the [corresponding example](#).

1.4 License

PyKrige uses the BSD 3-Clause License.

VARIOGRAM MODELS

PyKriging internally supports the six variogram models listed below. Additionally, the code supports user-defined variogram models via the ‘custom’ variogram model keyword argument.

- Gaussian Model

$$p \cdot \left(1 - e^{-\frac{d^2}{(r/3)^2}}\right) + n$$

- Exponential Model

$$p \cdot \left(1 - e^{-\frac{d}{r/3}}\right) + n$$

- Spherical Model

$$\begin{cases} p \cdot \left(\frac{3d}{2r} - \frac{d^3}{2r^3}\right) + n & d \leq r \\ p + n & d > r \end{cases}$$

- Linear Model

$$s \cdot d + n$$

Where s is the slope and n is the nugget.

- Power Model

$$s \cdot d^e + n$$

Where s is the scaling factor, e is the exponent (between 0 and 2), and n is the nugget term.

- Hole-Effect Model

$$p \cdot \left(1 - \left(1 - \frac{d}{r/3}\right) * e^{-\frac{d}{r/3}}\right) + n$$

Variables are defined as:

d = distance values at which to calculate the variogram

p = partial sill (psill = sill - nugget)

r = range

n = nugget

s = scaling factor or slope

e = exponent for power model

For stationary variogram models (gaussian, exponential, spherical, and hole-effect models), the partial sill is defined as the difference between the full sill and the nugget term. The sill represents the asymptotic maximum spatial variance at longest lags (distances). The range represents the distance at which the spatial variance has reached ~95% of the sill variance. The nugget effectively takes up ‘noise’ in measurements. It represents the random deviations from an overall

smooth spatial data trend. (The name *nugget* is an allusion to kriging's mathematical origin in gold exploration; the nugget effect is intended to take into account the possibility that when sampling you randomly hit a pocket gold that is anomalously richer than the surrounding area.)

For nonstationary models (linear and power models, with unbounded spatial variances), the nugget has the same meaning. The exponent for the power-law model should be between 0 and 2¹.

A few important notes:

The PyKrige user interface by default takes the full sill. This default behavior can be changed with a keyword flag, so that the user can supply the partial sill instead. The code internally uses the partial sill ($\text{psill} = \text{sill} - \text{nugget}$) rather than the full sill, as it's safer to perform automatic variogram estimation using the partial sill.

The exact definitions of the variogram models here may differ from those used elsewhere. Keep that in mind when switching from another kriging code over to PyKrige.

According to¹, the hole-effect variogram model is only correct for the 1D case. It's implemented here for completeness and should be used cautiously.

2.1 References

¹ P.K. Kitanidis, Introduction to Geostatistics: Applications in Hydrogeology, (Cambridge University Press, 1997) 272 p.

API REFERENCE

3.1 Kriging algorithms

<code>pykrige.ok.OrdinaryKriging(x, y, z[, ...])</code>	Convenience class for easy access to 2D Ordinary Kriging.
<code>pykrige.uk.UniversalKriging(x, y, z[, ...])</code>	Provides greater control over 2D kriging by utilizing drift terms.
<code>pykrige.ok3d.OrdinaryKriging3D(x, y, z, val)</code>	Three-dimensional ordinary kriging.
<code>pykrige.uk3d.UniversalKriging3D(x, y, z, val)</code>	Three-dimensional universal kriging.
<code>pykrige.rk.RegressionKriging([...])</code>	An implementation of Regression-Kriging.

3.1.1 pykrige.ok.OrdinaryKriging

```
class pykrige.ok.OrdinaryKriging(x, y, z, variogram_model='linear', variogram_parameters=None,
                                variogram_function=None, nlags=6, weight=False,
                                anisotropy_scaling=1.0, anisotropy_angle=0.0, verbose=False,
                                enable_plotting=False, enable_statistics=False,
                                coordinates_type='euclidean', exact_values=True, pseudo_inv=False,
                                pseudo_inv_type='pinv')
```

Bases: object

Convenience class for easy access to 2D Ordinary Kriging.

Parameters

- **x** (*array_like*) – X-coordinates of data points.
- **y** (*array_like*) – Y-coordinates of data points.
- **z** (*array-like*) – Values at data points.
- **variogram_model** (*str or GStools CovModel, optional*) – Specifies which variogram model to use; may be one of the following: linear, power, gaussian, spherical, exponential, hole-effect. Default is linear variogram model. To utilize a custom variogram model, specify ‘custom’; you must also provide `variogram_parameters` and `variogram_function`. Note that the hole-effect model is only technically correct for one-dimensional problems. You can also use a [GStools CovModel](#).
- **variogram_parameters** (*list or dict, optional*) – Parameters that define the specified variogram model. If not provided, parameters will be automatically calculated using a “soft” L1 norm minimization scheme. For variogram model parameters provided in a dict, the required dict keys vary according to the specified variogram model:

```
# linear
    {'slope': slope, 'nugget': nugget}
# power
    {'scale': scale, 'exponent': exponent, 'nugget': nugget}
# gaussian, spherical, exponential and hole-effect:
    {'sill': s, 'range': r, 'nugget': n}
# OR
    {'psill': p, 'range': r, 'nugget': n}
```

Note that either the full sill or the partial sill ($\text{psill} = \text{sill} - \text{nugget}$) can be specified in the dict. For variogram model parameters provided in a list, the entries must be as follows:

```
# linear
    [slope, nugget]
# power
    [scale, exponent, nugget]
# gaussian, spherical, exponential and hole-effect:
    [sill, range, nugget]
```

Note that the full sill (NOT the partial sill) must be specified in the list format. For a custom variogram model, the parameters are required, as custom variogram models will not automatically be fit to the data. Furthermore, the parameters must be specified in list format, in the order in which they are used in the callable function (see `variogram_function` for more information). The code does not check that the provided list contains the appropriate number of parameters for the custom variogram model, so an incorrect parameter list in such a case will probably trigger an esoteric exception somewhere deep in the code. NOTE that, while the list format expects the full sill, the code itself works internally with the partial sill.

- **variogram_function** (*callable, optional*) – A callable function that must be provided if `variogram_model` is specified as ‘custom’. The function must take only two arguments: first, a list of parameters for the variogram model; second, the distances at which to calculate the variogram model. The list provided in `variogram_parameters` will be passed to the function as the first argument.
- **nlags** (*int, optional*) – Number of averaging bins for the semivariogram. Default is 6.
- **weight** (*bool, optional*) – Flag that specifies if semivariance at smaller lags should be weighted more heavily when automatically calculating variogram model. The routine is currently hard-coded such that the weights are calculated from a logistic function, so weights at small lags are ~1 and weights at the longest lags are ~0; the center of the logistic weighting is hard-coded to be at 70% of the distance from the shortest lag to the largest lag. Setting this parameter to True indicates that weights will be applied. Default is False. (Kitanidis suggests that the values at smaller lags are more important in fitting a variogram model, so the option is provided to enable such weighting.)
- **anisotropy_scaling** (*float, optional*) – Scalar stretching value to take into account anisotropy. Default is 1 (effectively no stretching). Scaling is applied in the y-direction in the rotated data frame (i.e., after adjusting for the `anisotropy_angle`, if `anisotropy_angle` is not 0). This parameter has no effect if `coordinate_types` is set to ‘geographic’.
- **anisotropy_angle** (*float, optional*) – CCW angle (in degrees) by which to rotate coordinate system in order to take into account anisotropy. Default is 0 (no rotation). Note that the coordinate system is rotated. This parameter has no effect if `coordinate_types` is set to ‘geographic’.
- **verbose** (*bool, optional*) – Enables program text output to monitor kriging process. Default is False (off).

- **enable_plotting** (*bool*, *optional*) – Enables plotting to display variogram. Default is False (off).
- **enable_statistics** (*bool*, *optional*) – Default is False
- **coordinates_type** (*str*, *optional*) – One of ‘euclidean’ or ‘geographic’. Determines if the x and y coordinates are interpreted as on a plane (‘euclidean’) or as coordinates on a sphere (‘geographic’). In case of geographic coordinates, x is interpreted as longitude and y as latitude coordinates, both given in degree. Longitudes are expected in [0, 360] and latitudes in [-90, 90]. Default is ‘euclidean’.
- **exact_values** (*bool*, *optional*) – If True, interpolation provides input values at input locations. If False, interpolation accounts for variance/nugget within input values at input locations and does not behave as an exact-interpolator [2]. Note that this only has an effect if there is variance/nugget present within the input data since it is interpreted as measurement error. If the nugget is zero, the kriged field will behave as an exact interpolator.
- **pseudo_inv** (*bool*, *optional*) – Whether the kriging system is solved with the pseudo inverted kriging matrix. If *True*, this leads to more numerical stability and redundant points are averaged. But it can take more time. Default: False
- **pseudo_inv_type** (*str*, *optional*) – Here you can select the algorithm to compute the pseudo-inverse matrix:
 - “*pinv*”: use *pinv* from *scipy* which uses *lstsq*
 - “*pinvh*”: use *pinvh* from *scipy* which uses eigen-values
 Default: “*pinv*”

References

Methods

<code>display_variogram_model()</code>	Displays variogram model with the actual binned data.
<code>execute(style, xpoints, ypoints[, mask, ...])</code>	Calculates a kriged grid and the associated variance.
<code>get_epsilon_residuals()</code>	Returns the epsilon residuals for the variogram fit.
<code>get_statistics()</code>	Returns the Q1, Q2, and cR statistics for the variogram fit (in that order).
<code>get_variogram_points()</code>	Returns both the lags and the variogram function evaluated at each of them.
<code>plot_epsilon_residuals()</code>	Plots the epsilon residuals for the variogram fit.
<code>print_statistics()</code>	Prints out the Q1, Q2, and cR statistics for the variogram fit.
<code>switch_plotting()</code>	Allows user to switch plot display on/off.
<code>switch_verbose()</code>	Allows user to switch code talk-back on/off.
<code>update_variogram_model(variogram_model[, ...])</code>	Allows user to update variogram type and/or variogram model parameters.

`display_variogram_model()`

Displays variogram model with the actual binned data.

execute (*style*, *xpoints*, *ypoints*, *mask=None*, *backend='vectorized'*, *n_closest_points=None*)

Calculates a kriged grid and the associated variance.

Parameters

- **style** (*str*) – Specifies how to treat input kriging points. Specifying ‘grid’ treats xpoints and ypoints as two arrays of x and y coordinates that define a rectangular grid. Specifying ‘points’ treats xpoints and ypoints as two arrays that provide coordinate pairs at which to solve the kriging system. Specifying ‘masked’ treats xpoints and ypoints as two arrays of x and y coordinates that define a rectangular grid and uses mask to only evaluate specific points in the grid.
- **xpoints** (*array_like, shape (N,) or (N, 1)*) – If style is specific as ‘grid’ or ‘masked’, x-coordinates of MxN grid. If style is specified as ‘points’, x-coordinates of specific points at which to solve kriging system.
- **ypoints** (*array_like, shape (M,) or (M, 1)*) – If style is specified as ‘grid’ or ‘masked’, y-coordinates of MxN grid. If style is specified as ‘points’, y-coordinates of specific points at which to solve kriging system. Note that in this case, xpoints and ypoints must have the same dimensions (i.e., $M = N$).
- **mask** (*bool, array_like, shape (M, N), optional*) – Specifies the points in the rectangular grid defined by xpoints and ypoints that are to be excluded in the kriging calculations. Must be provided if style is specified as ‘masked’. False indicates that the point should not be masked, so the kriging system will be solved at the point. True indicates that the point should be masked, so the kriging system should will not be solved at the point.
- **backend** (*str, optional*) – Specifies which approach to use in kriging. Specifying ‘vectorized’ will solve the entire kriging problem at once in a vectorized operation. This approach is faster but also can consume a significant amount of memory for large grids and/or large datasets. Specifying ‘loop’ will loop through each point at which the kriging system is to be solved. This approach is slower but also less memory-intensive. Specifying ‘C’ will utilize a loop in Cython. Default is ‘vectorized’.
- **n_closest_points** (*int, optional*) – For kriging with a moving window, specifies the number of nearby points to use in the calculation. This can speed up the calculation for large datasets, but should be used with caution. As Kitanidis notes, kriging with a moving window can produce unexpected oddities if the variogram model is not carefully chosen.

Returns

- **zvalues** (*ndarray, shape (M, N) or (N, 1)*) – Z-values of specified grid or at the specified set of points. If style was specified as ‘masked’, zvalues will be a numpy masked array.
- **sigmasq** (*ndarray, shape (M, N) or (N, 1)*) – Variance at specified grid points or at the specified set of points. If style was specified as ‘masked’, sigmasq will be a numpy masked array.

`get_epsilon_residuals()`

Returns the epsilon residuals for the variogram fit.

`get_statistics()`

Returns the Q1, Q2, and cR statistics for the variogram fit (in that order). No arguments.

`get_variogram_points()`

Returns both the lags and the variogram function evaluated at each of them.

The evaluation of the variogram function and the lags are produced internally. This method is convenient when the user wants to access to the lags and the resulting variogram (according to the model provided) for further analysis.

Returns

lags (array) - the lags at which the variogram was evaluated
variogram (array) - the variogram

function evaluated at the lags

Return type

(tuple) tuple containing

plot_epsilon_residuals()

Plots the epsilon residuals for the variogram fit.

print_statistics()

Prints out the Q1, Q2, and cR statistics for the variogram fit. NOTE that ideally Q1 is close to zero, Q2 is close to 1, and cR is as small as possible.

switch_plotting()

Allows user to switch plot display on/off. Takes no arguments.

switch_verbose()

Allows user to switch code talk-back on/off. Takes no arguments.

update_variogram_model(*variogram_model*, *variogram_parameters=None*, *variogram_function=None*, *nlags=6*, *weight=False*, *anisotropy_scaling=1.0*, *anisotropy_angle=0.0*)

Allows user to update variogram type and/or variogram model parameters.

Parameters

- **variogram_model** (*str* or *GSTools CovModel*) – May be any of the variogram models listed above. May also be ‘custom’, in which case *variogram_parameters* and *variogram_function* must be specified. You can also use a [GSTools CovModel](#).
- **variogram_parameters** (*list* or *dict*, *optional*) – List or dict of variogram model parameters, as explained above. If not provided, a best fit model will be calculated as described above.
- **variogram_function** (*callable*, *optional*) – A callable function that must be provided if *variogram_model* is specified as ‘custom’. See above for more information.
- **nlags** (*int*, *optional*) – Number of averaging bins for the semivariogram. Default is 6.
- **weight** (*boolean*, *optional*) – Flag that specifies if semivariance at smaller lags should be weighted more heavily when automatically calculating the variogram model. See above for more information. True indicates that weights will be applied. Default is False.
- **anisotropy_scaling** (*float*, *optional*) – Scalar stretching value to take into account anisotropy. Default is 1 (effectively no stretching). Scaling is applied in the y-direction.
- **anisotropy_angle** (*float*, *optional*) – CCW angle (in degrees) by which to rotate coordinate system in order to take into account anisotropy. Default is 0 (no rotation).

eps = 1e-10

```
variogram_dict = {'exponential': <function exponential_variogram_model>,
'gaussian': <function gaussian_variogram_model>, 'hole-effect': <function
hole_effect_variogram_model>, 'linear': <function linear_variogram_model>, 'power':
<function power_variogram_model>, 'spherical': <function
spherical_variogram_model>}
```

3.1.2 pykrige.uk.UniversalKriging

```
class pykrige.uk.UniversalKriging(x, y, z, variogram_model='linear', variogram_parameters=None,
                                  variogram_function=None, nlags=6, weight=False,
                                  anisotropy_scaling=1.0, anisotropy_angle=0.0, drift_terms=None,
                                  point_drift=None, external_drift=None, external_drift_x=None,
                                  external_drift_y=None, specified_drift=None, functional_drift=None,
                                  verbose=False, enable_plotting=False, exact_values=True,
                                  pseudo_inv=False, pseudo_inv_type='pinv')
```

Bases: object

Provides greater control over 2D kriging by utilizing drift terms.

Parameters

- **x** (*array_like*) – X-coordinates of data points.
- **y** (*array_like*) – Y-coordinates of data points.
- **z** (*array_like*) – Values at data points.
- **variogram_model** (*str or GSTools CovModel, optional*) – Specified which variogram model to use; may be one of the following: linear, power, gaussian, spherical, exponential, hole-effect. Default is linear variogram model. To utilize a custom variogram model, specify 'custom'; you must also provide variogram_parameters and variogram_function. Note that the hole-effect model is only technically correct for one-dimensional problems. You can also use a [GSTools CovModel](#).
- **variogram_parameters** (*list or dict, optional*) – Parameters that define the specified variogram model. If not provided, parameters will be automatically calculated using a “soft” L1 norm minimization scheme. For variogram model parameters provided in a dict, the required dict keys vary according to the specified variogram model:

```
# linear
    {'slope': slope, 'nugget': nugget}
# power
    {'scale': scale, 'exponent': exponent, 'nugget': nugget}
# gaussian, spherical, exponential and hole-effect:
    {'sill': s, 'range': r, 'nugget': n}
# OR
    {'psill': p, 'range': r, 'nugget': n}
```

Note that either the full sill or the partial sill (psill = sill - nugget) can be specified in the dict. For variogram model parameters provided in a list, the entries must be as follows:

```
# linear
    [slope, nugget]
# power
    [scale, exponent, nugget]
# gaussian, spherical, exponential and hole-effect:
    [sill, range, nugget]
```

Note that the full sill (NOT the partial sill) must be specified in the list format. For a custom variogram model, the parameters are required, as custom variogram models will not automatically be fit to the data. Furthermore, the parameters must be specified in list format, in the order in which they are used in the callable function (see variogram_function for more information). The code does not check that the provided list contains the appropriate number

of parameters for the custom variogram model, so an incorrect parameter list in such a case will probably trigger an esoteric exception someplace deep in the code. NOTE that, while the list format expects the full sill, the code itself works internally with the partial sill.

- **variogram_function** (*callable, optional*) – A callable function that must be provided if `variogram_model` is specified as ‘custom’. The function must take only two arguments: first, a list of parameters for the variogram model; second, the distances at which to calculate the variogram model. The list provided in `variogram_parameters` will be passed to the function as the first argument.
- **nlags** (*int, optional*) – Number of averaging bins for the semivariogram. Default is 6.
- **weight** (*bool, optional*) – Flag that specifies if semivariance at smaller lags should be weighted more heavily when automatically calculating variogram model. The routine is currently hard-coded such that the weights are calculated from a logistic function, so weights at small lags are ~1 and weights at the longest lags are ~0; the center of the logistic weighting is hard-coded to be at 70% of the distance from the shortest lag to the largest lag. Setting this parameter to True indicates that weights will be applied. Default is False. (Kitanidis suggests that the values at smaller lags are more important in fitting a variogram model, so the option is provided to enable such weighting.)
- **anisotropy_scaling** (*float, optional*) – Scalar stretching value to take into account anisotropy. Default is 1 (effectively no stretching). Scaling is applied in the y-direction in the rotated data frame (i.e., after adjusting for the `anisotropy_angle`, if `anisotropy_angle` is not 0).
- **anisotropy_angle** (*float, optional*) – CCW angle (in degrees) by which to rotate coordinate system in order to take into account anisotropy. Default is 0 (no rotation). Note that the coordinate system is rotated.
- **drift_terms** (*list of strings, optional*) – List of drift terms to include in universal kriging. Supported drift terms are currently ‘regional_linear’, ‘point_log’, ‘external_Z’, ‘specified’, and ‘functional’.
- **point_drift** (*array_like, optional*) – Array-like object that contains the coordinates and strengths of the point-logarithmic drift terms. Array shape must be (N, 3), where N is the number of point drift terms. First column (index 0) must contain x-coordinates, second column (index 1) must contain y-coordinates, and third column (index 2) must contain the strengths of each point term. Strengths are relative, so only the relation of the values to each other matters. Note that the code will appropriately deal with point-logarithmic terms that are at the same coordinates as an evaluation point or data point, but Python will still kick out a warning message that an $\ln(0)$ has been encountered. If the problem involves anisotropy, the well coordinates will be adjusted and the drift values will be calculated in the adjusted data frame.
- **external_drift** (*array_like, optional*) – Gridded data used for the external Z scalar drift term. Must be shape (M, N), where M is in the y-direction and N is in the x-direction. Grid spacing does not need to be constant. If grid spacing is not constant, must specify the grid cell sizes. If the problem involves anisotropy, the external drift values are extracted based on the pre-adjusted coordinates (i.e., the original coordinate system).
- **external_drift_x** (*array_like, optional*) – X-coordinates for gridded external Z-scalar data. Must be shape (M,) or (M, 1), where M is the number of grid cells in the x-direction. The coordinate is treated as the center of the cell.
- **external_drift_y** (*array_like, optional*) – Y-coordinates for gridded external Z-scalar data. Must be shape (N,) or (N, 1), where N is the number of grid cells in the y-direction. The coordinate is treated as the center of the cell.

- **specified_drift** (*list of array-like objects, optional*) – List of arrays that contain the drift values at data points. The arrays must be shape (N,) or (N, 1), where N is the number of data points. Any number of specified-drift terms may be used.
- **functional_drift** (*list of callable objects, optional*) – List of callable functions that will be used to evaluate drift terms. The function must be a function of only the two spatial coordinates and must return a single value for each coordinate pair. It must be set up to be called with only two arguments, first an array of x values and second an array of y values. If the problem involves anisotropy, the drift values are calculated in the adjusted data frame.
- **verbose** (*bool, optional*) – Enables program text output to monitor kriging process. Default is False (off).
- **enable_plotting** (*boolean, optional*) – Enables plotting to display variogram. Default is False (off).
- **exact_values** (*bool, optional*) – If True, interpolation provides input values at input locations. If False, interpolation accounts for variance/nugget within input values at input locations and does not behave as an exact-interpolator [2]. Note that this only has an effect if there is variance/nugget present within the input data since it is interpreted as measurement error. If the nugget is zero, the kriged field will behave as an exact interpolator.
- **pseudo_inv** (*bool, optional*) – Whether the kriging system is solved with the pseudo inverted kriging matrix. If *True*, this leads to more numerical stability and redundant points are averaged. But it can take more time. Default: False
- **pseudo_inv_type** (*str, optional*) – Here you can select the algorithm to compute the pseudo-inverse matrix:
 - “*pinv*”: use *pinv* from *scipy* which uses *lstsq*
 - “*pinvh*”: use *pinvh* from *scipy* which uses eigen-valuesDefault: “*pinv*”

References

Methods

<code>display_variogram_model()</code>	Displays variogram model with the actual binned data.
<code>execute(style, xpoints, ypoints[, mask, ...])</code>	Calculates a kriged grid and the associated variance.
<code>get_epsilon_residuals()</code>	Returns the epsilon residuals for the variogram fit.
<code>get_statistics()</code>	Returns the Q1, Q2, and cR statistics for the variogram fit (in that order).
<code>get_variogram_points()</code>	Returns both the lags and the variogram function evaluated at each of them.
<code>plot_epsilon_residuals()</code>	Plots the epsilon residuals for the variogram fit.
<code>print_statistics()</code>	Prints out the Q1, Q2, and cR statistics for the variogram fit.
<code>switch_plotting()</code>	Allows user to switch plot display on/off.
<code>switch_verbose()</code>	Allows user to switch code talk-back on/off.
<code>update_variogram_model(variogram_model[, ...])</code>	Allows user to update variogram type and/or variogram model parameters.

display_variogram_model()

Displays variogram model with the actual binned data.

execute(*style, xpoints, ypoints, mask=None, backend='vectorized', specified_drift_arrays=None*)

Calculates a kriged grid and the associated variance. Includes drift terms.

Parameters

- **style** (*str*) – Specifies how to treat input kriging points. Specifying 'grid' treats xpoints and ypoints as two arrays of x and y coordinates that define a rectangular grid. Specifying 'points' treats xpoints and ypoints as two arrays that provide coordinate pairs at which to solve the kriging system. Specifying 'masked' treats xpoints and ypoints as two arrays of x and y coordinates that define a rectangular grid and uses mask to only evaluate specific points in the grid.
- **xpoints** (*array-like, shape (N,) or (N, 1)*) – If style is specific as 'grid' or 'masked', x-coordinates of MxN grid. If style is specified as 'points', x-coordinates of specific points at which to solve kriging system.
- **ypoints** (*array-like, shape (M,) or (M, 1)*) – If style is specified as 'grid' or 'masked', y-coordinates of MxN grid. If style is specified as 'points', y-coordinates of specific points at which to solve kriging system. Note that in this case, xpoints and ypoints must have the same dimensions (i.e., $M = N$).
- **mask** (*boolean array, shape (M, N), optional*) – Specifies the points in the rectangular grid defined by xpoints and ypoints that are to be excluded in the kriging calculations. Must be provided if style is specified as 'masked'. False indicates that the point should not be masked, so the kriging system will be solved at the point. True indicates that the point should be masked, so the kriging system should will not be solved at the point.
- **backend** (*str, optional*) – Specifies which approach to use in kriging. Specifying 'vectorized' will solve the entire kriging problem at once in a vectorized operation. This approach is faster but also can consume a significant amount of memory for large grids and/or large datasets. Specifying 'loop' will loop through each point at which the kriging system is to be solved. This approach is slower but also less memory-intensive. Default is 'vectorized'. Note that Cython backend is not supported for UK.
- **specified_drift_arrays** (*list of array-like objects, optional*) – Specifies the drift values at the points at which the kriging system is to be evaluated. Required if 'specified' drift provided in the list of drift terms when instantiating the UniversalKriging class. Must be a list of arrays in the same order as the list provided when instantiating the kriging object. Array(s) must be the same dimension as the specified grid or have the same number of points as the specified points; i.e., the arrays either must be shape (M, N), where M is the number of y grid-points and N is the number of x grid-points, or shape (M,) or (N, 1), where M is the number of points at which to evaluate the kriging system.

Returns

- **zvalues** (*ndarray, shape (M, N) or (N, 1)*) – Z-values of specified grid or at the specified set of points. If style was specified as 'masked', zvalues will be a numpy masked array.
- **sigmasq** (*ndarray, shape (M, N) or (N, 1)*) – Variance at specified grid points or at the specified set of points. If style was specified as 'masked', sigmasq will be a numpy masked array.

get_epsilon_residuals()

Returns the epsilon residuals for the variogram fit.

get_statistics()

Returns the Q1, Q2, and cR statistics for the variogram fit (in that order). No arguments.

get_variogram_points()

Returns both the lags and the variogram function evaluated at each of them.

The evaluation of the variogram function and the lags are produced internally. This method is convenient when the user wants to access to the lags and the resulting variogram (according to the model provided) for further analysis.

Returns

lags (array) - the lags at which the variogram was evaluated
variogram (array) - the variogram function evaluated at the lags

Return type

(tuple) tuple containing

plot_epsilon_residuals()

Plots the epsilon residuals for the variogram fit.

print_statistics()

Prints out the Q1, Q2, and cR statistics for the variogram fit. NOTE that ideally Q1 is close to zero, Q2 is close to 1, and cR is as small as possible.

switch_plotting()

Allows user to switch plot display on/off. Takes no arguments.

switch_verbose()

Allows user to switch code talk-back on/off. Takes no arguments.

update_variogram_model(*variogram_model*, *variogram_parameters=None*, *variogram_function=None*, *nlags=6*, *weight=False*, *anisotropy_scaling=1.0*, *anisotropy_angle=0.0*)

Allows user to update variogram type and/or variogram model parameters.

Parameters

- **variogram_model** (*str* or *GSTools CovModel*) – May be any of the variogram models listed above. May also be ‘custom’, in which case *variogram_parameters* and *variogram_function* must be specified. You can also use a [GSTools CovModel](#).
- **variogram_parameters** (*list* or *dict*, *optional*) – List or dict of variogram model parameters, as explained above. If not provided, a best fit model will be calculated as described above.
- **variogram_function** (*callable*, *optional*) – A callable function that must be provided if *variogram_model* is specified as ‘custom’. See above for more information.
- **nlags** (*int*, *optional*) – Number of averaging bins for the semivariogram. Default is 6.
- **weight** (*boolean*, *optional*) – Flag that specifies if semivariance at smaller lags should be weighted more heavily when automatically calculating the variogram model. See above for more information. True indicates that weights will be applied. Default is False.
- **anisotropy_scaling** (*float*, *optional*) – Scalar stretching value to take into account anisotropy. Default is 1 (effectively no stretching). Scaling is applied in the y-direction.
- **anisotropy_angle** (*float*, *optional*) – CCW angle (in degrees) by which to rotate coordinate system in order to take into account anisotropy. Default is 0 (no rotation).

```
UNBIAS = True
```

```
eps = 1e-10
```

```
variogram_dict = {'exponential': <function exponential_variogram_model>,  
'gaussian': <function gaussian_variogram_model>, 'hole-effect': <function  
hole_effect_variogram_model>, 'linear': <function linear_variogram_model>, 'power':  
<function power_variogram_model>, 'spherical': <function  
spherical_variogram_model>}
```

3.1.3 pykrige.ok3d.OrdinaryKriging3D

```
class pykrige.ok3d.OrdinaryKriging3D(x, y, z, val, variogram_model='linear',
                                     variogram_parameters=None, variogram_function=None, nlags=6,
                                     weight=False, anisotropy_scaling_y=1.0,
                                     anisotropy_scaling_z=1.0, anisotropy_angle_x=0.0,
                                     anisotropy_angle_y=0.0, anisotropy_angle_z=0.0, verbose=False,
                                     enable_plotting=False, exact_values=True, pseudo_inv=False,
                                     pseudo_inv_type='pinv')
```

Bases: object

Three-dimensional ordinary kriging.

Parameters

- **x** (*array_like*) – X-coordinates of data points.
- **y** (*array_like*) – Y-coordinates of data points.
- **z** (*array_like*) – Z-coordinates of data points.
- **val** (*array_like*) – Values at data points.
- **variogram_model** (*str or GSTools CovModel, optional*) – Specified which variogram model to use; may be one of the following: linear, power, gaussian, spherical, exponential, hole-effect. Default is linear variogram model. To utilize a custom variogram model, specify 'custom'; you must also provide `variogram_parameters` and `variogram_function`. Note that the hole-effect model is only technically correct for one-dimensional problems. You can also use a [GSTools CovModel](#).
- **variogram_parameters** (*list or dict, optional*) – Parameters that define the specified variogram model. If not provided, parameters will be automatically calculated using a “soft” L1 norm minimization scheme. For variogram model parameters provided in a dict, the required dict keys vary according to the specified variogram model:

```
# linear
    {'slope': slope, 'nugget': nugget}
# power
    {'scale': scale, 'exponent': exponent, 'nugget': nugget}
# gaussian, spherical, exponential and hole-effect:
    {'sill': s, 'range': r, 'nugget': n}
# OR
    {'psill': p, 'range': r, 'nugget': n}
```

Note that either the full sill or the partial sill (psill = sill - nugget) can be specified in the dict. For variogram model parameters provided in a list, the entries must be as follows:

```
# linear
    [slope, nugget]
# power
    [scale, exponent, nugget]
# gaussian, spherical, exponential and hole-effect:
    [sill, range, nugget]
```

Note that the full sill (NOT the partial sill) must be specified in the list format. For a custom variogram model, the parameters are required, as custom variogram models will not automatically be fit to the data. Furthermore, the parameters must be specified in list format, in the order in which they are used in the callable function (see `variogram_function` for more

information). The code does not check that the provided list contains the appropriate number of parameters for the custom variogram model, so an incorrect parameter list in such a case will probably trigger an esoteric exception someplace deep in the code. NOTE that, while the list format expects the full sill, the code itself works internally with the partial sill.

- **variogram_function** (*callable, optional*) – A callable function that must be provided if `variogram_model` is specified as ‘custom’. The function must take only two arguments: first, a list of parameters for the variogram model; second, the distances at which to calculate the variogram model. The list provided in `variogram_parameters` will be passed to the function as the first argument.
- **nlags** (*int, optional*) – Number of averaging bins for the semivariogram. Default is 6.
- **weight** (*boolean, optional*) – Flag that specifies if semivariance at smaller lags should be weighted more heavily when automatically calculating variogram model. The routine is currently hard-coded such that the weights are calculated from a logistic function, so weights at small lags are ~1 and weights at the longest lags are ~0; the center of the logistic weighting is hard-coded to be at 70% of the distance from the shortest lag to the largest lag. Setting this parameter to True indicates that weights will be applied. Default is False. (Kitanidis suggests that the values at smaller lags are more important in fitting a variogram model, so the option is provided to enable such weighting.)
- **anisotropy_scaling_y** (*float, optional*) – Scalar stretching value to take into account anisotropy in the y direction. Default is 1 (effectively no stretching). Scaling is applied in the y direction in the rotated data frame (i.e., after adjusting for the `anisotropy_angle_x/y/z`, if `anisotropy_angle_x/y/z` is/are not 0).
- **anisotropy_scaling_z** (*float, optional*) – Scalar stretching value to take into account anisotropy in the z direction. Default is 1 (effectively no stretching). Scaling is applied in the z direction in the rotated data frame (i.e., after adjusting for the `anisotropy_angle_x/y/z`, if `anisotropy_angle_x/y/z` is/are not 0).
- **anisotropy_angle_x** (*float, optional*) – CCW angle (in degrees) by which to rotate coordinate system about the x axis in order to take into account anisotropy. Default is 0 (no rotation). Note that the coordinate system is rotated. X rotation is applied first, then y rotation, then z rotation. Scaling is applied after rotation.
- **anisotropy_angle_y** (*float, optional*) – CCW angle (in degrees) by which to rotate coordinate system about the y axis in order to take into account anisotropy. Default is 0 (no rotation). Note that the coordinate system is rotated. X rotation is applied first, then y rotation, then z rotation. Scaling is applied after rotation.
- **anisotropy_angle_z** (*float, optional*) – CCW angle (in degrees) by which to rotate coordinate system about the z axis in order to take into account anisotropy. Default is 0 (no rotation). Note that the coordinate system is rotated. X rotation is applied first, then y rotation, then z rotation. Scaling is applied after rotation.
- **verbose** (*bool, optional*) – Enables program text output to monitor kriging process. Default is False (off).
- **enable_plotting** (*bool, optional*) – Enables plotting to display variogram. Default is False (off).
- **exact_values** (*bool, optional*) – If True, interpolation provides input values at input locations. If False, interpolation accounts for variance/nugget within input values at input locations and does not behave as an exact-interpolator [2]. Note that this only has an effect if there is variance/nugget present within the input data since it is interpreted as measurement error. If the nugget is zero, the kriged field will behave as an exact interpolator.

- **pseudo_inv** (bool, optional) – Whether the kriging system is solved with the pseudo inverted kriging matrix. If *True*, this leads to more numerical stability and redundant points are averaged. But it can take more time. Default: False
- **pseudo_inv_type** (str, optional) – Here you can select the algorithm to compute the pseudo-inverse matrix:
 - “*pinv*”: use *pinv* from *scipy* which uses *lstsq*
 - “*pinvh*”: use *pinvh* from *scipy* which uses eigen-valuesDefault: “*pinv*”

References

Methods

<code>display_variogram_model()</code>	Displays variogram model with the actual binned data.
<code>execute(style, xpoints, ypoints, zpoints[, ...])</code>	Calculates a kriged grid and the associated variance.
<code>get_epsilon_residuals()</code>	Returns the epsilon residuals for the variogram fit.
<code>get_statistics()</code>	Returns the Q1, Q2, and cR statistics for the variogram fit (in that order).
<code>plot_epsilon_residuals()</code>	Plots the epsilon residuals for the variogram fit.
<code>print_statistics()</code>	Prints out the Q1, Q2, and cR statistics for the variogram fit.
<code>switch_plotting()</code>	Allows user to switch plot display on/off.
<code>switch_verbose()</code>	Allows user to switch code talk-back on/off.
<code>update_variogram_model(variogram_model[, ...])</code>	Changes the variogram model and variogram parameters for the kriging system.

`display_variogram_model()`

Displays variogram model with the actual binned data.

execute(*style*, *xpoints*, *ypoints*, *zpoints*, *mask=None*, *backend='vectorized'*, *n_closest_points=None*)

Calculates a kriged grid and the associated variance.

This is now the method that performs the main kriging calculation. Note that currently measurements (i.e., *z* values) are considered ‘exact’. This means that, when a specified coordinate for interpolation is exactly the same as one of the data points, the variogram evaluated at the point is forced to be zero. Also, the diagonal of the kriging matrix is also always forced to be zero. In forcing the variogram evaluated at data points to be zero, we are effectively saying that there is no variance at that point (no uncertainty, so the value is ‘exact’).

In the future, the code may include an extra ‘exact_values’ boolean flag that can be adjusted to specify whether to treat the measurements as ‘exact’. Setting the flag to false would indicate that the variogram should not be forced to be zero at zero distance (i.e., when evaluated at data points). Instead, the uncertainty in the point will be equal to the nugget. This would mean that the diagonal of the kriging matrix would be set to the nugget instead of to zero.

Parameters

- **style** (str) – Specifies how to treat input kriging points. Specifying ‘grid’ treats *xpoints*, *ypoints*, and *zpoints* as arrays of *x*, *y*, and *z* coordinates that define a rectangular grid. Specifying ‘points’ treats *xpoints*, *ypoints*, and *zpoints* as arrays that provide coordinates at which to solve the kriging system. Specifying ‘masked’ treats *xpoints*, *ypoints*, and

zpoints as arrays of x, y, and z coordinates that define a rectangular grid and uses mask to only evaluate specific points in the grid.

- **xpoints** (*array-like, shape (N,) or (N, 1)*) – If style is specific as ‘grid’ or ‘masked’, x-coordinates of LxMxN grid. If style is specified as ‘points’, x-coordinates of specific points at which to solve kriging system.
- **ypoints** (*array-like, shape (M,) or (M, 1)*) – If style is specified as ‘grid’ or ‘masked’, y-coordinates of LxMxN grid. If style is specified as ‘points’, y-coordinates of specific points at which to solve kriging system. Note that in this case, xpoints, ypoints, and zpoints must have the same dimensions (i.e., $L = M = N$).
- **zpoints** (*array-like, shape (L,) or (L, 1)*) – If style is specified as ‘grid’ or ‘masked’, z-coordinates of LxMxN grid. If style is specified as ‘points’, z-coordinates of specific points at which to solve kriging system. Note that in this case, xpoints, ypoints, and zpoints must have the same dimensions (i.e., $L = M = N$).
- **mask** (*boolean array, shape (L, M, N), optional*) – Specifies the points in the rectangular grid defined by xpoints, ypoints, zpoints that are to be excluded in the kriging calculations. Must be provided if style is specified as ‘masked’. False indicates that the point should not be masked, so the kriging system will be solved at the point. True indicates that the point should be masked, so the kriging system should will not be solved at the point.
- **backend** (*str, optional*) – Specifies which approach to use in kriging. Specifying ‘vectorized’ will solve the entire kriging problem at once in a vectorized operation. This approach is faster but also can consume a significant amount of memory for large grids and/or large datasets. Specifying ‘loop’ will loop through each point at which the kriging system is to be solved. This approach is slower but also less memory-intensive. Default is ‘vectorized’.
- **n_closest_points** (*int, optional*) – For kriging with a moving window, specifies the number of nearby points to use in the calculation. This can speed up the calculation for large datasets, but should be used with caution. As Kitanidis notes, kriging with a moving window can produce unexpected oddities if the variogram model is not carefully chosen.

Returns

- **kvalues** (*ndarray, shape (L, M, N) or (N, 1)*) – Interpolated values of specified grid or at the specified set of points. If style was specified as ‘masked’, kvalues will be a numpy masked array.
- **sigmasq** (*ndarray, shape (L, M, N) or (N, 1)*) – Variance at specified grid points or at the specified set of points. If style was specified as ‘masked’, sigmasq will be a numpy masked array.

get_epsilon_residuals()

Returns the epsilon residuals for the variogram fit.

get_statistics()

Returns the Q1, Q2, and cR statistics for the variogram fit (in that order). No arguments.

plot_epsilon_residuals()

Plots the epsilon residuals for the variogram fit.

print_statistics()

Prints out the Q1, Q2, and cR statistics for the variogram fit. NOTE that ideally Q1 is close to zero, Q2 is close to 1, and cR is as small as possible.

switch_plotting()

Allows user to switch plot display on/off. Takes no arguments.

switch_verbose()

Allows user to switch code talk-back on/off. Takes no arguments.

update_variogram_model(*variogram_model*, *variogram_parameters=None*, *variogram_function=None*, *nlags=6*, *weight=False*, *anisotropy_scaling_y=1.0*, *anisotropy_scaling_z=1.0*, *anisotropy_angle_x=0.0*, *anisotropy_angle_y=0.0*, *anisotropy_angle_z=0.0*)

Changes the variogram model and variogram parameters for the kriging system.

Parameters

- **variogram_model** (*str* or *GSTools CovModel*) – May be any of the variogram models listed above. May also be ‘custom’, in which case *variogram_parameters* and *variogram_function* must be specified. You can also use a [GSTools CovModel](#).
- **variogram_parameters** (*list* or *dict*, *optional*) – List or dict of variogram model parameters, as explained above. If not provided, a best fit model will be calculated as described above.
- **variogram_function** (*callable*, *optional*) – A callable function that must be provided if *variogram_model* is specified as ‘custom’. See above for more information.
- **nlags** (*int*, *optional*) – Number of averaging bins for the semivariogram. Default is 6.
- **weight** (*bool*, *optional*) – Flag that specifies if semivariance at smaller lags should be weighted more heavily when automatically calculating variogram model. See above for more information. True indicates that weights will be applied. Default is False.
- **anisotropy_scaling_y** (*float*, *optional*) – Scalar stretching value to take into account anisotropy in y-direction. Default is 1 (effectively no stretching). See above for more information.
- **anisotropy_scaling_z** (*float*, *optional*) – Scalar stretching value to take into account anisotropy in z-direction. Default is 1 (effectively no stretching). See above for more information.
- **anisotropy_angle_x** (*float*, *optional*) – Angle (in degrees) by which to rotate coordinate system about the x axis in order to take into account anisotropy. Default is 0 (no rotation). See above for more information.
- **anisotropy_angle_y** (*float*, *optional*) – Angle (in degrees) by which to rotate coordinate system about the y axis in order to take into account anisotropy. Default is 0 (no rotation). See above for more information.
- **anisotropy_angle_z** (*float*, *optional*) – Angle (in degrees) by which to rotate coordinate system about the z axis in order to take into account anisotropy. Default is 0 (no rotation). See above for more information.

eps = 1e-10

```
variogram_dict = {'exponential': <function exponential_variogram_model>,
'gaussian': <function gaussian_variogram_model>, 'hole-effect': <function
hole_effect_variogram_model>, 'linear': <function linear_variogram_model>, 'power':
<function power_variogram_model>, 'spherical': <function
spherical_variogram_model>}
```

3.1.4 pykrige.uk3d.UniversalKriging3D

```
class pykrige.uk3d.UniversalKriging3D(x, y, z, val, variogram_model='linear',
                                       variogram_parameters=None, variogram_function=None,
                                       nlags=6, weight=False, anisotropy_scaling_y=1.0,
                                       anisotropy_scaling_z=1.0, anisotropy_angle_x=0.0,
                                       anisotropy_angle_y=0.0, anisotropy_angle_z=0.0,
                                       drift_terms=None, specified_drift=None, functional_drift=None,
                                       verbose=False, enable_plotting=False, exact_values=True,
                                       pseudo_inv=False, pseudo_inv_type='pinv')
```

Bases: object

Three-dimensional universal kriging.

Parameters

- **x** (*array_like*) – X-coordinates of data points.
- **y** (*array_like*) – Y-coordinates of data points.
- **z** (*array_like*) – Z-coordinates of data points.
- **val** (*array_like*) – Values at data points.
- **variogram_model** (*str or GStools CovModel, optional*) – Specified which variogram model to use; may be one of the following: linear, power, gaussian, spherical, exponential, hole-effect. Default is linear variogram model. To utilize a custom variogram model, specify 'custom'; you must also provide `variogram_parameters` and `variogram_function`. Note that the hole-effect model is only technically correct for one-dimensional problems. You can also use a [GStools CovModel](#).
- **variogram_parameters** (*list or dict, optional*) – Parameters that define the specified variogram model. If not provided, parameters will be automatically calculated using a “soft” L1 norm minimization scheme. For variogram model parameters provided in a dict, the required dict keys vary according to the specified variogram model:

```
# linear
    {'slope': slope, 'nugget': nugget}
# power
    {'scale': scale, 'exponent': exponent, 'nugget': nugget}
# gaussian, spherical, exponential and hole-effect:
    {'sill': s, 'range': r, 'nugget': n}
# OR
    {'psill': p, 'range': r, 'nugget': n}
```

Note that either the full sill or the partial sill (psill = sill - nugget) can be specified in the dict. For variogram model parameters provided in a list, the entries must be as follows:

```
# linear
    [slope, nugget]
# power
    [scale, exponent, nugget]
# gaussian, spherical, exponential and hole-effect:
    [sill, range, nugget]
```

Note that the full sill (NOT the partial sill) must be specified in the list format. For a custom variogram model, the parameters are required, as custom variogram models will not automatically be fit to the data. Furthermore, the parameters must be specified in list format, in

the order in which they are used in the callable function (see `variogram_function` for more information). The code does not check that the provided list contains the appropriate number of parameters for the custom variogram model, so an incorrect parameter list in such a case will probably trigger an esoteric exception someplace deep in the code. NOTE that, while the list format expects the full sill, the code itself works internally with the partial sill.

- **`variogram_function`** (*callable, optional*) – A callable function that must be provided if `variogram_model` is specified as ‘custom’. The function must take only two arguments: first, a list of parameters for the variogram model; second, the distances at which to calculate the variogram model. The list provided in `variogram_parameters` will be passed to the function as the first argument.
- **`nlags`** (*int, optional*) – Number of averaging bins for the semivariogram. Default is 6.
- **`weight`** (*bool, optional*) – Flag that specifies if semivariance at smaller lags should be weighted more heavily when automatically calculating variogram model. The routine is currently hard-coded such that the weights are calculated from a logistic function, so weights at small lags are ~1 and weights at the longest lags are ~0; the center of the logistic weighting is hard-coded to be at 70% of the distance from the shortest lag to the largest lag. Setting this parameter to True indicates that weights will be applied. Default is False. (Kitanidis suggests that the values at smaller lags are more important in fitting a variogram model, so the option is provided to enable such weighting.)
- **`anisotropy_scaling_y`** (*float, optional*) – Scalar stretching value to take into account anisotropy in the y direction. Default is 1 (effectively no stretching). Scaling is applied in the y direction in the rotated data frame (i.e., after adjusting for the `anisotropy_angle_x/y/z`, if `anisotropy_angle_x/y/z` is/are not 0).
- **`anisotropy_scaling_z`** (*float, optional*) – Scalar stretching value to take into account anisotropy in the z direction. Default is 1 (effectively no stretching). Scaling is applied in the z direction in the rotated data frame (i.e., after adjusting for the `anisotropy_angle_x/y/z`, if `anisotropy_angle_x/y/z` is/are not 0).
- **`anisotropy_angle_x`** (*float, optional*) – CCW angle (in degrees) by which to rotate coordinate system about the x axis in order to take into account anisotropy. Default is 0 (no rotation). Note that the coordinate system is rotated. X rotation is applied first, then y rotation, then z rotation. Scaling is applied after rotation.
- **`anisotropy_angle_y`** (*float, optional*) – CCW angle (in degrees) by which to rotate coordinate system about the y axis in order to take into account anisotropy. Default is 0 (no rotation). Note that the coordinate system is rotated. X rotation is applied first, then y rotation, then z rotation. Scaling is applied after rotation.
- **`anisotropy_angle_z`** (*float, optional*) – CCW angle (in degrees) by which to rotate coordinate system about the z axis in order to take into account anisotropy. Default is 0 (no rotation). Note that the coordinate system is rotated. X rotation is applied first, then y rotation, then z rotation. Scaling is applied after rotation.
- **`drift_terms`** (*list of strings, optional*) – List of drift terms to include in three-dimensional universal kriging. Supported drift terms are currently ‘regional_linear’, ‘specified’, and ‘functional’.
- **`specified_drift`** (*list of array-like objects, optional*) – List of arrays that contain the drift values at data points. The arrays must be shape (N,) or (N, 1), where N is the number of data points. Any number of specified-drift terms may be used.
- **`functional_drift`** (*list of callable objects, optional*) – List of callable functions that will be used to evaluate drift terms. The function must be a function of only the three spatial coordinates and must return a single value for each coordinate triplet. It must be

set up to be called with only three arguments, first an array of x values, the second an array of y values, and the third an array of z values. If the problem involves anisotropy, the drift values are calculated in the adjusted data frame.

- **verbose** (*boolean, optional*) – Enables program text output to monitor kriging process. Default is False (off).
- **enable_plotting** (*boolean, optional*) – Enables plotting to display variogram. Default is False (off).
- **exact_values** (*bool, optional*) – If True, interpolation provides input values at input locations. If False, interpolation accounts for variance/nugget within input values at input locations and does not behave as an exact-interpolator [2]. Note that this only has an effect if there is variance/nugget present within the input data since it is interpreted as measurement error. If the nugget is zero, the kriged field will behave as an exact interpolator.
- **pseudo_inv** (*bool, optional*) – Whether the kriging system is solved with the pseudo inverted kriging matrix. If *True*, this leads to more numerical stability and redundant points are averaged. But it can take more time. Default: False
- **pseudo_inv_type** (*str, optional*) – Here you can select the algorithm to compute the pseudo-inverse matrix:
 - “*pinv*”: use *pinv* from *scipy* which uses *lstsq*
 - “*pinvh*”: use *pinvh* from *scipy* which uses eigen-values
 Default: “*pinv*”

References

Methods

<code>display_variogram_model()</code>	Displays semivariogram and variogram model.
<code>execute(style, xpoints, ypoints, zpoints[, ...])</code>	Calculates a kriged grid and the associated variance.
<code>get_epsilon_residuals()</code>	Returns the epsilon residuals for the variogram fit.
<code>get_statistics()</code>	Returns the Q1, Q2, and cR statistics for the variogram fit (in that order).
<code>plot_epsilon_residuals()</code>	Plots the epsilon residuals for the variogram fit.
<code>print_statistics()</code>	Prints out the Q1, Q2, and cR statistics for the variogram fit.
<code>switch_plotting()</code>	Enables/disable variogram plot display.
<code>switch_verbose()</code>	Enables/disables program text output.
<code>update_variogram_model(variogram_model[, ...])</code>	Changes the variogram model and variogram parameters for the kriging system.

`display_variogram_model()`

Displays semivariogram and variogram model.

execute(*style, xpoints, ypoints, zpoints, mask=None, backend='vectorized', specified_drift_arrays=None*)

Calculates a kriged grid and the associated variance.

This is now the method that performs the main kriging calculation. Note that currently measurements (i.e., z values) are considered ‘exact’. This means that, when a specified coordinate for interpolation is exactly the same as one of the data points, the variogram evaluated at the point is forced to be zero. Also, the diagonal of the kriging matrix is also always forced to be zero. In forcing the variogram evaluated at data

points to be zero, we are effectively saying that there is no variance at that point (no uncertainty, so the value is 'exact').

In the future, the code may include an extra 'exact_values' boolean flag that can be adjusted to specify whether to treat the measurements as 'exact'. Setting the flag to false would indicate that the variogram should not be forced to be zero at zero distance (i.e., when evaluated at data points). Instead, the uncertainty in the point will be equal to the nugget. This would mean that the diagonal of the kriging matrix would be set to the nugget instead of to zero.

Parameters

- **style** (*str*) – Specifies how to treat input kriging points. Specifying 'grid' treats xpoints, ypoints, and zpoints as arrays of x, y, and z coordinates that define a rectangular grid. Specifying 'points' treats xpoints, ypoints, and zpoints as arrays that provide coordinates at which to solve the kriging system. Specifying 'masked' treats xpoints, ypoints, and zpoints as arrays of x, y, and z coordinates that define a rectangular grid and uses mask to only evaluate specific points in the grid.
- **xpoints** (*array_like, shape (N,) or (N, 1)*) – If style is specific as 'grid' or 'masked', x-coordinates of LxMxN grid. If style is specified as 'points', x-coordinates of specific points at which to solve kriging system.
- **ypoints** (*array_like, shape (M,) or (M, 1)*) – If style is specified as 'grid' or 'masked', y-coordinates of LxMxN grid. If style is specified as 'points', y-coordinates of specific points at which to solve kriging system. Note that in this case, xpoints, ypoints, and zpoints must have the same dimensions (i.e., $L = M = N$).
- **zpoints** (*array_like, shape (L,) or (L, 1)*) – If style is specified as 'grid' or 'masked', z-coordinates of LxMxN grid. If style is specified as 'points', z-coordinates of specific points at which to solve kriging system. Note that in this case, xpoints, ypoints, and zpoints must have the same dimensions (i.e., $L = M = N$).
- **mask** (*boolean array, shape (L, M, N), optional*) – Specifies the points in the rectangular grid defined by xpoints, ypoints, zpoints that are to be excluded in the kriging calculations. Must be provided if style is specified as 'masked'. False indicates that the point should not be masked, so the kriging system will be solved at the point. True indicates that the point should be masked, so the kriging system will not be solved at the point.
- **backend** (*string, optional*) – Specifies which approach to use in kriging. Specifying 'vectorized' will solve the entire kriging problem at once in a vectorized operation. This approach is faster but also can consume a significant amount of memory for large grids and/or large datasets. Specifying 'loop' will loop through each point at which the kriging system is to be solved. This approach is slower but also less memory-intensive. Default is 'vectorized'.
- **specified_drift_arrays** (*list of array-like objects, optional*) – Specifies the drift values at the points at which the kriging system is to be evaluated. Required if 'specified' drift provided in the list of drift terms when instantiating the UniversalKriging3D class. Must be a list of arrays in the same order as the list provided when instantiating the kriging object. Array(s) must be the same dimension as the specified grid or have the same number of points as the specified points; i.e., the arrays either must be shape (L, M, N), where L is the number of z grid-points, M is the number of y grid-points, and N is the number of x grid-points, or shape (N,) or (N, 1), where N is the number of points at which to evaluate the kriging system.

Returns

- **kvalues** (*ndarray, shape (L, M, N) or (N,) or (N, 1)*) – Interpolated values of specified grid or at the specified set of points. If style was specified as 'masked', kvalues will be a numpy

masked array.

- **sigmasq** (*ndarray, shape (L, M, N) or (N,) or (N, 1)*) – Variance at specified grid points or at the specified set of points. If style was specified as ‘masked’, sigmasq will be a numpy masked array.

get_epsilon_residuals()

Returns the epsilon residuals for the variogram fit. No arguments.

get_statistics()

Returns the Q1, Q2, and cR statistics for the variogram fit (in that order). No arguments.

plot_epsilon_residuals()

Plots the epsilon residuals for the variogram fit. No arguments.

print_statistics()

Prints out the Q1, Q2, and cR statistics for the variogram fit. NOTE that ideally Q1 is close to zero, Q2 is close to 1, and cR is as small as possible.

switch_plotting()

Enables/disable variogram plot display. No arguments.

switch_verbose()

Enables/disables program text output. No arguments.

update_variogram_model (*variogram_model, variogram_parameters=None, variogram_function=None, nlags=6, weight=False, anisotropy_scaling_y=1.0, anisotropy_scaling_z=1.0, anisotropy_angle_x=0.0, anisotropy_angle_y=0.0, anisotropy_angle_z=0.0*)

Changes the variogram model and variogram parameters for the kriging system.

Parameters

- **variogram_model** (*str or GStools CovModel*) – May be any of the variogram models listed above. May also be ‘custom’, in which case *variogram_parameters* and *variogram_function* must be specified. You can also use a [GStools CovModel](#).
- **variogram_parameters** (*list or dict, optional*) – List or dict of variogram model parameters, as explained above. If not provided, a best fit model will be calculated as described above.
- **variogram_function** (*callable, optional*) – A callable function that must be provided if *variogram_model* is specified as ‘custom’. See above for more information.
- **nlags** (*int, optional*) – Number of averaging bins for the semivariogram. Default is 6.
- **weight** (*boolean, optional*) – Flag that specifies if semivariance at smaller lags should be weighted more heavily when automatically calculating variogram model. See above for more information. True indicates that weights will be applied. Default is False.
- **anisotropy_scaling_y** (*float, optional*) – Scalar stretching value to take into account anisotropy in y-direction. Default is 1 (effectively no stretching). See above for more information.
- **anisotropy_scaling_z** (*float, optional*) – Scalar stretching value to take into account anisotropy in z-direction. Default is 1 (effectively no stretching). See above for more information.
- **anisotropy_angle_x** (*float, optional*) – Angle (in degrees) by which to rotate coordinate system about the x axis in order to take into account anisotropy. Default is 0 (no rotation). See above for more information.

- **anisotropy_angle_y** (*float, optional*) – Angle (in degrees) by which to rotate coordinate system about the y axis in order to take into account anisotropy. Default is 0 (no rotation). See above for more information.
- **anisotropy_angle_z** (*float, optional*) – Angle (in degrees) by which to rotate coordinate system about the z axis in order to take into account anisotropy. Default is 0 (no rotation). See above for more information.

UNBIAS = True

eps = 1e-10

```
variogram_dict = {'exponential': <function exponential_variogram_model>,
                  'gaussian': <function gaussian_variogram_model>, 'hole-effect': <function
hole_effect_variogram_model>, 'linear': <function linear_variogram_model>, 'power':
<function power_variogram_model>, 'spherical': <function
spherical_variogram_model>}
```


3.1.5 pykrige.rk.RegressionKriging

```
class pykrige.rk.RegressionKriging(regression_model=SVR(), method='ordinary',
                                   variogram_model='linear', n_closest_points=10, nlags=6,
                                   weight=False, verbose=False, exact_values=True, pseudo_inv=False,
                                   pseudo_inv_type='pinv', variogram_parameters=None,
                                   variogram_function=None, anisotropy_scaling=(1.0, 1.0),
                                   anisotropy_angle=(0.0, 0.0, 0.0), enable_statistics=False,
                                   coordinates_type='euclidean', drift_terms=None, point_drift=None,
                                   ext_drift_grid=(None, None, None), functional_drift=None)
```

Bases: object

An implementation of Regression-Kriging.

As described here: <https://en.wikipedia.org/wiki/Regression-Kriging>

Parameters

- **regression_model** (*machine learning model instance from sklearn*) –
- **method** (*str, optional*) – type of kriging to be performed
- **variogram_model** (*str, optional*) – variogram model to be used during Kriging
- **n_closest_points** (*int*) – number of closest points to be used during Ordinary Kriging
- **nlags** (*int*) – see OK/UK class description
- **weight** (*bool*) – see OK/UK class description
- **verbose** (*bool*) – see OK/UK class description
- **exact_values** (*bool*) – see OK/UK class description
- **variogram_parameters** (*list or dict*) – see OK/UK class description
- **variogram_function** (*callable*) – see OK/UK class description
- **anisotropy_scaling** (*tuple*) – single value for 2D (UK/OK) and two values in 3D (UK3D/OK3D)
- **anisotropy_angle** (*tuple*) – single value for 2D (UK/OK) and three values in 3D (UK3D/OK3D)
- **enable_statistics** (*bool*) – see OK class description
- **coordinates_type** (*str*) – see OK/UK class description
- **drift_terms** (*list of strings*) – see UK/UK3D class description
- **point_drift** (*array_like*) – see UK class description
- **ext_drift_grid** (*tuple*) – Holding the three values `external_drift`, `external_drift_x` and `external_drift_z` for the UK class
- **functional_drift** (*list of callable*) – see UK/UK3D class description

Methods

<code>fit(p, x, y)</code>	Fit the regression method and also Kriging the residual.
<code>krige_residual(x, **kwargs)</code>	Calculate the residuals.
<code>predict(p, x, **kwargs)</code>	Predict.
<code>score(p, x, y[, sample_weight])</code>	Overloading default regression score method.

`fit(p, x, y)`

Fit the regression method and also Kriging the residual.

Parameters

- **p** (*ndarray*) – (Ns, d) array of predictor variables (Ns samples, d dimensions) for regression
- **x** (*ndarray*) – ndarray of (x, y) points. Needs to be a (Ns, 2) array corresponding to the lon/lat, for example 2d regression kriging. array of Points, (x, y, z) pairs of shape (N, 3) for 3d kriging
- **y** (*ndarray*) – array of targets (Ns,)

`krige_residual(x, **kwargs)`

Calculate the residuals.

Parameters

x (*ndarray*) – ndarray of (x, y) points. Needs to be a (Ns, 2) array corresponding to the lon/lat, for example.

Returns

residual – kriged residual values

Return type

ndarray

`predict(p, x, **kwargs)`

Predict.

Parameters

- **p** (*ndarray*) – (Ns, d) array of predictor variables (Ns samples, d dimensions) for regression
- **x** (*ndarray*) – ndarray of (x, y) points. Needs to be a (Ns, 2) array corresponding to the lon/lat, for example. array of Points, (x, y, z) pairs of shape (N, 3) for 3d kriging

Returns

pred – The expected value of ys for the query inputs, of shape (Ns,).

Return type

ndarray

`score(p, x, y, sample_weight=None, **kwargs)`

Overloading default regression score method.

Parameters

- **p** (*ndarray*) – (Ns, d) array of predictor variables (Ns samples, d dimensions) for regression
- **x** (*ndarray*) – ndarray of (x, y) points. Needs to be a (Ns, 2) array corresponding to the lon/lat, for example. array of Points, (x, y, z) pairs of shape (N, 3) for 3d kriging

- **y** (*ndarray*) – array of targets (N_s ,)

3.2 Wrappers

<code>pykrige.rk.Krige([method, variogram_model, ...])</code>	A scikit-learn wrapper class for Ordinary and Universal Kriging.
---	--

3.2.1 pykrige.rk.Krige

```
class pykrige.rk.Krige(method='ordinary', variogram_model='linear', nlags=6, weight=False,
                        n_closest_points=10, verbose=False, exact_values=True, pseudo_inv=False,
                        pseudo_inv_type='pinv', variogram_parameters=None, variogram_function=None,
                        anisotropy_scaling=(1.0, 1.0), anisotropy_angle=(0.0, 0.0, 0.0),
                        enable_statistics=False, coordinates_type='euclidean', drift_terms=None,
                        point_drift=None, ext_drift_grid=(None, None, None), functional_drift=None)
```

Bases: `RegressorMixin`, `BaseEstimator`

A scikit-learn wrapper class for Ordinary and Universal Kriging.

This works with both `Grid/RandomSearchCv` for finding the best Krige parameters combination for a problem.

Parameters

- **method** (*str*, *optional*) – type of kriging to be performed
- **variogram_model** (*str*, *optional*) – variogram model to be used during Kriging
- **nlags** (*int*) – see OK/UK class description
- **weight** (*bool*) – see OK/UK class description
- **n_closest_points** (*int*) – number of closest points to be used during Ordinary Kriging
- **verbose** (*bool*) – see OK/UK class description
- **exact_values** (*bool*) – see OK/UK class description
- **variogram_parameters** (*list* or *dict*) – see OK/UK class description
- **variogram_function** (*callable*) – see OK/UK class description
- **anisotropy_scaling** (*tuple*) – single value for 2D (UK/OK) and two values in 3D (UK3D/OK3D)
- **anisotropy_angle** (*tuple*) – single value for 2D (UK/OK) and three values in 3D (UK3D/OK3D)
- **enable_statistics** (*bool*) – see OK class description
- **coordinates_type** (*str*) – see OK/UK class description
- **drift_terms** (*list of strings*) – see UK/UK3D class description
- **point_drift** (*array_like*) – see UK class description
- **ext_drift_grid** (*tuple*) – Holding the three values `external_drift`, `external_drift_x` and `external_drift_z` for the UK class
- **functional_drift** (*list of callable*) – see UK/UK3D class description

Methods

<code>execute(points, *args, **kwargs)</code>	Execute.
<code>fit(x, y, *args, **kwargs)</code>	Fit the current model.
<code>get_params([deep])</code>	Get parameters for this estimator.
<code>predict(x, *args, **kwargs)</code>	Predict.
<code>score(X, y[, sample_weight])</code>	Return the coefficient of determination of the prediction.
<code>set_params(**params)</code>	Set the parameters of this estimator.

execute(*points*, **args*, ***kwargs*)

Execute.

Parameters

points (*dict*) –

Returns

- *Prediction array*
- *Variance array*

fit(*x*, *y*, **args*, ***kwargs*)

Fit the current model.

Parameters

- **x** (*ndarray*) – array of Points, (x, y) pairs of shape (N, 2) for 2d kriging array of Points, (x, y, z) pairs of shape (N, 3) for 3d kriging
- **y** (*ndarray*) – array of targets (N,)

get_params(*deep=True*)

Get parameters for this estimator.

Parameters

deep (*bool*, *default=True*) – If True, will return the parameters for this estimator and contained subobjects that are estimators.

Returns

params – Parameter names mapped to their values.

Return type

dict

predict(*x*, **args*, ***kwargs*)

Predict.

Parameters

x (*ndarray*) – array of Points, (x, y) pairs of shape (N, 2) for 2d kriging array of Points, (x, y, z) pairs of shape (N, 3) for 3d kriging

Return type

Prediction array

score(*X*, *y*, *sample_weight=None*)

Return the coefficient of determination of the prediction.

The coefficient of determination R^2 is defined as $(1 - \frac{u}{v})$, where u is the residual sum of squares $((y_true - y_pred)** 2).sum()$ and v is the total sum of squares $((y_true - y_true.mean()) ** 2).sum()$. The best possible score is 1.0 and it can be negative (because the model can be arbitrarily worse). A constant model that always predicts the expected value of y , disregarding the input features, would get a R^2 score of 0.0.

Parameters

- **X** (*array-like of shape (n_samples, n_features)*) – Test samples. For some estimators this may be a precomputed kernel matrix or a list of generic objects instead with shape $(n_samples, n_samples_fitted)$, where `n_samples_fitted` is the number of samples used in the fitting for the estimator.
- **y** (*array-like of shape (n_samples,) or (n_samples, n_outputs)*) – True values for X .
- **sample_weight** (*array-like of shape (n_samples,)*, *default=None*) – Sample weights.

Returns

score – R^2 of `self.predict(X)` wrt. y .

Return type

float

Notes

The R^2 score used when calling `score` on a regressor uses `multioutput='uniform_average'` from version 0.23 to keep consistent with default value of `r2_score()`. This influences the `score` method of all the multioutput regressors (except for `MultiOutputRegressor`).

`set_params(**params)`

Set the parameters of this estimator.

The method works on simple estimators as well as on nested objects (such as `Pipeline`). The latter have parameters of the form `<component>__<parameter>` so that it's possible to update each component of a nested object.

Parameters

****params** (*dict*) – Estimator parameters.

Returns

self – Estimator instance.

Return type

estimator instance

3.3 Tools

<code>pykrige.kriging_tools.write_asc_grid(x, y, z)</code>	Writes gridded data to ASCII grid file (*.asc).
<code>pykrige.kriging_tools.read_asc_grid(filename)</code>	Reads ASCII grid file (*.asc).

3.3.1 pykrige.kriging_tools.write_asc_grid

`pykrige.kriging_tools.write_asc_grid(x, y, z, filename='output.asc', no_data=-999.0, style=1)`

Writes gridded data to ASCII grid file (*.asc).

This is useful for exporting data to a GIS program.

Parameters

- **x** (*array_like, shape (N,) or (N, 1)*) – X-coordinates of grid points at center of cells.
- **y** (*array_like, shape (M,) or (M, 1)*) – Y-coordinates of grid points at center of cells.
- **z** (*array_like, shape (M, N)*) – Gridded data values. May be a masked array.
- **filename** (*string, optional*) – Name of output *.asc file. Default name is 'output.asc'.
- **no_data** (*float, optional*) – no data value to be used
- **style** (*int, optional*) – Determines how to write the *.asc file header. Specifying 1 writes out DX, DY, XLLCENTER, YLLCENTER. Specifying 2 writes out CELLSIZE (note DX must be the same as DY), XLLCORNER, YLLCORNER. Default is 1.

3.3.2 pykrige.kriging_tools.read_asc_grid

`pykrige.kriging_tools.read_asc_grid(filename, footer=0)`

Reads ASCII grid file (*.asc).

Parameters

- **filename** (*str*) – Name of *.asc file.
- **footer** (*int, optional*) – Number of lines at bottom of *.asc file to skip.

Returns

- **grid_array** (*numpy array, shape (M, N)*) – (M, N) array of grid values, where M is number of Y-coordinates and N is number of X-coordinates. The array entry corresponding to the lower-left coordinates is at index [M, 0], so that the array is oriented as it would be in X-Y space.
- **x** (*numpy array, shape (N,)*) – 1D array of N X-coordinates.
- **y** (*numpy array, shape (M,)*) – 1D array of M Y-coordinates.
- **CELLSIZE** (*tuple or float*) – Either a two-tuple of (x-cell size, y-cell size), or a float that specifies the uniform cell size.
- **NODATA** (*float*) – Value that specifies which entries are not actual data.

EXAMPLES

4.1 Universal Kriging Example

In this example we apply a regional linear trend to the kriging system.

```
import matplotlib.pyplot as plt
import numpy as np

from pykrige.uk import UniversalKriging

data = np.array(
    [
        [0.3, 1.2, 0.47],
        [1.9, 0.6, 0.56],
        [1.1, 3.2, 0.74],
        [3.3, 4.4, 1.47],
        [4.7, 3.8, 1.74],
    ]
)

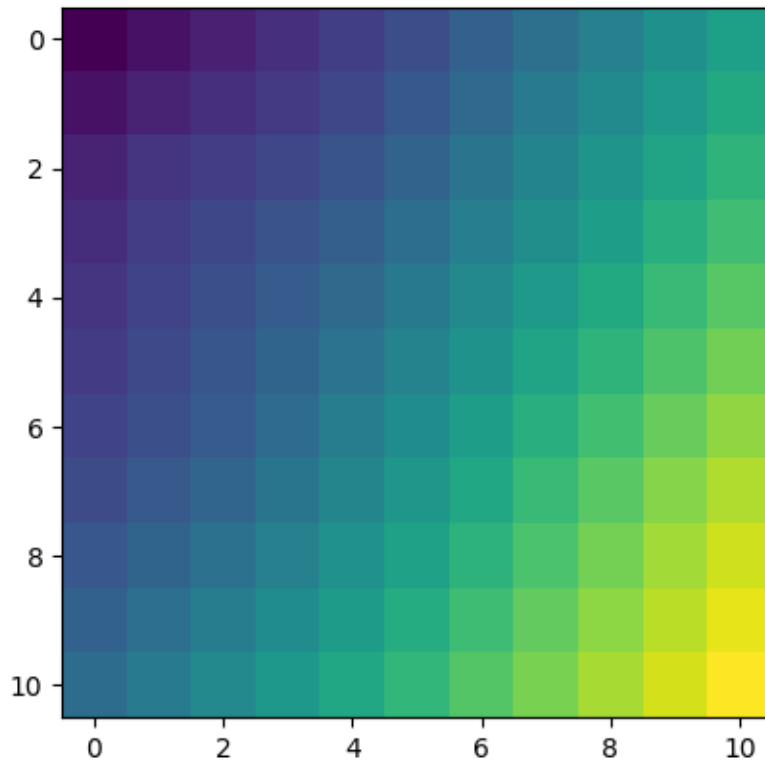
gridx = np.arange(0.0, 5.5, 0.5)
gridy = np.arange(0.0, 5.5, 0.5)
```

Create the universal kriging object. Required inputs are the X-coordinates of the data points, the Y-coordinates of the data points, and the Z-values of the data points. Variogram is handled as in the ordinary kriging case. `drift_terms` is a list of the drift terms to include; currently supported terms are 'regional_linear', 'point_log', and 'external_Z'. Refer to `UniversalKriging.__doc__` for more information.

```
UK = UniversalKriging(
    data[:, 0],
    data[:, 1],
    data[:, 2],
    variogram_model="linear",
    drift_terms=["regional_linear"],
)
```

Creates the kriged grid and the variance grid. Allows for kriging on a rectangular grid of points, on a masked rectangular grid of points, or with arbitrary points. (See `UniversalKriging.__doc__` for more information.)

```
z, ss = UK.execute("grid", gridx, gridy)
plt.imshow(z)
plt.show()
```



Total running time of the script: (0 minutes 0.154 seconds)

4.2 Ordinary Kriging Example

First we will create a 2D dataset together with the associated x, y grids.

```
import matplotlib.pyplot as plt
import numpy as np

import pykrige.kriging_tools as kt
from pykrige.ok import OrdinaryKriging

data = np.array([
    [0.3, 1.2, 0.47],
    [1.9, 0.6, 0.56],
    [1.1, 3.2, 0.74],
    [3.3, 4.4, 1.47],
```

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```

        [4.7, 3.8, 1.74],
    ]
)

gridx = np.arange(0.0, 5.5, 0.5)
gridy = np.arange(0.0, 5.5, 0.5)

```

Create the ordinary kriging object. Required inputs are the X-coordinates of the data points, the Y-coordinates of the data points, and the Z-values of the data points. If no variogram model is specified, defaults to a linear variogram model. If no variogram model parameters are specified, then the code automatically calculates the parameters by fitting the variogram model to the binned experimental semivariogram. The verbose kwarg controls code talk-back, and the enable_plotting kwarg controls the display of the semivariogram.

```

OK = OrdinaryKriging(
    data[:, 0],
    data[:, 1],
    data[:, 2],
    variogram_model="linear",
    verbose=False,
    enable_plotting=False,
)

```

Creates the kriged grid and the variance grid. Allows for kriging on a rectangular grid of points, on a masked rectangular grid of points, or with arbitrary points. (See OrdinaryKriging.__doc__ for more information.)

```

z, ss = OK.execute("grid", gridx, gridy)

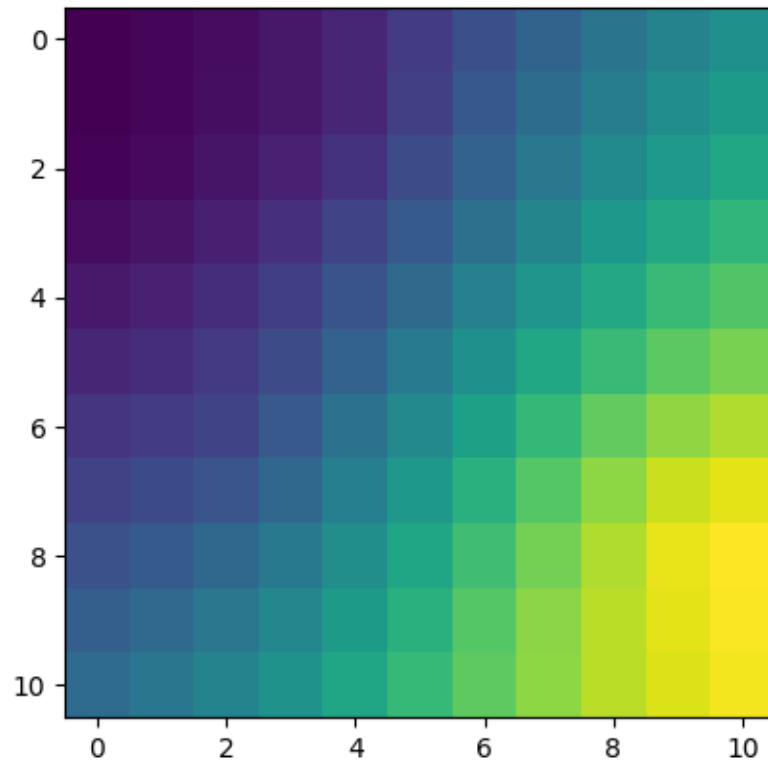
```

Writes the kriged grid to an ASCII grid file and plot it.

```

kt.write_asc_grid(gridx, gridy, z, filename="output.asc")
plt.imshow(z)
plt.show()

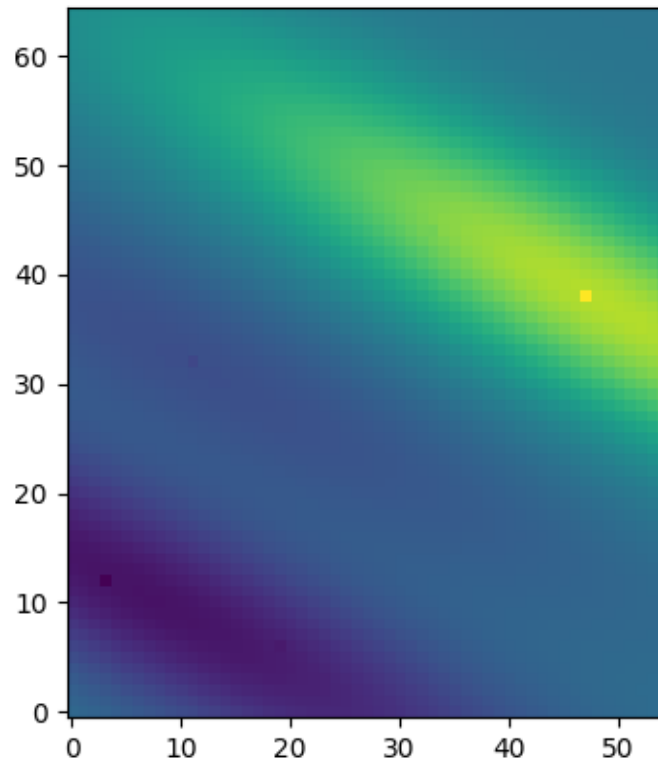
```



Total running time of the script: (0 minutes 0.109 seconds)

4.3 GStools Interface

Example how to use the PyKriging routines with a GStools CovModel.



```
import gstools as gs
import numpy as np
from matplotlib import pyplot as plt

from pykrige.ok import OrdinaryKriging

# conditioning data
data = np.array(
    [
        [0.3, 1.2, 0.47],
        [1.9, 0.6, 0.56],
        [1.1, 3.2, 0.74],
        [3.3, 4.4, 1.47],
        [4.7, 3.8, 1.74],
    ]
)

# grid definition for output field
gridx = np.arange(0.0, 5.5, 0.1)
gridy = np.arange(0.0, 6.5, 0.1)
# a GSTools based covariance model
cov_model = gs.Gaussian(dim=2, len_scale=4, anis=0.2, angles=-0.5, var=0.5, nugget=0.1)
# ordinary kriging with pykrige
OK1 = OrdinaryKriging(data[:, 0], data[:, 1], data[:, 2], cov_model)
```

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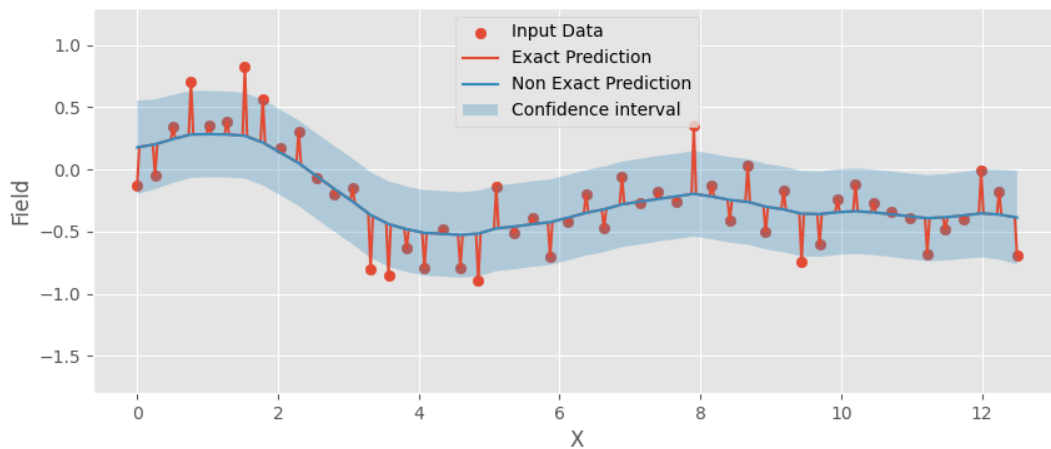
(continued from previous page)

```
z1, ss1 = OK1.execute("grid", gridx, gridy)
plt.imshow(z1, origin="lower")
plt.show()
```

Total running time of the script: (0 minutes 0.116 seconds)

4.4 Exact Values

PyKriging demonstration and usage as a non-exact interpolator in 1D.



```
import matplotlib.pyplot as plt
import numpy as np

from pykrige.ok import OrdinaryKriging

plt.style.use("ggplot")

np.random.seed(42)

x = np.linspace(0, 12.5, 50)
xpred = np.linspace(0, 12.5, 393)
y = np.sin(x) * np.exp(-0.25 * x) + np.random.normal(-0.25, 0.25, 50)

# compare OrdinaryKriging as an exact and non exact interpolator
uk = OrdinaryKriging(
    x, np.zeros(x.shape), y, variogram_model="linear", exact_values=False
)
uk_exact = OrdinaryKriging(x, np.zeros(x.shape), y, variogram_model="linear")

y_pred, y_std = uk.execute("grid", xpred, np.array([0.0]), backend="loop")
y_pred_exact, y_std_exact = uk_exact.execute(
    "grid", xpred, np.array([0.0]), backend="loop"
)
```

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```

y_pred = np.squeeze(y_pred)
y_std = np.squeeze(y_std)

y_pred_exact = np.squeeze(y_pred_exact)
y_std_exact = np.squeeze(y_std_exact)

fig, ax = plt.subplots(1, 1, figsize=(10, 4))

ax.scatter(x, y, label="Input Data")
ax.plot(xpred, y_pred_exact, label="Exact Prediction")
ax.plot(xpred, y_pred, label="Non Exact Prediction")

ax.fill_between(
    xpred,
    y_pred - 3 * y_std,
    y_pred + 3 * y_std,
    alpha=0.3,
    label="Confidence interval",
)
ax.legend(loc=9)
ax.set_ylim(-1.8, 1.3)
ax.legend(loc=9)
plt.xlabel("X")
plt.ylabel("Field")
plt.show()

```

Total running time of the script: (0 minutes 0.193 seconds)

4.5 Regression kriging

An example of regression kriging

```

=====
regression model: SVR
Finished learning regression model
Finished kriging residuals
Regression Score:  -0.03405385545698292
RK score:  0.6706182225388981
=====
regression model: RandomForestRegressor
Finished learning regression model
Finished kriging residuals
Regression Score:  0.7041419269689924
RK score:  0.7415694880137507
=====
regression model: LinearRegression
/home/docs/checkouts/readthedocs.org/user_builds/pykrige/envs/v1.7.0/lib/python3.7/site-
↳ packages/sklearn/linear_model/_base.py:145: FutureWarning: 'normalize' was deprecated_

```

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```

↳ in version 1.0 and will be removed in 1.2.
If you wish to scale the data, use Pipeline with a StandardScaler in a preprocessing
↳ stage. To reproduce the previous behavior:

from sklearn.pipeline import make_pipeline

model = make_pipeline(StandardScaler(with_mean=False), LinearRegression())

If you wish to pass a sample_weight parameter, you need to pass it as a fit parameter to
↳ each step of the pipeline as follows:

kwargs = {s[0] + '__sample_weight': sample_weight for s in model.steps}
model.fit(X, y, **kwargs)

FutureWarning,
Finished learning regression model
Finished kriging residuals
Regression Score: 0.5277968398381674
RK score: 0.6036605153133717

```

```

import sys

from sklearn.datasets import fetch_california_housing
from sklearn.ensemble import RandomForestRegressor
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import train_test_split
from sklearn.svm import SVR

from pykrige.rk import RegressionKriging

svr_model = SVR(C=0.1, gamma="auto")
rf_model = RandomForestRegressor(n_estimators=100)
lr_model = LinearRegression(normalize=True, copy_X=True, fit_intercept=False)

models = [svr_model, rf_model, lr_model]

try:
    housing = fetch_california_housing()
except PermissionError:
    # this dataset can occasionally fail to download on Windows
    sys.exit(0)

# take the first 5000 as Kriging is memory intensive
p = housing["data"][:5000, :-2]
x = housing["data"][:5000, -2:]
target = housing["target"][:5000]

```

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```

p_train, p_test, x_train, x_test, target_train, target_test = train_test_split(
    p, x, target, test_size=0.3, random_state=42
)

for m in models:
    print("=" * 40)
    print("regression model:", m.__class__.__name__)
    m_rk = RegressionKriging(regression_model=m, n_closest_points=10)
    m_rk.fit(p_train, x_train, target_train)
    print("Regression Score: ", m_rk.regression_model.score(p_test, target_test))
    print("RK score: ", m_rk.score(p_test, x_test, target_test))

```

Total running time of the script: (0 minutes 6.993 seconds)

4.6 Classification kriging

An example of classification kriging

```

=====
classification model: SVC
Finished learning classification model
Finished kriging residuals
Classification Score:  0.212
CK score:  0.6566666666666666
=====
classification model: RandomForestClassifier
Finished learning classification model
Finished kriging residuals
Classification Score:  0.5766666666666667
CK score:  0.5946666666666667
=====
classification model: LogisticRegression
Finished learning classification model
Finished kriging residuals
Classification Score:  0.5246666666666666
CK score:  0.6546666666666666

```

```

import sys

from sklearn.datasets import fetch_california_housing
from sklearn.ensemble import RandomForestClassifier
from sklearn.linear_model import LogisticRegression
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import KBinsDiscretizer
from sklearn.svm import SVC

```

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```

from pykrige.ck import ClassificationKriging

svc_model = SVC(C=0.1, gamma="auto", probability=True)
rf_model = RandomForestClassifier(n_estimators=100)
lr_model = LogisticRegression(max_iter=10000)

models = [svc_model, rf_model, lr_model]

try:
    housing = fetch_california_housing()
except PermissionError:
    # this dataset can occasionally fail to download on Windows
    sys.exit(0)

# take the first 5000 as Kriging is memory intensive
p = housing["data"][:5000, :-2]
x = housing["data"][:5000, :-2]
target = housing["target"][:5000]
discretizer = KBinsDiscretizer(encode="ordinal")
target = discretizer.fit_transform(target.reshape(-1, 1))

p_train, p_test, x_train, x_test, target_train, target_test = train_test_split(
    p, x, target, test_size=0.3, random_state=42
)

for m in models:
    print("=" * 40)
    print("classification model:", m.__class__.__name__)
    m_ck = ClassificationKriging(classification_model=m, n_closest_points=10)
    m_ck.fit(p_train, x_train, target_train)
    print(
        "Classification Score: ", m_ck.classification_model.score(p_test, target_test)
    )
    print("CK score: ", m_ck.score(p_test, x_test, target_test))

```

Total running time of the script: (0 minutes 25.383 seconds)

4.7 Geometric example

A small example script showing the usage of the ‘geographic’ coordinates type for ordinary kriging on a sphere.

```

import numpy as np
from matplotlib import pyplot as plt

from pykrige.ok import OrdinaryKriging

# Make this example reproducible:
np.random.seed(89239413)

```

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```

# Generate random data following a uniform spatial distribution
# of nodes and a uniform distribution of values in the interval
# [2.0, 5.5]:
N = 7
lon = 360.0 * np.random.random(N)
lat = 180.0 / np.pi * np.arcsin(2 * np.random.random(N) - 1)
z = 3.5 * np.random.rand(N) + 2.0

# Generate a regular grid with 60° longitude and 30° latitude steps:
grid_lon = np.linspace(0.0, 360.0, 7)
grid_lat = np.linspace(-90.0, 90.0, 7)

# Create ordinary kriging object:
OK = OrdinaryKriging(
    lon,
    lat,
    z,
    variogram_model="linear",
    verbose=False,
    enable_plotting=False,
    coordinates_type="geographic",
)

# Execute on grid:
z1, ss1 = OK.execute("grid", grid_lon, grid_lat)

# Create ordinary kriging object ignoring curvature:
OK = OrdinaryKriging(
    lon, lat, z, variogram_model="linear", verbose=False, enable_plotting=False
)

# Execute on grid:
z2, ss2 = OK.execute("grid", grid_lon, grid_lat)

```

Print data at equator (last longitude index will show periodicity):

```

print("Original data:")
print("Longitude:", lon.astype(int))
print("Latitude: ", lat.astype(int))
print("z:          ", np.array_str(z, precision=2))
print("\nKrige at 60° latitude:\n=====")
print("Longitude:", grid_lon)
print("Value:      ", np.array_str(z1[5, :], precision=2))
print("Sigma²:     ", np.array_str(ss1[5, :], precision=2))
print("\nIgnoring curvature:\n=====")
print("Value:      ", np.array_str(z2[5, :], precision=2))
print("Sigma²:     ", np.array_str(ss2[5, :], precision=2))

```

```

Original data:
Longitude: [122 166  92 138  86 122 136]
Latitude:  [-46 -36 -25 -73 -25  50 -29]
z:          [2.75 3.36 2.24 3.07 3.37 5.25 2.82]

```

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Kriging at 60° latitude:

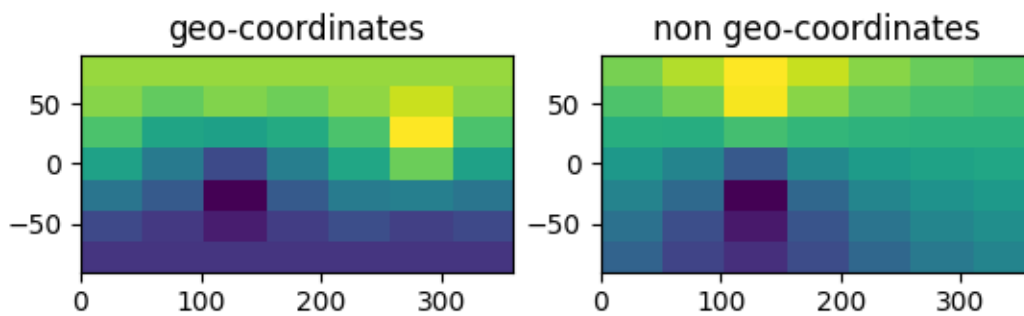
```
=====
Longitude: [ 0.  60. 120. 180. 240. 300. 360.]
Value:     [5.29 5.11 5.27 5.17 5.35 5.63 5.29]
Sigma²:    [2.22 1.32 0.42 1.21 2.07 2.48 2.22]
```

Ignoring curvature:

```
=====
Value:     [4.55 4.72 5.25 4.82 4.61 4.53 4.48]
Sigma²:    [3.79 2.   0.39 1.85 3.54 5.46 7.53]
```

We can see that the data point at longitude 122, latitude 50 correctly dominates the kriged results, since it is the closest node in spherical distance metric, as longitude differences scale with $\cos(\text{latitude})$. When kriging using longitude / latitude linearly, the value for grid points with longitude values further away as longitude is now incorrectly weighted equally as latitude.

```
fig, (ax1, ax2) = plt.subplots(1, 2)
ax1.imshow(z1, extent=[0, 360, -90, 90], origin="lower")
ax1.set_title("geo-coordinates")
ax2.imshow(z2, extent=[0, 360, -90, 90], origin="lower")
ax2.set_title("non geo-coordinates")
plt.show()
```



Total running time of the script: (0 minutes 0.145 seconds)

4.8 Three-Dimensional Kriging Example

```
import numpy as np
from matplotlib import pyplot as plt

from pykrige.ok3d import OrdinaryKriging3D
from pykrige.uk3d import UniversalKriging3D

data = np.array(
    [
        [0.1, 0.1, 0.3, 0.9],
        [0.2, 0.1, 0.4, 0.8],
        [0.1, 0.3, 0.1, 0.9],
        [0.5, 0.4, 0.4, 0.5],
        [0.3, 0.3, 0.2, 0.7],
    ]
)

gridx = np.arange(0.0, 0.6, 0.05)
gridy = np.arange(0.0, 0.6, 0.01)
gridz = np.arange(0.0, 0.6, 0.1)
```

Create the 3D ordinary kriging object and solves for the three-dimension kriged volume and variance. Refer to OrdinaryKriging3D.__doc__ for more information.

```
ok3d = OrdinaryKriging3D(
    data[:, 0], data[:, 1], data[:, 2], data[:, 3], variogram_model="linear"
)
k3d1, ss3d = ok3d.execute("grid", gridx, gridy, gridz)
```

Create the 3D universal kriging object and solves for the three-dimension kriged volume and variance. Refer to UniversalKriging3D.__doc__ for more information.

```
uk3d = UniversalKriging3D(
    data[:, 0],
    data[:, 1],
    data[:, 2],
    data[:, 3],
    variogram_model="linear",
    drift_terms=["regional_linear"],
)
k3d2, ss3d = uk3d.execute("grid", gridx, gridy, gridz)
```

To use the generic ‘specified’ drift term, the user must provide the drift values at each data point and at every grid point. The following example is equivalent to using a linear drift in all three spatial dimensions. Refer to UniversalKriging3D.__doc__ for more information.

```
zg, yg, xg = np.meshgrid(gridz, gridy, gridx, indexing="ij")
uk3d = UniversalKriging3D(
    data[:, 0],
```

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```

    data[:, 1],
    data[:, 2],
    data[:, 3],
    variogram_model="linear",
    drift_terms=["specified"],
    specified_drift=[data[:, 0], data[:, 1], data[:, 2]],
)
k3d3, ss3d = uk3d.execute(
    "grid", gridx, gridy, gridz, specified_drift_arrays=[xg, yg, zg]
)

```

To use the generic ‘functional’ drift term, the user must provide a callable function that takes only the spatial dimensions as arguments. The following example is equivalent to using a linear drift only in the x-direction. Refer to `UniversalKriging3D.__doc__` for more information.

```

func = lambda x, y, z: x
uk3d = UniversalKriging3D(
    data[:, 0],
    data[:, 1],
    data[:, 2],
    data[:, 3],
    variogram_model="linear",
    drift_terms=["functional"],
    functional_drift=[func],
)
k3d4, ss3d = uk3d.execute("grid", gridx, gridy, gridz)

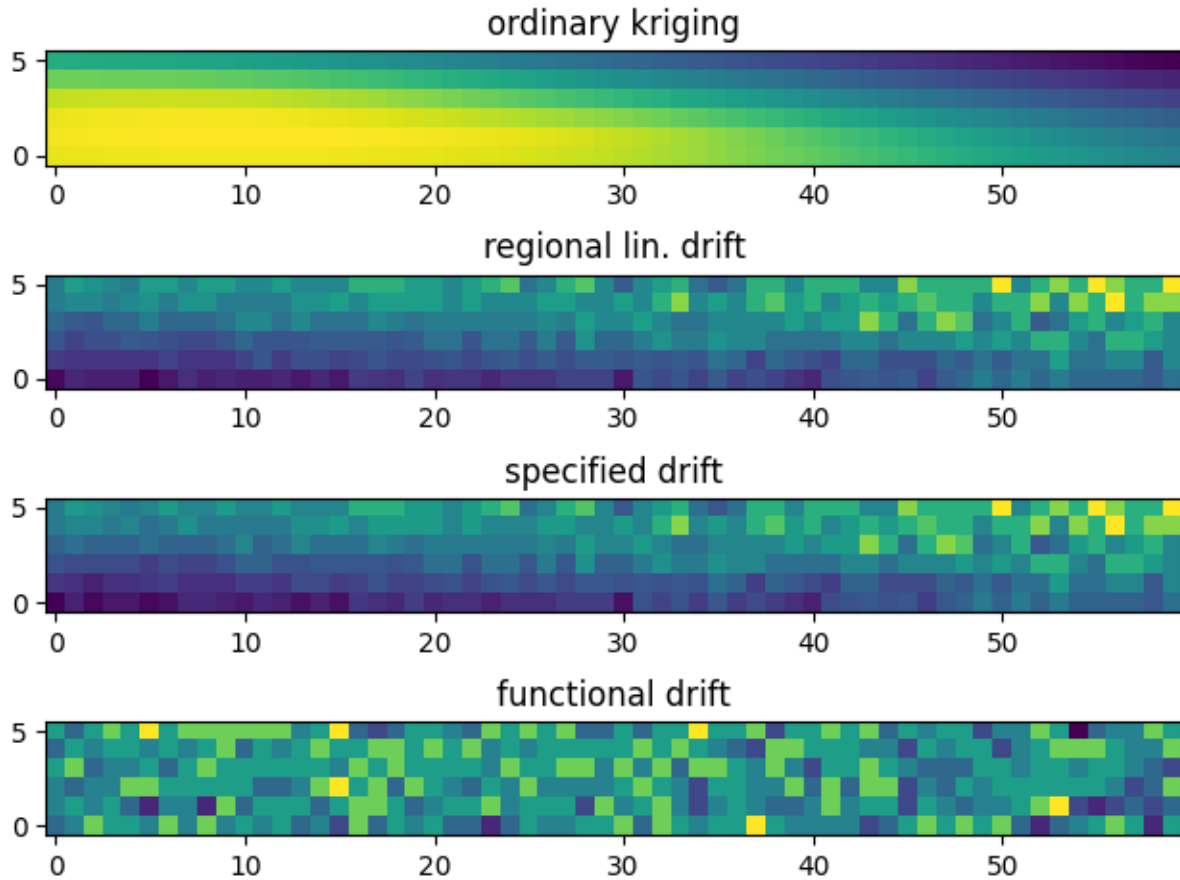
```

Note that the use of the ‘specified’ and ‘functional’ generic drift capabilities is essentially identical in the two-dimensional universal kriging class (except for a difference in the number of spatial coordinates for the passed drift functions). See `UniversalKriging.__doc__` for more information.

```

fig, (ax1, ax2, ax3, ax4) = plt.subplots(4)
ax1.imshow(k3d1[:, :, 0], origin="lower")
ax1.set_title("ordinary kriging")
ax2.imshow(k3d2[:, :, 0], origin="lower")
ax2.set_title("regional lin. drift")
ax3.imshow(k3d3[:, :, 0], origin="lower")
ax3.set_title("specified drift")
ax4.imshow(k3d4[:, :, 0], origin="lower")
ax4.set_title("functional drift")
plt.tight_layout()
plt.show()

```



Total running time of the script: (0 minutes 0.354 seconds)

4.9 Krige CV

Searching for optimal kriging parameters with cross validation

```
Fitting 5 folds for each of 8 candidates, totalling 40 fits
n_closest_points will be ignored for UniversalKriging
n_closest_points will be ignored for UniversalKriging
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n_closest_points will be ignored for UniversalKriging
n_closest_points will be ignored for UniversalKriging
n_closest_points will be ignored for UniversalKriging
```

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[illegible]

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```
n_closest_points will be ignored for UniversalKriging
n_closest_points will be ignored for UniversalKriging
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n_closest_points will be ignored for UniversalKriging
n_closest_points will be ignored for UniversalKriging
n_closest_points will be ignored for UniversalKriging
n_closest_points will be ignored for UniversalKriging
best_score R2 = -0.097
best_params = {'method': 'universal3d', 'variogram_model': 'power'}
```

CV results::

```
- mean_test_score : [-0.25558625 -0.25299609 -0.27330461 -0.26203314 -0.11692599 -0.
-0.09740768
-0.15082744 -0.12514965]
- mean_train_score : [1. 1. 1. 1. 1. 1. 1. 1.]
- param_method : ['ordinary3d' 'ordinary3d' 'ordinary3d' 'ordinary3d' 'universal3d'
'universal3d' 'universal3d' 'universal3d']
- param_variogram_model : ['linear' 'power' 'gaussian' 'spherical' 'linear' 'power'
'gaussian'
'spherical']
```

```
import numpy as np
from sklearn.model_selection import GridSearchCV

from pykrige.rk import Krige

# 2D Kriging param opt
```

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```

param_dict = {
    "method": ["ordinary", "universal"],
    "variogram_model": ["linear", "power", "gaussian", "spherical"],
    # "nlags": [4, 6, 8],
    # "weight": [True, False]
}

estimator = GridSearchCV(Krige(), param_dict, verbose=True, return_train_score=True)

# dummy data
X = np.random.randint(0, 400, size=(100, 2)).astype(float)
y = 5 * np.random.rand(100)

# run the gridsearch
estimator.fit(X=X, y=y)

if hasattr(estimator, "best_score_"):
    print("best_score R² = {:.3f}".format(estimator.best_score_))
    print("best_params = ", estimator.best_params_)

print("\nCV results:")
if hasattr(estimator, "cv_results_"):
    for key in [
        "mean_test_score",
        "mean_train_score",
        "param_method",
        "param_variogram_model",
    ]:
        print(" - {} : {}".format(key, estimator.cv_results_[key]))

# 3D Kriging param opt

param_dict3d = {
    "method": ["ordinary3d", "universal3d"],
    "variogram_model": ["linear", "power", "gaussian", "spherical"],
    # "nlags": [4, 6, 8],
    # "weight": [True, False]
}

estimator = GridSearchCV(Krige(), param_dict3d, verbose=True, return_train_score=True)

# dummy data
X3 = np.random.randint(0, 400, size=(100, 3)).astype(float)
y = 5 * np.random.rand(100)

# run the gridsearch
estimator.fit(X=X3, y=y)

if hasattr(estimator, "best_score_"):

```

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```

print("best_score R2 = {:.3f}".format(estimator.best_score_))
print("best_params = ", estimator.best_params_)

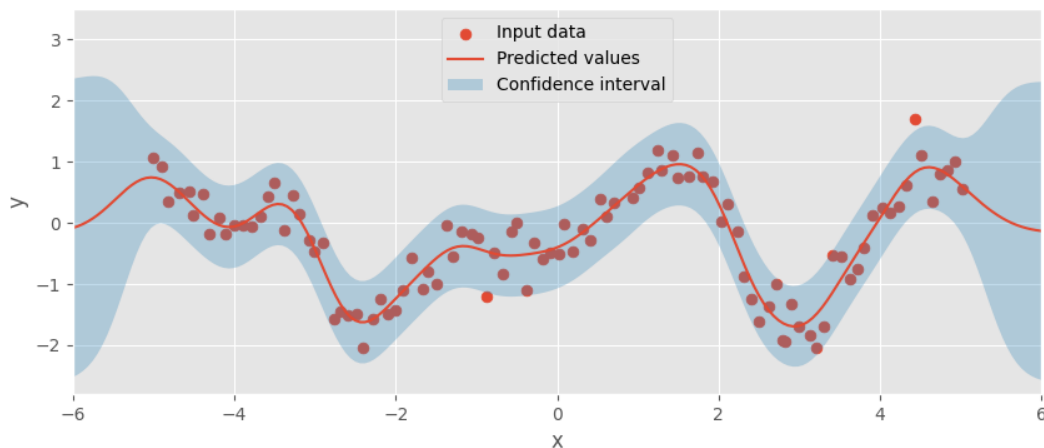
print("\nCV results:")
if hasattr(estimator, "cv_results_"):
    for key in [
        "mean_test_score",
        "mean_train_score",
        "param_method",
        "param_variogram_model",
    ]:
        print(" - {} : {}".format(key, estimator.cv_results_[key]))

```

Total running time of the script: (0 minutes 4.968 seconds)

4.10 1D Kriging

An example of 1D kriging with PyKrige



```

import matplotlib.pyplot as plt
import numpy as np

from pykrige import OrdinaryKriging

plt.style.use("ggplot")

# fmt: off
# Data taken from
# https://blog.dominodatalab.com/fitting-gaussian-process-models-python/
X, y = np.array([
    [-5.01, 1.06], [-4.90, 0.92], [-4.82, 0.35], [-4.69, 0.49], [-4.56, 0.52],
    [-4.52, 0.12], [-4.39, 0.47], [-4.32, -0.19], [-4.19, 0.08], [-4.11, -0.19],
    [-4.00, -0.03], [-3.89, -0.03], [-3.78, -0.05], [-3.67, 0.10], [-3.59, 0.44],
    [-3.50, 0.66], [-3.39, -0.12], [-3.28, 0.45], [-3.20, 0.14], [-3.07, -0.28],

```

(continues on next page)

(continued from previous page)

```

    [-3.01,-0.46], [-2.90,-0.32], [-2.77,-1.58], [-2.69,-1.44], [-2.60,-1.51],
    [-2.49,-1.50], [-2.41,-2.04], [-2.28,-1.57], [-2.19,-1.25], [-2.10,-1.50],
    [-2.00,-1.42], [-1.91,-1.10], [-1.80,-0.58], [-1.67,-1.08], [-1.61,-0.79],
    [-1.50,-1.00], [-1.37,-0.04], [-1.30,-0.54], [-1.19,-0.15], [-1.06,-0.18],
    [-0.98,-0.25], [-0.87,-1.20], [-0.78,-0.49], [-0.68,-0.83], [-0.57,-0.15],
    [-0.50, 0.00], [-0.38,-1.10], [-0.29,-0.32], [-0.18,-0.60], [-0.09,-0.49],
    [0.03, -0.50], [0.09, -0.02], [0.20, -0.47], [0.31, -0.11], [0.41, -0.28],
    [0.53, 0.40], [0.61, 0.11], [0.70, 0.32], [0.94, 0.42], [1.02, 0.57],
    [1.13, 0.82], [1.24, 1.18], [1.30, 0.86], [1.43, 1.11], [1.50, 0.74],
    [1.63, 0.75], [1.74, 1.15], [1.80, 0.76], [1.93, 0.68], [2.03, 0.03],
    [2.12, 0.31], [2.23, -0.14], [2.31, -0.88], [2.40, -1.25], [2.50, -1.62],
    [2.63, -1.37], [2.72, -0.99], [2.80, -1.92], [2.83, -1.94], [2.91, -1.32],
    [3.00, -1.69], [3.13, -1.84], [3.21, -2.05], [3.30, -1.69], [3.41, -0.53],
    [3.52, -0.55], [3.63, -0.92], [3.72, -0.76], [3.80, -0.41], [3.91, 0.12],
    [4.04, 0.25], [4.13, 0.16], [4.24, 0.26], [4.32, 0.62], [4.44, 1.69],
    [4.52, 1.11], [4.65, 0.36], [4.74, 0.79], [4.84, 0.87], [4.93, 1.01],
    [5.02, 0.55]
]).T
# fmt: on

X_pred = np.linspace(-6, 6, 200)

# pykrige doesn't support 1D data for now, only 2D or 3D
# adapting the 1D input to 2D
uk = OrdinaryKriging(X, np.zeros(X.shape), y, variogram_model="gaussian")

y_pred, y_std = uk.execute("grid", X_pred, np.array([0.0]))

y_pred = np.squeeze(y_pred)
y_std = np.squeeze(y_std)

fig, ax = plt.subplots(1, 1, figsize=(10, 4))
ax.scatter(X, y, s=40, label="Input data")

ax.plot(X_pred, y_pred, label="Predicted values")
ax.fill_between(
    X_pred,
    y_pred - 3 * y_std,
    y_pred + 3 * y_std,
    alpha=0.3,
    label="Confidence interval",
)
ax.legend(loc=9)
ax.set_xlabel("x")
ax.set_ylabel("y")
ax.set_xlim(-6, 6)
ax.set_ylim(-2.8, 3.5)
plt.show()

```

Total running time of the script: (0 minutes 0.174 seconds)

CHANGELOG

5.1 Version 1.7.0

August 18, 2022

New features

- added support for GStools latlon models (#245)

Changes

- drop Python 3.6 support (setuptools>60 needs py>3.7) (#242)
- move `setup.cfg` content to `pyproject.toml` (PEP 621) (#242)
- move to `src/` based package structure (better for testing, building and structure) (#242)
- build wheels for apple silicon (#242)
- apply isort and add check to CI (#242)
- fix documentation (#242, #252)

Bug fixes

- fix `AttributeError: 'UniversalKriging' object has no attribute 'external_Z_array'` (#247)
- remove deprecated scipy (v1.9) method `pinv2` (#237)
- correcting partial sill in C backend (#226)

5.2 Version 1.6.1

September 02, 2021

New features

- IO routines for `zmap` files (#199)
- `write_asc_grid` got new keyword `no_data` (#199)

Changes

- now using a `pyproject.toml` file (#211)
- now using a single `main` branch in the repository (#212)
- Fixed typos (#188, #189)

Bug fixes

- `write_asc_grid` was too strict about dx (#197)

5.3 Version 1.6.0

April 04, 2021

New features

- added Classification Kriging (#165, #184)
- added wheels for Python 3.9 (#175)

Changes

- moved scikit-learn compat-class `Krige` to `pykrige.compat` (#165)
- dropped Python 3.5 support (#183)
- moved CI to GitHub-Actions (#175, #183)
- Fixed Typo in `02_kriging3D.py` example (#182)

5.4 Version 1.5.1

August 20, 2020

New features

- update Regression Kriging class to be compatible with all kriging features (#158)
- added option to enable/disable “exact values” to all kriging routines (#153)
- added option to use the pseudo-inverse in all kriging routines (#151)

Changes

- removed compat-layer for sklearn (#157)
- updated examples in documentation

5.5 Version 1.5.0

April 04, 2020

New features

- support for GStools covariance models (#125)
- pre-build wheels for py35-py38 on Linux, Windows and MacOS (#142)
- `GridSerachCV` from the `compat` module sets `iid=False` by default (if present in sklearn) to be future prove (iid will be deprecated) (#144)

Changes

- dropped py2* and py<3.5 support (#142)
- installation now requires cython (#142)
- codebase was formatted with black (#144)

- internally use of scipys lapack/blas bindings (#142)
- PyKrige is now part of the GeoStat-Framework

5.6 Version 1.4.1

January 13, 2019

New features

- Added method to obtain variogram model points. PR#94 by [Daniel Mejía Raigosa](#)

Bug fixes

- Fixed OrdinaryKriging readme example. PR#107 by [Harry Matchette-Downes](#)
- Fixed kriging matrix not being calculated correctly for geographic coordinates. PR99 by [Mike Rilee](#)

5.7 Version 1.4.0

April 24, 2018

New features

- Regression kriging algorithm. PR #27 by [Sudipta Basaks](#).
- Support for spherical coordinates. PR #23 by [Malte Ziebarth](#)
- Kriging parameter tuning with scikit-learn. PR #24 by [Sudipta Basaks](#).
- Variogram model parameters can be specified using a list or a dict. Allows for directly feeding in the partial sill rather than the full sill. PR #47 by [Benjamin Murphy](#).

Enhancements

- Improved memory usage in variogram calculations. PR #42 by [Sudipta Basaks](#).
- Added benchmark scripts. PR #36 by [Roman Yurchak](#)
- Added an extensive example using the meusegrids dataset. PR #28 by [kvanlombeek](#).

Bug fixes

- Statistics calculations in 3D kriging. PR #45 by [Will Chang](#).
- Automatic variogram estimation robustified. PR #47 by [Benjamin Murphy](#).

5.8 Version 1.3.1

December 10, 2016

- More robust setup for building Cython extensions

5.9 Version 1.3.0

October 23, 2015

- Added support for Python 3.
- Updated the setup script to handle problems with trying to build the Cython extensions. If the appropriate compiler hasn't been installed on Windows, then the extensions won't work (see [this discussion of using Cython extensions on Windows] for how to deal with this problem). The setup script now attempts to build the Cython extensions and automatically falls back to pure Python if the build fails. **NOTE that the Cython extensions currently are not set up to work in Python 3** (see [discussion in issue #10]), so they are not built when installing with Python 3. This will be changed in the future.
- [closed issue #2]: <https://github.com/GeoStat-Framework/PyKrige/issues/2>
- [this discussion of using Cython extensions on Windows]: <https://github.com/cython/cython/wiki/CythonExtensionsOnWindows>
- [discussion in issue #10]: <https://github.com/GeoStat-Framework/PyKrige/issues/10>

5.10 Version 1.2.0

August 1, 2015

- Updated the execution portion of each class to streamline processing and reduce redundancy in the code.
- Integrated kriging with a moving window for two-dimensional ordinary kriging. Thanks to Roman Yurchak for this addition. This can be very useful for working with very large datasets, as it limits the size of the kriging matrix system. However, note that this approach can also produce unexpected oddities if the spatial covariance of the data does not decay quickly or if the window is too small. (See Kitanidis 1997 for a discussion of potential problems in kriging with a moving window; also see [closed issue #2] for a brief note about important considerations when kriging with a moving window.)
- Integrated a Cython backend for two-dimensional ordinary kriging. Again, thanks to Roman Yurchak for this addition. Note that currently the Cython backend is only implemented for two-dimensional ordinary kriging; it is not implemented in any of the other kriging classes. (I'll gladly accept any pull requests to extend the Cython backend to the other classes.)
- Implemented two new generic drift capabilities that should allow for use of arbitrary user-designed drifts. These generic drifts are referred to as 'specified' and 'functional' in the code. They are available for both two-dimensional and three-dimensional universal kriging (see below). With the 'specified' drift capability, the user specifies the values of the drift term at every data point and every point at which the kriging system is to be evaluated. With the 'functional' drift capability, the user provides callable function(s) of the two or three spatial coordinates that define the drift term(s). The functions must only take the spatial coordinates as arguments. An arbitrary number of 'specified' or 'functional' drift terms may be used. See `UniversalKriging.__doc__` or `UniversalKriging3D.__doc__` for more information.
- Made a few changes to how the drift terms are implemented when the problem is anisotropic. The regional linear drift is applied in the adjusted coordinate frame. For the point logarithmic drift, the point coordinates are transformed into the adjusted coordinate frame and the drift values are calculated in the transformed frame. The external scalar drift values are extracted using the original (i.e., unadjusted) coordinates. Any functions that are used with the 'functional' drift capability are evaluated in the adjusted coordinate frame. Specified drift values are not adjusted as they are taken to be for the exact points provided.
- Added support for three-dimensional universal kriging. The previous three-dimensional kriging class has been renamed `OrdinaryKriging3D` within module `ok3d`, and the new class is called `UniversalKriging3D` within module `uk3d`. See `UniversalKriging3D.__doc__` for usage information. A regional linear drift ('regional_linear')

is the only code-internal drift that is currently supported, but the ‘specified’ and ‘functional’ generic drift capabilities are also implemented here (see above). The regional linear drift is applied in all three spatial dimensions.

5.11 Version 1.1.0

May 25, 2015

- Added support for two different approaches to solving the entire kriging problem. One approach solves for the specified grid or set of points in a single vectorized operation; this method is default. The other approach loops through the specified points and solves the kriging system at each point. In both of these techniques, the kriging matrix is set up and inverted only once. In the vectorized approach, the rest of the kriging system (i.e., the RHS matrix) is set up as a single large array, and the whole system is solved with a single call to `numpy.dot()`. This approach is faster, but it can consume a lot of RAM for large datasets and/or large grids. In the looping approach, the rest of the kriging system (the RHS matrix) is set up at each point, and the kriging system at that point is solved with a call to `numpy.dot()`. This approach is slower, but it does not take as much memory. The approach can be specified by using the `backend` kwarg in the `execute()` method: ‘vectorized’ (default) for the vectorized approach, ‘loop’ for the looping approach. Thanks to Roman Yurchak for these changes and optimizations.
- Added support for implementing custom variogram models. To do so, set `variogram_model` to ‘custom’. You must then also specify `variogram_parameters` as well as `variogram_function`, which must be a callable object that takes only two arguments, first a list of function parameters and then the distances at which to evaluate the variogram model. Note that currently the code will not automatically fit the custom variogram model to the data. You must provide the `variogram_parameters`, which will be passed to the callable `variogram_function` as the first argument.
- Modified anisotropy rotation so that coordinate system is rotated CCW by specified angle. The sense of rotation for 2D kriging is now the opposite of what it was before.
- Added support for 3D kriging. This is now available as class `Krige3D` in `pykrige.k3d`. The usage is essentially the same as with the two-dimensional kriging classes, except for a few extra arguments that must be passed during instantiation and when calling `Krige3D.execute()`. See `Krige3D.__doc__` for more information.

5.12 Version 1.0.3

February 15, 2015

- Fixed a problem with the tests that are performed to see if the kriging system is to be solved at a data point. (Tests are completed in order to determine whether to force the kriging solution to converge to the true data value.)
- Changed setup script.

5.13 Version 1.0

January 25, 2015

- Changed license to New BSD.
- Added support for point-specific and masked-grid kriging. Note that the arguments for the `OrdinaryKriging.execute()` and `UniversalKriging.execute()` methods have changed.
- Changed semivariogram binning procedure.
- Boosted execution speed by almost an order of magnitude.

- Fixed some problems with the external drift capabilities.
- Added more comprehensive testing script.
- Fixed slight problem with `read_asc_grid()` function in `kriging_tools`. Also made some code improvements to both the `write_asc_grid()` and `read_asc_grid()` functions in `kriging_tools`.

5.14 Version 0.2.0

November 23, 2014

- Consolidated backbone functions into a single module in order to reduce redundancy in the code. `OrdinaryKriging` and `UniversalKriging` classes now import and call the core module for the standard functions.
- Fixed a few glaring mistakes in the code.
- Added more documentation.

5.15 Version 0.1.2

October 27, 2014

- First complete release.

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INDEX

D

`display_variogram_model()`
(*pykrige.ok.OrdinaryKriging* method), 9
`display_variogram_model()`
(*pykrige.ok3d.OrdinaryKriging3D* method), 20
`display_variogram_model()`
(*pykrige.uk.UniversalKriging* method), 14
`display_variogram_model()`
(*pykrige.uk3d.UniversalKriging3D* method), 25

E

`eps` (*pykrige.ok.OrdinaryKriging* attribute), 11
`eps` (*pykrige.ok3d.OrdinaryKriging3D* attribute), 22
`eps` (*pykrige.uk.UniversalKriging* attribute), 17
`eps` (*pykrige.uk3d.UniversalKriging3D* attribute), 28
`execute()` (*pykrige.ok.OrdinaryKriging* method), 9
`execute()` (*pykrige.ok3d.OrdinaryKriging3D* method), 20
`execute()` (*pykrige.rk.Krige* method), 33
`execute()` (*pykrige.uk.UniversalKriging* method), 15
`execute()` (*pykrige.uk3d.UniversalKriging3D* method), 25

F

`fit()` (*pykrige.rk.Krige* method), 33
`fit()` (*pykrige.rk.RegressionKriging* method), 30

G

`get_epsilon_residuals()`
(*pykrige.ok.OrdinaryKriging* method), 10
`get_epsilon_residuals()`
(*pykrige.ok3d.OrdinaryKriging3D* method), 21
`get_epsilon_residuals()`
(*pykrige.uk.UniversalKriging* method), 15
`get_epsilon_residuals()`
(*pykrige.uk3d.UniversalKriging3D* method), 27
`get_params()` (*pykrige.rk.Krige* method), 33
`get_statistics()` (*pykrige.ok.OrdinaryKriging* method), 10
`get_statistics()` (*pykrige.ok3d.OrdinaryKriging3D* method), 21

`get_statistics()` (*pykrige.uk.UniversalKriging* method), 15
`get_statistics()` (*pykrige.uk3d.UniversalKriging3D* method), 27
`get_variogram_points()`
(*pykrige.ok.OrdinaryKriging* method), 10
`get_variogram_points()`
(*pykrige.uk.UniversalKriging* method), 16

K

Krige (class in *pykrige.rk*), 32
`krige_residual()` (*pykrige.rk.RegressionKriging* method), 30

O

OrdinaryKriging (class in *pykrige.ok*), 7
OrdinaryKriging3D (class in *pykrige.ok3d*), 18

P

`plot_epsilon_residuals()`
(*pykrige.ok.OrdinaryKriging* method), 11
`plot_epsilon_residuals()`
(*pykrige.ok3d.OrdinaryKriging3D* method), 21
`plot_epsilon_residuals()`
(*pykrige.uk.UniversalKriging* method), 16
`plot_epsilon_residuals()`
(*pykrige.uk3d.UniversalKriging3D* method), 27
`predict()` (*pykrige.rk.Krige* method), 33
`predict()` (*pykrige.rk.RegressionKriging* method), 30
`print_statistics()` (*pykrige.ok.OrdinaryKriging* method), 11
`print_statistics()` (*pykrige.ok3d.OrdinaryKriging3D* method), 21
`print_statistics()` (*pykrige.uk.UniversalKriging* method), 16
`print_statistics()` (*pykrige.uk3d.UniversalKriging3D* method), 27

R

`read_asc_grid()` (in module *pykrige.kriging_tools*), 35
RegressionKriging (class in *pykrige.rk*), 29

S

`score()` (*pykrige.rk.Krige method*), 33
`score()` (*pykrige.rk.RegressionKriging method*), 30
`set_params()` (*pykrige.rk.Krige method*), 34
`switch_plotting()` (*pykrige.ok.OrdinaryKriging method*), 11
`switch_plotting()` (*pykrige.ok3d.OrdinaryKriging3D method*), 21
`switch_plotting()` (*pykrige.uk.UniversalKriging method*), 16
`switch_plotting()` (*pykrige.uk3d.UniversalKriging3D method*), 27
`switch_verbose()` (*pykrige.ok.OrdinaryKriging method*), 11
`switch_verbose()` (*pykrige.ok3d.OrdinaryKriging3D method*), 21
`switch_verbose()` (*pykrige.uk.UniversalKriging method*), 16
`switch_verbose()` (*pykrige.uk3d.UniversalKriging3D method*), 27

U

`UNBIAS` (*pykrige.uk.UniversalKriging attribute*), 16
`UNBIAS` (*pykrige.uk3d.UniversalKriging3D attribute*), 28
`UniversalKriging` (*class in pykrige.uk*), 12
`UniversalKriging3D` (*class in pykrige.uk3d*), 23
`update_variogram_model()`
 (*pykrige.ok.OrdinaryKriging method*), 11
`update_variogram_model()`
 (*pykrige.ok3d.OrdinaryKriging3D method*), 22
`update_variogram_model()`
 (*pykrige.uk.UniversalKriging method*), 16
`update_variogram_model()`
 (*pykrige.uk3d.UniversalKriging3D method*), 27

V

`variogram_dict` (*pykrige.ok.OrdinaryKriging attribute*), 11
`variogram_dict` (*pykrige.ok3d.OrdinaryKriging3D attribute*), 22
`variogram_dict` (*pykrige.uk.UniversalKriging attribute*), 17
`variogram_dict` (*pykrige.uk3d.UniversalKriging3D attribute*), 28

W

`write_asc_grid()` (*in module pykrige.kriging_tools*), 35